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Interactive Comment

Interactive comment on "Turbulence dissipation rate derivation for meandering occurrences in a stable planetary boundary layer" *by* G. A. Degrazia et al.

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INTERACTIVE DISCUSSION.

Replay to Anonymous referee 2's comments:

First of all, we would like to sincerely thank the reviewer for the deep and serious analysis he gave to the manuscript.

Specific comments

We agree with the suggestions presented by the referee. Therefore, in the introduction of the paper we obtain a simplified description of a two-dimensional wind field, characterized by a rotation of period $T_M = \frac{2\pi}{q}$, where T_M is the meandering period of the order of 10^3 *s*. In fact, when we assume the simplified condition that the horizontal and vertical gradients of the turbulent moment fluxes can be neglected the turbulent effect disappears and the solutions as given by

$$\bar{u}(t) = \alpha_1 cos(qt)$$

and

$$\bar{v}(t) = \alpha_1 sin(qt),$$

represent a two-dimensional prototype wind field that describes the horizontal wind meandering behavior.

Therefore, the right correlation function associated to this two-dimensional wind field is given by

$$\rho_{L_{u,v}} = cos(qt) + isin(qt),$$

and only if an average over a long time period is taken, the correlation function can be the same for both the horizontal components (*u* and *v*). In this case, the autocorrelation function is proportional to $cos(qt)$, where τ is the lag time. These considerations that were made by the referee will be introduced in the new version of the paper.

Concerning to the use of the Eq.(2) to calculate the lateral spatial variance σ_y^2 , employing Taylor Statistical Diffusion Theory, it is important to note that in his classical article describing turbulent diffusion Frenkiel exhibits the behavior of Eq. (12) for the distinct values of the loop parameter *m*. In Frenkiel's curves, as times go on, the length scale of the cloud contaminants approaches the wavelength of the horizontal oscillations and,

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in this situation the movements of equidistant points of the center of the cloud become out of phase and, as a consequence, the spatial variance from Frenkiel model (Eq.(12)) increases and decreases, and the variance fluctuates around a constant mean value with a decreasing amplitude. On the other hand, the Frenkiel's dispersion factor curves increase continuously for *t* approaches infinite. This can be explained as follows. For large times Frenkiel's formula considers the contribution of the fully developed turbulence in widening the plume. Furthermore, the result that a plume becomes smaller at a certain time, stays, in contrast with classical diffusion theory where the mass flux of a tracer is always direct along the concentration gradient, which prohibits a sudden increase in concentration in absence of a source. This effect derives from the fact that two phenomena are acting simultaneously; meandering which a large scale oscillating process and turbulence. Meandering causes the wave-like body motion of a cloud of tracers, and the effect of turbulence is to spread these tracers around the cloud. Therefore, from the application point of view (heuristic arguments), the Frenkiel formulation presents an empirical flexibility that allows represent the physical properties of a fully developed turbulence as well as hybrid flow case, in which turbulence and meandering occurrences coexist.

Concerning to the formula $T = \frac{\tau}{1+(q\tau)^2}$, which has been derived substituting $\rho_L = exp(-s/\tau)cos(qs)$ in the relation $T = \int_0^\infty \rho_L(s)ds$, the following considerations are relevant: firstly, it is important to note that the original formula as proposed by Frenkiel and that is identical to the Eq. (2) satisfies the condition $T_{L_v} = \int_0^\infty \rho_{L_v}(s)ds$, so that the Frenkiel formula is valid as long as T_{L_v} is well defined. Differently, the relation $\rho_L = exp(-s/\tau)cos(qs)$ does not satisfies the condition (definition) that $T_{L_v} = \int_0^\infty \rho_{L_v}(s)ds$. As a consequence it is ill defined to represent a turbulent autocorrelation function (Manomaiphiboon and Russel (2003), Hinze (1975)).

From the above considerations, it is suggested that Frenkiel's formula can be used for evaluating dispersion for some reasons:

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1. Frenkiel's form recovers the classical results obtained from Taylor Statistical Diffusion Theory, when the meandering effects are not considered (i.e., by setting $m = 0$);
2. Frenkiel's form satisfies the definition $T_{L_v} = \int_0^\infty \rho_{L_v}(s)ds$;
3. The lateral dispersion parameter, $\sigma_y(t)$, calculated from Frenkiel relation continues growing for large travel times. This means that the contribution of small-scale turbulence acts to spread the tracers around the cloud.

Finally, the development to obtain the Eq. (15) that represents the most important result in this study is given by following analysis:

The development is made in terms of the turbulent energy spectrum and filter functions and follows arguments given by Tennekes (1982). To proceed, we start from the lateral dispersion parameter, as given by Pasquill and Smith (1983):

$$\sigma_y^2 = \sigma_v^2 t^2 \int_0^\infty F_{L_v}(n) \frac{sin^2(n\pi t)}{(n\pi t)^2} dn \tag{1}$$

where F_{L_v} is the Lagrangian turbulent energy spectrum normalized by the lateral velocity variance σ_v^2, n is the frequency in *Hz* and $\frac{sin^2(n\pi t)}{(n\pi t)^2}$ is a low-pass filter function. For a very short time interval after the instant of release, when $t << T_{L_v}$, $sin(n\pi t) \approx n\pi t$ and (1) can be written as:

$$lim_{t\to 0}\sigma_y^2 = \sigma_v^2 t^2 \int_0^\infty F_{L_v}(n) lim_{t\to 0} \frac{sin^2(n\pi t)}{(n\pi t)^2} dn = \sigma_v^2 t^2 \tag{2}$$

The above expression shows that all turbulent spectrum frequencies contribute to the growth of the spatial variance σ_y^2. Therefore, at the short-time limit considered in Eq.(2), the low-pass filter operator is ineffective to cut the high-frequency harmonics.

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On the other hand, for large diffusion travel times ($t >> T_{L_v}$), the following asymptotic eddy diffusivity can be obtained from (1):

$$\frac{d}{dt}\left(\frac{1}{2}\sigma_y^2\right) = lim_{t\rightarrow\infty}\frac{\sigma_v^2}{2\pi}\int_0^\infty F_{L_v}(n)\frac{sin(2\pi nt)}{n}dn$$

$$= \frac{\sigma_v^2}{4}\int_{-\infty}^{\infty} F_{L_v}\left(\frac{g}{2\pi}\right)\delta(g)dg = \sigma_v^2 T_{L_v}$$

where $g = 2\pi n$, $\delta(g)$ is the Dirac delta function and $T_{L_v} = F_{L_v}(0)/4$.

Integration of Eq.(3) yields the classical result obtained from Taylor Statistical Diffusion Theory for $t \rightarrow \infty$,i.e. $\sigma_y^2 = 2\sigma_v^2 T_{L_v} t$.

The above analysis shows that, as time proceeds, the filter function begins to remove the energy associated to high frequencies in the turbulent spectrum. This behavior becomes evident by the presence of Dirac's delta function, an extremely restrictive operator, which selects only the very low frequency components of the turbulent spectrum. At this point, we perform a MacLaurin series expansion of the expression given by Eq.(12) (see article). In this case, for $t < T_{L_v}$:

$$\sigma_y^2(t) = 2\sigma_v^2\left[\frac{t^2}{2} - \frac{t^3}{6\left(1+m^2\right)T_{L_v}} + ...\right] = \sigma_v^2 t^2 - \frac{\sigma_v^2 t^3}{3\left(1+m^2\right)T_{L_v}} + ... \qquad (3)$$

Comparison of equations (4) and (2), shows that the negative term in the right hand side of (4) contributes to the decrease of the lateral dispersion parameter. Physically, it results from the suppression of a number of degrees of freedom of the turbulent field, associated to the high-frequency harmonics. Therefore, it is plausible to associate the term $\frac{\sigma_v^2 t^3}{3(1+m^2)T_{L_v}}$ to the inertial subrange high-frequency eddies. This relationship can be established through the use of the Lagrangian structure function D_{L_v}(the ensemble average of the square of the change in Lagrangian velocity), the Lagrangian autocorrelation function ρ_{L_v} and the inertial subrange Lagrangian turbulent spectrum (Monin and Yaglom, 1975):

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$$D_{L_v} = 2\sigma_v^2 \left[1 - \rho_{L_v}\right] = 2\int_0^\infty \left[1 - cos(2\pi n\tau)\right] S_{L_v}(n)dn \tag{4}$$

where $S_{L_v}(n)$ is the inertial subrange Lagrangian turbulent spectrum, given by (Corssin, 1963; Hanna, 1981):

$$S_{L_v}(n) = \frac{B_0}{2\pi}\epsilon n^{-2} \tag{5}$$

where B_0 is a constant.

Substitution of (6) in (5) leads to:

$$D_{L_v} = 2\sigma_v^2 \left[1 - \rho_{L_v}(\tau)\right] = C_0\epsilon\tau \tag{6}$$

where $C_0 = B_0\pi$.

Eq.(7) establishes the relationship for the Lagrangian autocorrelation function ρ_{L_v} in terms of inertial subrange parameters:

$$\rho_{L_v}(\tau) = 1 - \frac{C_0\epsilon\tau}{2\sigma_v^2} \tag{7}$$

Substitution of (8) in Taylor Statistical Diffusion Theory (Eq. (11) - see article) results $\sigma_y^2 = \sigma_v^2 t^2 - C_0\epsilon t^3/6$, which after comparison to the originated expression (4), leads to Eq. (15) (see article):

$$\epsilon = \frac{2}{(1+m^2)C_0}\frac{\sigma_v^2}{T_{L_v}}$$

The above analysis shows that the presence of the wind meandering, characterized by greater values of the looping parameter *m*, reduces the dissipation in the inertial

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subrange. Indeed, this fact agrees with the observed spectral curves obtained by Anfossi *et al* (2005). These spectra show that in the presence of meandering phenomena there is less kinetic energy density in the inertial subrange.

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