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Comment

Interactive comment on “Long-memory processes in global ozone and temperature variations” by C. Varotsos and D. Kirk-Davidoff

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Reply to the Comment by H. Rust

In his comment [1] on Varotsos and Kirk-Davidoff [2], hereinafter VK, Rust points out that simple inspection of a log-log plot of a detrended fluctuation analysis (DFA) cannot always distinguish between a first-order autoregressive (AR1) process and a two-region power-law process. Irrespective of the precise origin of the two scaling regions in DFA (which as discussed below correspond to power law behavior of the spectrum (either exact [5] or apparent (see Fig.1a http://www.uoa.gr/~nsarlis/Figure1_Reply_to_the_Comment_by_H._Rust.html)), we used DFA analysis because it makes very clear the different behavior of the tropical and extratropical time series as the time scale increases. It is not essential to our claims or to the interest of the climate community that the behavior shown in Fig. 2a of VK be appropriately described

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as self-similar, with two power laws in two different frequency domains. Our point is that at time scales longer than about two years, the behavior in the tropics and in the extratropics is different and that DFA analysis makes this very clear.

However, we do in fact believe that a two-region power-law description better suits the data presented in Figures 2a, 2c and 3a. Here, we clarify that linear models like the AR1 process that was used as an example in Rust [1], or even a superposition of such processes, though it might resemble the DFA behavior of Fig. 2a of VK, clearly violates the statistics of the data analyzed.

Let us first summarize the properties of AR1: The first order autoregressive process with parameter a is [3]:

$$X_t = aX_{t-1} + e_t \quad (1)$$

where e_t is a Gaussian white noise with zero mean and variance σ^2 . The process is stationary only when $|a| < 1$. For all times t , X_t is a Gaussian variable with the variance [3] $\sigma^2/(1 - a^2)$ and exhibits the power spectrum (see Fig.1a http://www.uoa.gr/~nsarlis/Figure1_Reply_to_the_Comment_by_H._Rust.html) given by [3]

$$P(f) = 2\sigma^2/[1 + a^2 - 2a \cos(2\pi f)] \quad (2)$$

The following arguments are now in order:

First, a prerequisite for AR1 modelling to be true is that the data, to be modelled, are Gaussian. In the case of the TOZ data of Fig. 2a of VK, however, the Kolmogorov-Smirnov-Liliefors (KSL) test on normal distribution yields that the hypothesis (H_0) that the data are normally distributed versus, is rejected at a significance level of 1%. Furthermore, even if one considers a superposition of a finite number of AR1 processes with varying a 's the corresponding KSL test yields that the hypothesis H_0 is accepted with a significance level of 10%. In other words, it was the lack of Gaussianity that prohibited the application of such linear models to our data. Second, even when disregarding the significant point mentioned above, and model the TOZ data of Fig.2a of VK

with an AR1 process, one obtains by applying the TISEAN [4] package $a = 0.81$. Such an a -value, however, is found to be highly unlikely if we accept the AR1 model suggested in the example used by Rust[1], i.e., $a = 0.75$. This results from a Monte-Carlo study in which we run 104 realizations of the AR1 ($a = 0.75$) processes and estimated that the probability to obtain [4] $a \geq 0.81$ is less than 1%.

The variation of the DFA exponent α with scale has been studied by Talkner and Weber [5]. In their Fig.3 they show that a power law model is fully consistent with the well established relation $\alpha = (\beta + 1)/2$, even when β varies in different frequency bands; β being the exponent of the power spectrum. A similar phenomenon has been observed in the case of dichotomous electric signals [6-8]. In this context, the case of Markovian Random Telegraph Signal has been studied in detail [8] (compare their Fig.4 and Fig.1 http://www.uoa.gr/~nsarlis/Figure1_Reply_to_the_Comment_by_H._Rust.html). In Fig.1b http://www.uoa.gr/~nsarlis/Figure1_Reply_to_the_Comment_by_H._Rust.html, we present a few examples that summarize the behavior of the DFA for AR1 processes with $a = 0.75$, i.e., the one suggested by Rust [1]. The DFA slope varies almost continuously within the limits determined by the power spectrum of Fig.1a http://www.uoa.gr/~nsarlis/Figure1_Reply_to_the_Comment_by_H._Rust.html.

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FIGURE CAPTION http://www.uoa.gr/~nsarlis/Figure1_Reply_to_the_Comment_by_H._Rust.html

FIG.1: (a) The normalized power spectrum $P(f)/P(0)$ of AR1 processes for various values of the parameter a . The straight lines correspond to $1/f^\beta$ spectra, which are drawn at the points where $P(f)$ apparently exhibits power-law behavior. The corresponding (short scale DFA exponents) α_1 are also included for the readers convenience. (b) Examples of the DFA for $a=0.75$ and $N=480$ as the one proposed in Rust [1]. The DFA slope varies continuously between the values consistent ($\alpha=(\beta+1)/2$, see Ref.[5]) with the apparent power-law behavior α_1 in (a) and $\alpha_2=0.5$ corresponding to the white spectrum of (a) at low frequencies (large scales).

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