

## ***Interactive comment on “Long-memory processes in global ozone and temperature variations” by C. Varotsos and D. Kirk-Davidoff***

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Received and published: 12 June 2006

Long memory or long-range dependent processes are frequently claimed to underly the dynamics of observed time series. Many of the works reaching those claims are based on detrended fluctuation analysis (DFA) or related scaling analysis techniques. The general procedure of these heuristic approaches is to describe the behaviour of a certain characteristic with a power-law. Although this is surely a useful ansatz in many respects, its validity should be checked for every application. Especially, when two or more power-law relations in a different range of scales are used to describe the result. Despite the difficulties to interpret, a power-law is frequently preferred over more complex concepts allowing a physical interpretation because of the small number of parameters. When using multiple power-laws for different “scaling regions”, this

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argument is corrupted.

In the manuscript "Long-memory processes in global ozone and temperature variations" the authors describe the result of a DFA analysis for about 480 data points with two power-laws, Figs. 2a and 2c (Varotsos and Kirk-Davidoff). They conclude long-range dependence for the underlying process with a "stronger memory" for small scales on the basis of two straight line fits in a log-log plot.

Inferring long-range dependence is a challenging task and one can be misled with seemingly easy to apply methods like DFA. For a discussion on some pitfalls refer to [Maraun et al.(2004)Maraun, Rust, and Timmer] and [Metzler(2003)].

As already noted in [Metzler(2003)], self-similarity is a fascinating concept and less exiting alternatives to describe some observed behaviour are sometimes not taken into account. This can be illustrated with an example. Consider an simple Gaussian stochastic process of first order (AR[1]) [Brockwell and Davis(1991)]

$$X_t = aX_{t-1} + \eta_t \quad (1)$$

with  $\eta_t$  being Gaussian white noise with zero mean and unit variance. The autoregressive parameter is set to  $a = 0.75$ . A DFA analysis for a realisation of this process with length  $N = 480$  yields a very similar plot to Fig. 2a in Varotsos and Kirk-Davidoff, cf. Fig.1<sup>1</sup>.

In this case one power-law is not sufficient to describe the overall behaviour. As Varotsos and Kirk-Davidoff do in their manuscript, one could choose two power-laws to achieve a better representation. Following their line of argumentation, one would falsely conclude a superposition of two self-similar processes for this realisation of an AR[1] model. Hence, obtaining a satisfactory fit in the log-log plot does not allow to infer self-similarity. It only means that DFA is not capable to bring out the difference between an AR[1] process and the sum of two self-similar process within this short range of scales.

<sup>1</sup>Figure is at <http://www.pik-potsdam.de/~hrust/AR-example1.pdf>

It is furthermore not generally possible to relate DFA results for a finite range of scales to the ACF. The relation of the scaling exponent to the decay of the ACF only holds asymptotically for large lags, cf. Maraun 2004a. Since this asymptotic relation is the only way of translating DFA results to the ACF, one is naturally led to fit power-laws. A wide range of more flexible models including also long-range dependence is offered by linear time series analysis. They can be used to escape the procrustean bed of self-similarity.

This example should not be misunderstood as an attempt to advocate AR[1] processes as models for the data under consideration. It only emphasises the lack of specificity of an DFA analysis leading to long-memory seeming an ubiquitous phenomenon. A broad review on alternative methods related to long-range dependence can be found in e.g. [Beran(1994), Doukhan et al.(2003) Doukhan, Oppenheim, and Taqqu].

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Interactive comment on Atmos. Chem. Phys. Discuss., 6, 4325, 2006.

ACPD

6, S1182–S1185, 2006

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