

**Divergence of
sun-rays by
atmospheric
refraction**

R. Uhl and T. Reddmann

Divergence of sun-rays by atmospheric refraction at large solar zenith angles

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Abstract

For the determination of photolysis rates at large zenith angles it has been demonstrated that refraction by the earth's atmosphere must be taken into account. In fact, due to the modified optical path the optical transmittance is thereby increased in most instances. Here we show that in addition the divergence of sun-rays, which is also caused by refraction but which reduces the direct solar irradiance, should not be neglected. Our calculations are based on a spherically symmetric atmosphere and include extinction by Rayleigh scattering, ozone, and background aerosol. For rays with 10 km tangent altitude the divergence yields a reduction of about 10% to 40% at solar zenith angles of 91° to 96° . Moreover, we find that the divergence effect can completely cancel the relative enhancement caused by the increase of transmittance.

1. Introduction

It is well known that refraction by the earth's atmosphere may be important for photochemical calculations near the terminator. For example, [Anderson and Lloyd \(1990\)](#) and [DeMajistre et al. \(1995\)](#) present detailed calculations to the effect of refraction on the optical path. Besides the lengthening of the sunlit day, at large solar zenith angles ($>90^\circ$) the inclusion of refraction reduces the optical depth of the direct beam in most cases, and therefore the radiation is enhanced. [Trentmann et al. \(2003\)](#) show substantial enhancement of photodissociation frequencies for some substances at large solar zenith angles. [Balluch and Lary \(1997\)](#) quantify the effect of refraction on stratospheric chemistry for a seven day period and find changes of concentration for many substances in the order of several percent.

As it is shown in Fig. 1, there is another effect of refraction on the solar irradiance: as sun-rays reaching deeper layers in the atmosphere are more refracted, refraction causes a divergence of the direct solar beam, which is related to the apparent flatness of the solar disc. This additional divergence effect reduces the direct irradiance in the

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atmosphere and therefore competes against the enhancement caused by the changed optical path. In order to quantify this effect, which is not included in the studies cited above, we have performed ray tracing calculations for large solar zenith angles, which account for both the change of optical path and divergence effect.

2. Ray tracing

2.1. Refracting atmosphere

A spherically symmetric atmosphere is taken as a basis. The mass density of air, ρ , depending on (geometric) altitude z is taken from the MSIS model of [Hedin \(1991\)](#) (official release NRLMSISE-00) for four typical atmospheric conditions, in order to test the influence of the temperature profile. The corresponding input data (besides the 10.7 cm solar flux indices equal to 150 and the planetary magnetic Ap indices equal to 4) and temperature profiles are shown in [Fig. 2](#).

The refractive index n_s for standard sea-level air with mass density $\rho_s=1.225 \text{ kg/m}^3$ is taken from [Edlén \(1966\)](#) (Eq. 1), whereas for fixed wavelength the refractivity $n-1$ is assumed to be just proportional to ρ , i.e.

$$n - 1 = \frac{(n_s - 1) \rho}{\rho_s};$$

cf. again [Edlén \(1966\)](#). Finally, we use earth's radius $r_0=6370 \text{ km}$.

2.2. Refraction equations

We describe a single refracted sun-ray by the distance $r=r_0+z$ to the earth's centre and the deviation ε from its original direction, both depending on the solar zenith angle χ ; see [Fig. 1](#).

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To characterise the curvature of the ray at any point with $\chi = \chi_0$, we suppose temporarily that the atmosphere is horizontal plane parallel there. Logarithmic differentiation of Snell's law $n \sin(\chi_0 - \varepsilon) = \text{constant}$ gives

$$\frac{1}{n} \frac{dn}{d\chi} - \cot(\chi - \varepsilon) \frac{d\varepsilon}{d\chi} = 0 \quad (1)$$

at $\chi = \chi_0$. Of course, this equation holds also for the spherically symmetric atmosphere and along the whole ray. Dividing by

$$- \frac{1}{r} \frac{dr}{d\chi} = \cot(\chi - \varepsilon) \quad (2)$$

as $\chi - \varepsilon$ is the apparent solar zenith angle, we obtain the basic equation

$$\frac{d\varepsilon}{d\chi} = - \frac{r}{n} \frac{dn}{dr} \quad (3)$$

Moreover, we use the fact that the quantity

$$a = n r \sin(\chi - \varepsilon) \quad (4)$$

is constant on each refracted ray. In fact, logarithmic differentiation yields

$$\frac{1}{a} \frac{da}{d\chi} = \frac{1}{n} \frac{dn}{d\chi} + \frac{1}{r} \frac{dr}{d\chi} + \cot(\chi - \varepsilon) \left(1 - \frac{d\varepsilon}{d\chi} \right) = 0$$

by Eqs. (1) and (2). Cf. also [Born and Wolf \(1980\)](#) (§ 3.2 (7)), for instance. Obviously, a is the distance of earth's centre to the corresponding unrefracted ray as is indicated in Fig. 1.

2.3. Integration

Note that if a is specified, r can be calculated to given χ and ε via Eq. (4) by, for instance, iterating the fixed point equation

$$r = \frac{a}{n(r) \sin(\chi - \varepsilon)}, \quad (5)$$

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starting with the approximation $r=a/\sin(\chi-\varepsilon)$. Thus we may consider Eq. (3), where the right hand side depends on r , as an ordinary differential equation for $\varepsilon(\chi)$. The approximation $\varepsilon=0$ at smaller χ (before the ray penetrates the deeper atmosphere) may be taken as initial value.

- 5 A numerical integration for many values of a yields the corresponding refracted rays, of which some are indicated in Fig. 3 for wavelength 550 nm. This figure shows the refraction induced decrease d of altitude, which is given by

$$d = \frac{a}{\sin \chi} - r$$

as in Fig. 1.

10 3. Reduction of direct irradiance

3.1. Divergence

The divergence of neighbouring refracted rays as in Fig. 1 is related to a decrease of the direct irradiance. The corresponding reduction factor $D < 1$ is approximately the area ratio of two surfaces of revolution which are orthogonal to the rays: an annulus with radius a and small width Δa , and the corresponding cone-like surface with radius $r \sin \chi$ and width $\sin(\chi-\varepsilon) \Delta r$; or precisely,

$$15 D = \frac{a}{r \sin \chi \sin(\chi - \varepsilon) \frac{\partial r}{\partial a}} = \frac{n}{\frac{\partial r}{\partial a} \sin \chi}$$

by Eq. (4), regarding $r=r(\chi, a)$ here.

- 20 This reduction factor is calculated approximately with $\partial r / \partial a$ replaced by $\Delta r / \Delta a$ (with two neighbouring refracted rays as in Fig. 1) and plotted in Fig. 4 for wavelength 550 nm. As the refractivity $n-1$ for other wavelengths between 400 nm and 1000 nm differs by less than 2%, this and the preceding figure would be nearly the same for every other wavelength in this range.

3.2. Extinction

We use the extinction coefficient $\alpha = \alpha(\text{Ray}) + \alpha(\text{O}_3) + \alpha(\text{aer})$ which is composed as follows; see also Fig. 5.

The Rayleigh scattering coefficient $\alpha_s(\text{Ray}) = 0.01149 \text{ km}^{-1}$ for wavelength 550 nm and standard sea-level air is taken from Bucholtz (1995) (Table 2), whereas for fixed wavelength the coefficient $\alpha(\text{Ray})$ is assumed to be just proportional to mass density ρ of air, i.e. $\alpha(\text{Ray}) = \alpha_s(\text{Ray})\rho/\rho_s$. Absorption by ozone is taken into account as $\alpha(\text{O}_3) = \sigma(\text{O}_3)N(\text{O}_3)$ with the particle density profile $N(\text{O}_3)$ of the U.S. Standard Atmosphere (1976) (Table 18, exponentially inter- and extrapolated) and with the constant cross section $\sigma(\text{O}_3) = 3.5 \cdot 10^{-25} \text{ m}^2$ for wavelength 550 nm; cf. Burrows et al. (1999). Finally, we use the simple aerosol extinction profile

$$\alpha(\text{aer}) = 0.0015 \text{ km}^{-1} \exp\left(-\left(\frac{z}{15 \text{ km}}\right)^2\right),$$

which corresponds approximately to background conditions in the lower stratosphere. Clouds are not taken into account.

The extinction reduces the direct radiation at any point by the factor

$$T = \exp\left(-\int \alpha ds\right),$$

the transmittance, with a line integral along a refracted ray where

$$\frac{ds}{d\chi} = \frac{r}{\sin(\chi - \varepsilon)}.$$

To evaluate the impact of refraction on the direct irradiance, also the transmittance in case of no refraction, T_{straight} , is calculated with a similar line integral to the same point but along an unrefracted ray.

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3.3. Total reduction

The irradiance of a direct beam is reduced simultaneously by divergence and extinction. Its total reduction factor DT is plotted in Fig. 6.

5 The influence of refraction on the direct irradiance is specified by the corresponding changing factor DT/T_{straight} as shown in Fig. 7.

For comparison also the refraction induced changing factor T/T_{straight} of transmittance is plotted in Fig. 8.

10 It describes the impact of refraction if the divergence effect is neglected. Note that at those points where $DT/T_{\text{straight}} < 1$, the direct irradiance is more reduced by divergence than increased by change of transmittance.

4. Results and discussion

15 As shown in Fig. 4, the solar irradiance of the direct beam is substantially reduced by the divergence due to the atmosphere. Along rays with a tangent altitude of about 10 km and which are not affected by clouds or aerosol of tropospheric origin, the reduction exceeds 5% at about 90° solar zenith angle and can reach about 40% at 96° solar zenith angle. Comparing the results for the different meteorological conditions, the polar and equatorial temperature profiles for 21 March differ substantially. Obviously, the steep and highly variable temperature profile in the tropics disturbs the reduction field compared with the uniform decreasing irradiation in the case of polar spring. In general, along rays with 10 km tangent altitude the reduction exceeds 10% within a band of about 2° from the terminator.

20 The total reduction caused by divergence and extinction, as shown in Fig. 6 for 550 nm, is obviously dominated by the extinction, as expected. For this wavelength Rayleigh scattering and ozone absorption essentially contribute to the extinction as follows from Fig. 5. For large solar zenith angles and deep rays the reduction of the direct beam is so high that scattered radiation may dominate the radiation field. Inspec-

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tion of Fig. 8 of [Dahlback and Stamnes \(1991\)](#) shows that about for a tangent altitude of 10 km the contribution of direct beam and scattered radiation are equal for wavelengths at 450 nm under clear sky conditions. As at 550 nm the extinction coefficient of Rayleigh scattering is only about half as large, we conclude that the direct beam dominates for rays with tangent altitude in the stratosphere. This is also in accordance with the photochemical calculations of [DeMajistre et al. \(1995\)](#).

We now turn to the question how the divergence effect compares with the change of transmittance caused by refraction. Again only rays with a tangent altitude higher than about 10 km need to be considered for the same reason as given above, so that Figs. 7 and 8 are sufficient for the discussion despite they do not show data at those points which are only met by refracted rays. In addition, these are rays which do not penetrate the troposphere and are not affected by clouds. Note that this is the part of the atmosphere where refraction causes a prolonged sunlit day. With that restriction, the band with an increase of the irradiance of more than 5% in Fig. 8 is more than halved by the divergence effect. In addition, radiation in the visual part of the solar spectrum is more important for photochemistry in the lower stratosphere where UV is mostly absorbed. Here the divergence effect narrows the band of enhancement even more. At about 20 km altitude and 93° solar zenith angle, there is even a total reduction of more than 5%.

Further calculations within the visual part of the spectrum show that at wavelengths greater than 550 nm the divergence effect counteracts the refraction induced enhancement of transmittance even more effective than just described. In particular, the region where divergence overcompensates the enhancement of transmittance (i.e. where $DT/T_{\text{straight}} < 1$) sinks in comparison with that one in Fig. 7, e.g. by about 3 and 6 km for 600 and 750 nm, respectively. Conversely, for 500, 450, and 400 nm this region is raised by about 3, 5, and 8 km, respectively, and this holds approximately for the three surfaces $DT/T_{\text{straight}} = 1.5, 3, 10$, too. Thus, at smaller wavelengths the divergence effect has a smaller impact.

We point out that our calculations do not take into account the angular extension of

the solar disc. This could be implemented by some weighted averaging of the reduction factor DT with respect to a varying solar zenith angle, according to the apparent solar diameter of about 0.5° .

5. Conclusions

5 Our study of the refraction induced divergence effect of the direct solar irradiation in the visual part of the spectrum show that this effect should be included if refraction is considered at all in photochemical studies. For rays in the stratosphere and background aerosol conditions the divergence effect narrows the refraction induced enhancement to an inclined band, which shrinks as wavelength is increased from about 10 to 15 km thickness at 400 nm and which vanishes at about 800 nm. Especially in the lower strato-
10 sphere, it may be justified to neglect refraction for photochemical calculation for larger wavelengths. Whereas high aerosol loading after volcanic eruptions would lead to a higher change of transmittance by refraction, the increase of scattering and the vertical filamentation probably makes special calculations necessary.

15 *Acknowledgements.* This work is a contribution to the project TEDDY (contract no. 6-056-0179) of the AFO 2000 framework and partly funded by the Bundesministerium für Bildung und Forschung.

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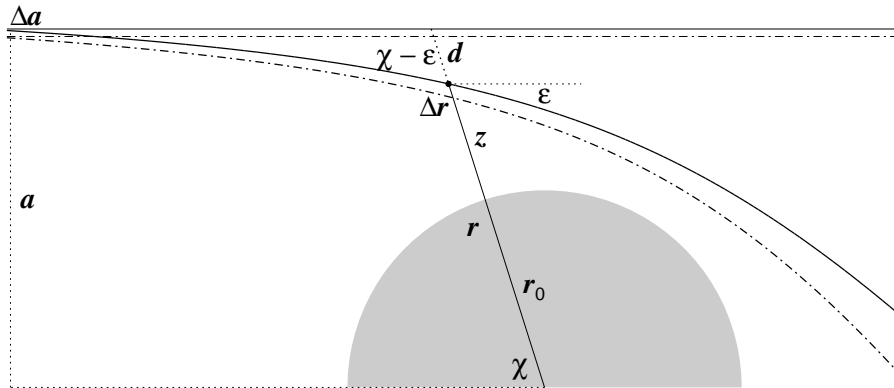


Fig. 1. Two neighbouring rays including/neglecting refraction.

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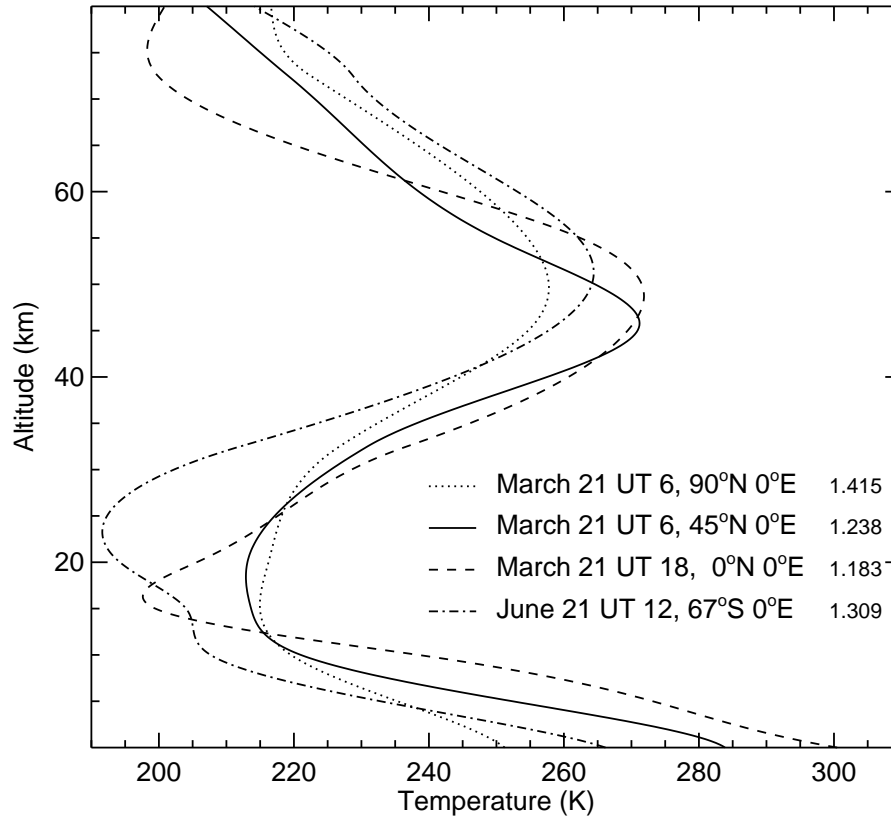


Fig. 2. The temperature profiles of the four used mass density profiles of air. At the right margin the mass density at altitude 0 is quoted in kg/m^3 .

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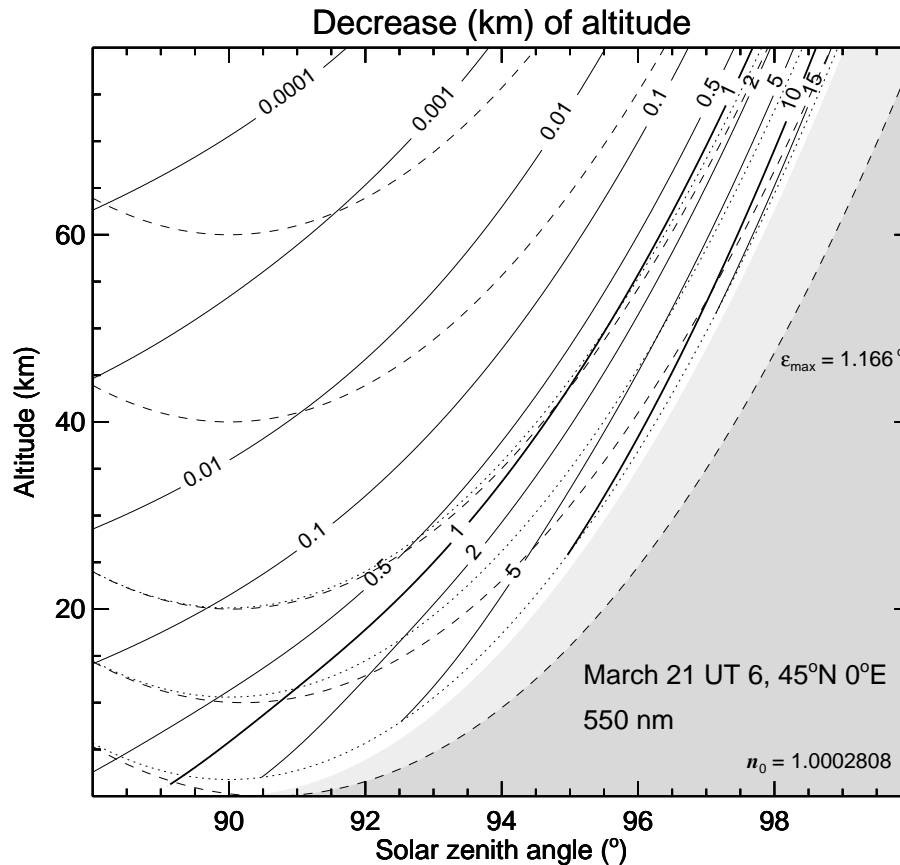


Fig. 3. The solid lines give the refraction induced decrease d (km) of altitude in relation to the respective point on the unrefracted ray. The dashed lines indicate the refracted rays with tangent altitudes 0, 10, 20, 40, 60 km. Three of the corresponding unrefracted rays are dotted. The earth's shadow is shown including/neglecting refraction. In addition, the refractive index n_0 at altitude 0 and the final direction deviation ε_{\max} of the ray with tangent altitude 0 are quoted.

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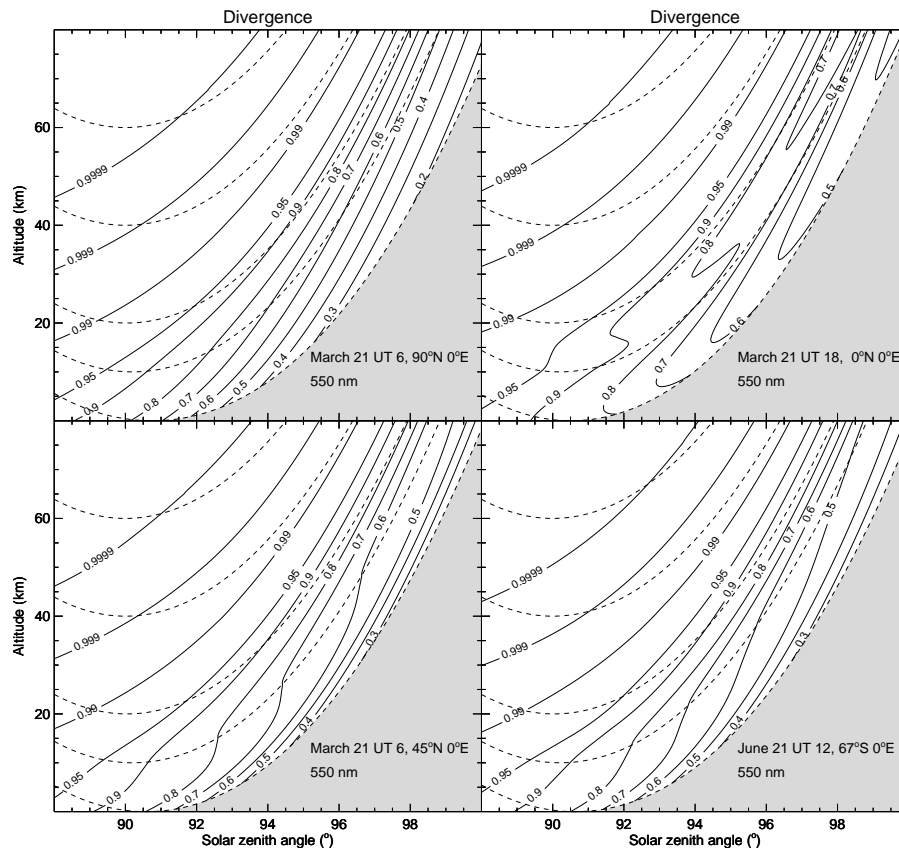


Fig. 4. The reduction factor D of direct irradiance due to divergence (in relation to the respective point on the refracted ray) for four atmospheric conditions.

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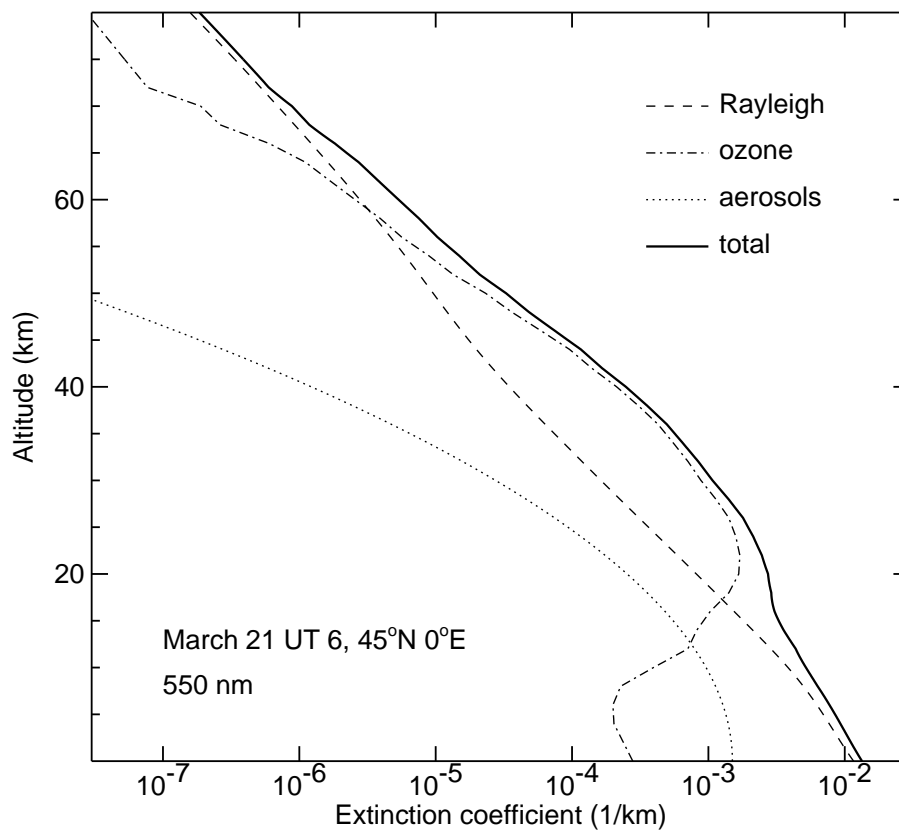


Fig. 5. The extinction coefficients $\alpha(\text{Ray})$, $\alpha(\text{O}_3)$, $\alpha(\text{aer})$, α .

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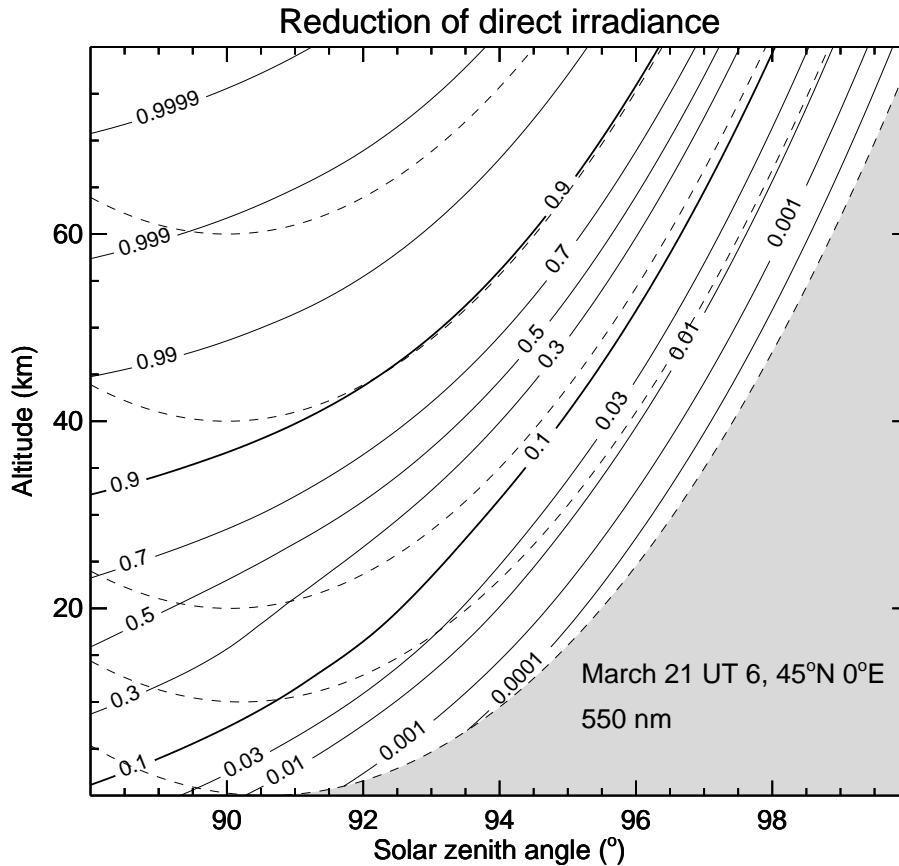


Fig. 6. The total reduction factor DT of direct irradiance due to divergence and extinction.

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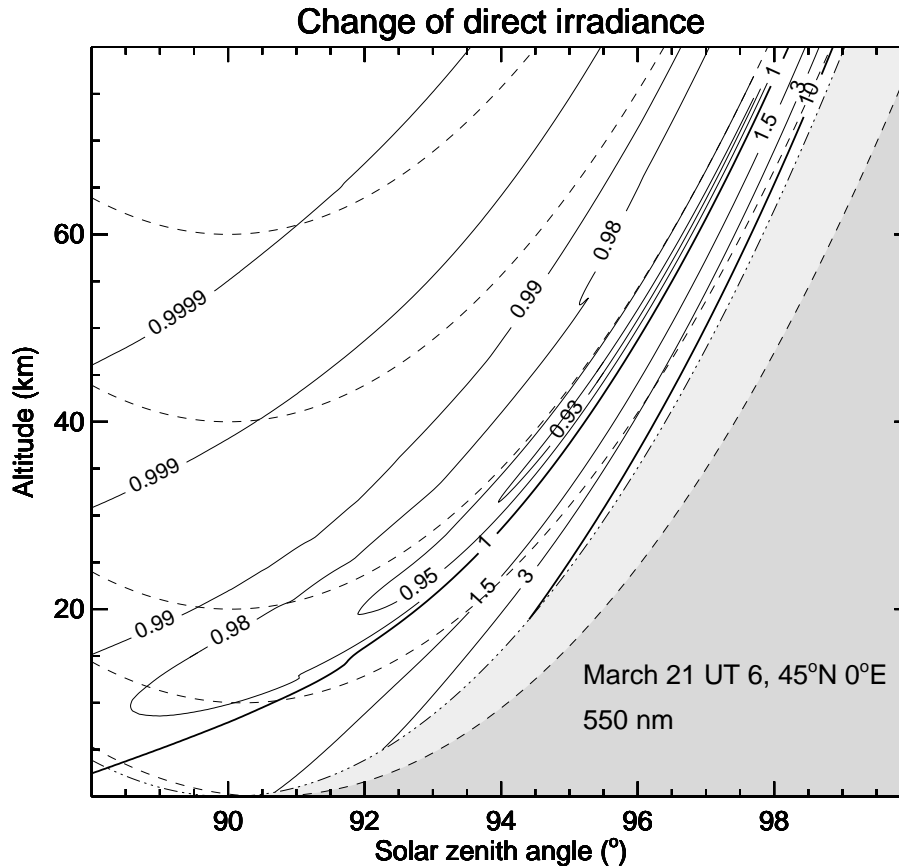


Fig. 7. The refraction induced changing factor DT/T_{straight} of direct irradiance.

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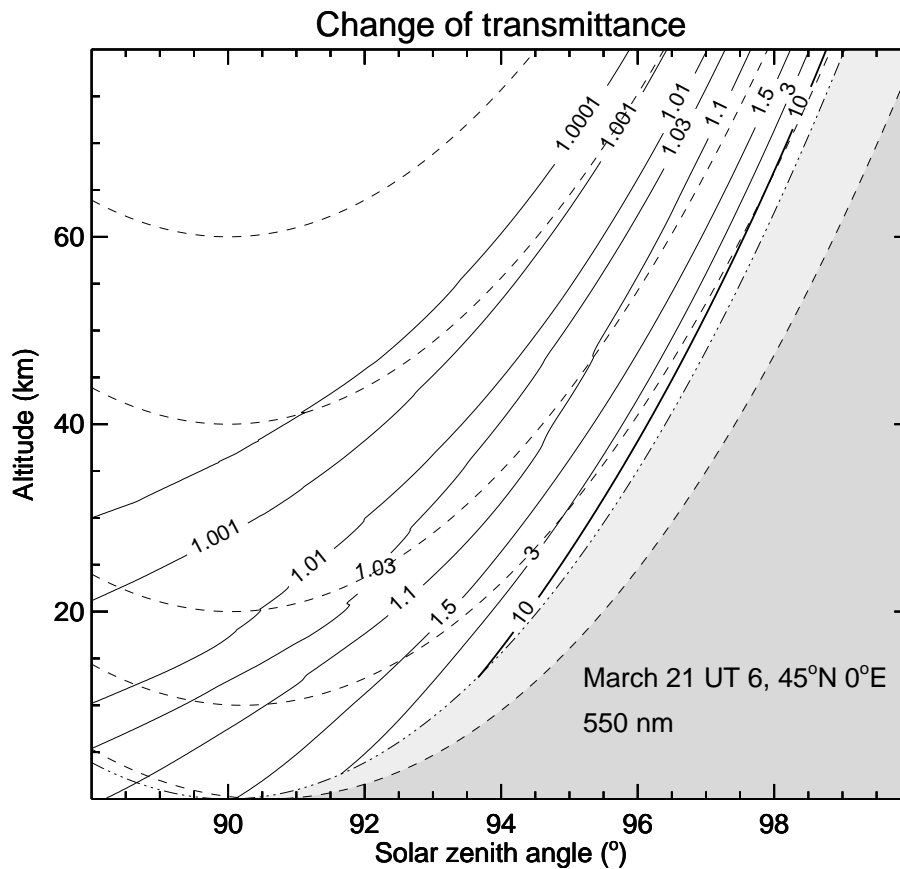


Fig. 8. The refraction induced changing factor T/T_{straight} of transmittance.

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