Replies to reviewer comment #2

RC C422:
Title: Temporal and spatial scaling impacts on extreme precipitation
Authors: Eggert et al.

We thank the anonymous reviewer for the insightful comments, which we feel have helped improve the clarity of the manuscript! Our point-by-point replies (blue) to the reviewer comments (black) are given below.

Reviewer #2

Considering that the average reader who is interested in this work (and this work has potentially many practical users) it would be nice to explain in general terms what a self-affine process is. The references mentioned deal with rather specific papers, with detailed mathematical analysis, which are not easy to read, and general information on a self-affine process was not specific enough. Finally, I understood this as a change from more linear precipitation structures at larger scales to more circular structures at small scales. If this is the case, or else, it would be nice to show this with a conceptual figure.

We have rewritten the introduction and discussion and conclusions chapter to be more accessible for the more practical users, while keeping much of the details of the results chapter for the more theoretically inclined readers. We added a general explanation of the term self-affine to the paper and also rephrased parts of the already given information in order to make the text more comprehensive. Additionally we added a conceptual figure showing the concept of the Taylor hypothesis together with the two major assumptions made in order to use this hypothesis for our analyzes (frozen in time, no variability perpendicular to the advection direction).
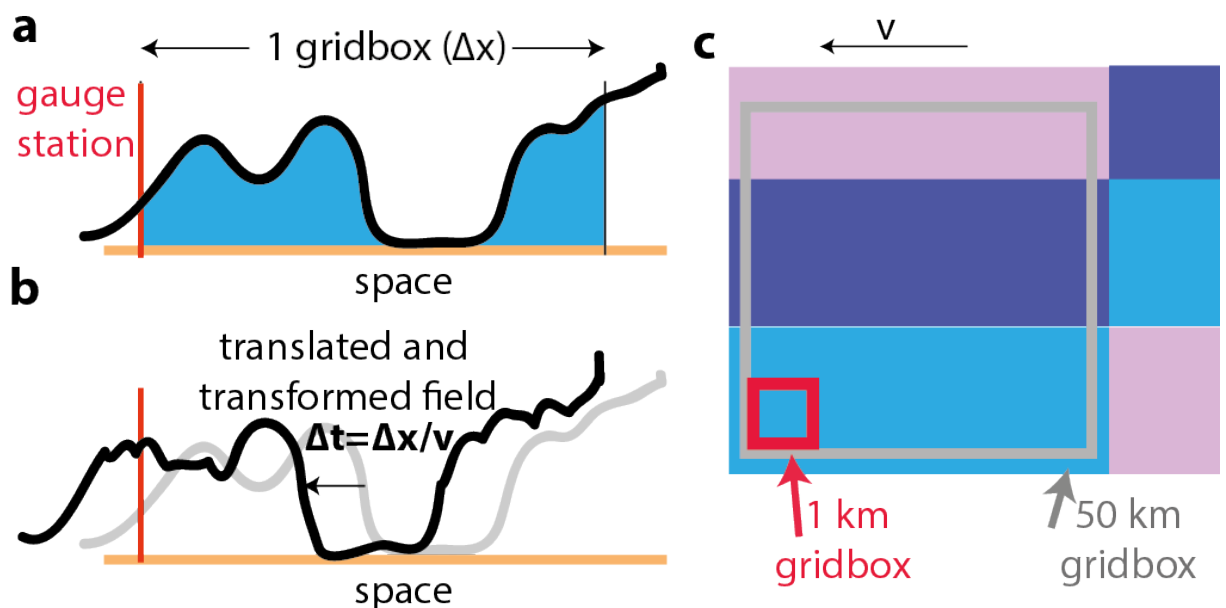

Figure 6. Schematic illustration of the Taylor hypothesis. (a) One-dimensional case, showing space, gridbox width and precipitation intensity (black curve); the location of a gauge station is marked in red. (b) Similar to (a), but illustrating how the curve may

change due to small scale dynamics after a time interval Δt = Δx/v, with v the atmospheric advection velocity. (c) Two-dimensional inhomogeneity (different colors indicate different intensities) perpendicular to the advection direction (direction indicated by the thin arrow). Small (red) and large (gray) gridboxes as marked.

In general I have difficulties in understanding to concept of optimal resolution, and I also do not fully understand the implications for this in term of model resolution and model output. This may be my misunderstanding, but I think the manuscript may benefit from explaining a number of points more clearly. I few points where I am puzzled are:

In the discussion, I do not see the points made at page. 2178, lines 12 to 20. I may have missed the point here, but you are arguing that the statistics of the 11 km, 5 minute output is similar to the statistics of 1 km and 25 minute output, right ? In general, there is a similarity between the statistics at different time and spatial aggregation as shown also in Figures 9 and 10. I agree to that, but I do not see the point that this implies that the combination of 11 km and 25 minutes is optimal. Optimal in the sense that it follows Eq. 6 appears a mathematical construct and I do not fully understand how these practical implications follow from this.

Also, at page 2171 line 10 you are stating that the optimal temporal resolution of stratiform events should be 3.6 times higher resolved than in the original data set to yield consistency between temporal and spatial information. I am not sure what you exactly mean by this. Somehow this goes against intuition as stratiform events are characterized by both relatively small spatial and temporal dependencies.

> We understand that the word „optimal" was not a good choice and leads to confusions. Therefore we rephrased the section and added more information to explain what we mean. We also added more detailed information on how the results should be interpreted. You are right that stratiform events are characterized by both relatively small spatial and temporal dependencies. Here we only looked at the different ratios (area reduction / duration reduction) not at absolute values.

> **Comparing the relevance of space compared to time aggregation.** We can distinguish the behavior of spatial and temporal aggregation using two kinds of approaches (Deidda, 2000). The first approach would be to regard precipitation as a self-similar process (simple scaling). In this case the Taylor-hypothesis (Taylor, 1938) would be obeyed, and temporal variations can be reinterpreted as spatial variations that are advected over a fixed location by a large-scale flow that has a constant value over the observed temporal and spatial scales.

> Following the notion of "frozen turbulence", intensity change due to spatial aggregation can then be calculated from the intensity changes that result due to temporal aggregation multiplied by a constant velocity u, i.e. Δx ≈ Δt · u. This would only hold, if precipitation extremes could be seen as objects of temporally constant characteristics that are translated by large scale advection. If we also assume spatial inhomogeneity only to occur in the advection direction, a gauge station could be used to measure the precipitation intensities that fall over an area (Fig. 6a).

> The second approach would assume that the spatial and temporal aggregation behavior of precipitation extremes would behave like a self-affine process (a process where the ratio of scales is changing as one of the scales changes). In this case, the simple linear relation that connects changes due to time aggregation with changes due to spatial aggregation through an advection velocity, generally does not hold anymore (e.g. due to temporal (Fig. 6b) or spatial inhomogeneity (Fig. 6c). A multifractal analysis is needed, where in short, the "velocity" itself would become a function of the respective spatial and temporal scales. If this function is known, it is possible also for self-affine processes to

connect spatial and temporal scales. Proper understanding of the relationship between spatial and temporal aggregation is e.g. crucial for precipitation downscaling and bias correction methods (Wood et al., 2004; Piani et al., 2010a, b).

Our goal here is to characterize the differences in scaling of convective and stratiform extremes: Comparing the intensity reduction due to time aggregation for the 1 km dataset (Fig. 3a, left column) with the intensity reduction that results from spatial aggregation at a temporal resolution of 5 min (bottom row), a 4 km spatial aggregation is comparable to that of spatial aggregation for roughly 15 min. Similarly, for stratiform precipitation (Fig. 4a) we find that 6 km spatial aggregation corresponds to 15 min temporally. There is hence a dependence on the precipitation type, a relation we now analyze.

Figure 7a shows for each horizontal resolution the matching temporal resolution that achieves similar intensity reduction. We describe the relation between temporal and spatial aggregation at a fixed Δx by

$$f_{\Delta x}(\Delta t) = |\, I(\Delta t, 1km) - I(5min, \Delta x)\,| \qquad\qquad (4)$$

We now define $\varphi_{\Delta x}$ as the minimum value of $f_{\Delta x}$ w.r.t. Δt:

$$\varphi_{\Delta x} = \min f_{\Delta x}(\Delta t) \qquad\qquad (5)$$

The best matching time window Δt for a given Δx can be determined using the inverse function of $f_{\Delta x}$: $\Delta t = f^{-1}(\varphi)$. In practice, we determine Δt by an iterative numerical procedure, using first an interpolation between available resolutions for better approximation. The result for several high percentiles is shown for both precipitation types over Germany for the entire year on a log-log plot (Fig. 7a), i.e. straight lines represented power laws. If the Taylor-hypothesis were obeyed, the exponent would equal unity.

Within the limitations of the relatively noisy data, we find that the data represents a straight line over most of the analyzed spatial range and can be fitted to a power law function $\Delta t = a \times \Delta x^b$ with fitting parameters a and b, or by using dimensionless variables (i.e. defining $\chi \equiv \Delta x/\Delta x_0$, $\tau \equiv \Delta t/\Delta t_0$ and $\tilde{a} \equiv a\Delta x^b_0/\Delta t_0$), we have

$$\tau = \tilde{a}\,\chi^b,) \qquad\qquad (6)$$

with fitting parameters $\tilde{a}$ and b. The parameter $\tilde{a}$ is a scaling parameter and describes the Δt_0 corresponding to Δx_0. χ^b describes how the relevance of space compared to the time aggregation changes with resolution.

In Fig. 7a and b, the best-fit for the 99th intensity percentile is shown for convective and stratiform precipitation. We find that b is similar for both types (generally between 1.17 and 1.32), a departure from unity that should be confirmed by other data sources than the radar data at hand. An exponent b > 1 indicates, that extreme precipitation is self-affine (self-similarity would require b = 1). The fractal properties of precipitation were already highlighted in earlier studies and are found to be a result of the hierarchical structure of precipitation fields (Schertzer and Lovejoy, 1987) with cells that are embedded in small mesoscale areas which in turn occur in clusters in large-scale synoptic areas Austin and Houze Jr. (1972).

Table 1 displays ã and b for the different percentiles shown in Fig. 7a (non-dimensional). We find that for convective precipitation ã is near 0.5. Within the error bars there is no obvious dependence on percentile. This is also the case for the stratiform type, besides for the 99.9th percentile, which has higher ã and lower b values.

Since the values of b are similar for both precipitation types (Table 1), the difference between the matching temporal resolution of stratiform and convective events is kept constant over the entire range of Δx analyzed. We find that the different scaling between the two precipitation types mainly results from the varying ã.

Note also the kink in the observed curves in Fig. 7a at about 6 km, where a change of slope is observed. To show that this kink is a manifestation of the scale mismatch, we aggregate data spatially to 2 km (3 km for stratiform) horizontal resolution and re-plot (Fig. 7b). Due to this procedure the kink almost vanished. This test shows that aligning resolutions according to Eq. (6) allows smooth scaling.

For further analysis, and to make contact to the Taylor-hypothesis, we use the ratio of the the matching Δx and Δt to calculate the mean *effective* advection velocity, which we call v_{eff}. We define:

$$v_{eff}(\chi) \equiv \chi/\tau = \chi^{1-b}/\tilde{a}.) \qquad\qquad (7)$$

This velocity is not obviously the same as the velocity obtained by tracking algorithms, such as in (Moseley et al., 2013), as v_{eff} combines all reasons for changes caused by aggregation. The main sources for these changes are advection of the precipitation field out of the grid box, temporal inhomogeneity caused by the temporal evolution of the precipitation event (Figure 6b) and horizontal inhomogeneity perpendicular to the advection direction, that will increase the area reduction factors (Figure 6c).

Figure 7c shows v_{eff} calculated for different Δx for the 95th, 98th, 99th and 99.9th percentile, using data without seasonal distinctions over Germany. v_{eff} lies in the same range as the velocities calculated by Deidda (2000) and Moseley et al. (2013) who calculated the velocities using tracking techniques. This shows that advection is likely the major source for changes due to temporal and horizontal aggregation. Low v_{eff} for horizontal resolutions below about 2 to 4 km are again a result of the mismatch of the 5 min temporal resolution and the 1 km spatial resolution explained above.

Note the deviating value of ã for the 99.9th percentile of stratiform precipitation. This could be explained by mesoscale stratiform systems with embedded convection, i.e. systems that are somewhat intermediate between stratiform and convective events. The corresponding graph (Fig. 7c) shows intermediary behavior, connecting the curves of convective precipitation (low Δx) to those of stratiform precipitation at high Δx. Due to substantial noise at high spatial resolution it is not possible to identify if v_{eff} shows a constant behavior (b = 1) at the high resolutions, therefore the results in Zawadzki (1973) and Waymire et al. (1984) indicating the Taylor-hypothesis to hold for time scales less than 40 min can neither be confirmed nor rejected.

Realizing that v_{eff} combines all sources for changes caused by aggregation enables a simplified view on the aggregation process. In a similar way as in Deidda (2000) we can use v_{eff} to generalize the Taylor-hypothesis for a self-affine process, by using v_{eff} instead of a constant velocity to describe the relation between space and time. Following the Taylor- hypothesis we can now interpret the matching temporal and spatial scales from Figure 7a as the mean time that is needed to advect the information about the

precipitation field over the matching horizontal scale (implicitly including all other sources of aggregation changes as described above). For example the typical timescale for a convective precipitation area to cross a grid box with a 10km grid-size, a typical resolution of state of the art climate models, would be about 40 min. For a stratiform precipitation event the information about the precipitation field is already captured after about 20 to 25 min. Reasons for the lower effective advection velocity might be that stratiform events are statistically more homogeneous than convective events which results in a shorter period to capture the structure of the event. Also, convective events often occur at high pressure weather conditions where low wind velocities might entail lower advection velocities.

Aggregation effects at a specific resolution will always be a combination of duration and area reduction factors. Connecting space and time scales using v_{eff} allows the association of temporal and spatial scales, shown in Fig. 7a. If, for a given spatial resolution, a larger temporal output period is used as indicated by Figure 7a, the event will on average be advected beyond the grid box area, leading to high duration reduction factors (a "smearing out").

Finally, I do not understand why a_tilde (as defined in eq 6) is not 1, since the ARF and DRF are equal at the reference resolution (1 km, 5 minutes) by construction. Does this perhaps imply that the effective resolution of the rain radar data is not 1 km and 5 minute, or that there is a mismatch between spatial and temporal scale in the radar data too. Is this what you want to say with Figure 6b? And is this also the reason why in Figure 9 the lower left point does appear to be an outlier (or is characterized by a very strong decay in pdf overlap at lower time and larger spatial resolution).

You are right, at the reference resolution of 1 km, 5 min we find that the temporal aggregation most likely lead to stronger intensity reductions than the spatial aggregation. This is what we show with Figure 7b (before 6b). The reason why in Figure 10 (before 9) the lower left point does appear to be an outlier is more likely an artifact from the data binning.

Minor points:

p 2163, l 27: I did not understand "convective together with mixed conditions"

We rephrase to make the txt more comprehensive:

For time resolutions longer than three hours, two 3 hourly time slices have to be considered. Here we classify the precipitation event as *stratiform* or *convective* only, if the type is identified at least at one of the time slices and the other time slice was not identified as the opposite type of event.

p 2165, line 26 and further. This is a nice example of explaining why these statistics are similar. But, the argument of the propagation speed should not enter the spatial averaging in this simple example since the averaged intensity over the grid cell (as long as the cell is within the grid box, and this is only where the propagation speed is important) does NOT depend on the propagation speed (at any time the area of precipitation is 10x10 km).

You are right, we corrected this example in the text. Without the propagation speed the size of the events needs to be a few hundred meters larger than the 10 km in order to

have an exact match with the passage over a location example. Since this is an idealized example that uses only approximate values, we feel that the example is still valid.

According to Berg et al. (2013) and Moseley et al. (2013) the average convective event has a lifetime of approximately 30 min, a spatial extent of ~ 10 km and a propagation speed of $\sim 10 \text{ ms}^{-1}$. When using a 50 km grid box and 5 min temporal resolution, the event will move about 3 km, therefore it can be assumed that the event stays in one grid box. It will affect roughly $(10\text{x}10)/(50\text{x}50) \approx 4$ % of the cell at a time. When an event of ~ 10 km cross section moves over a location with ~ 10 m/s, its passage over the location would last ~ 1000 s, which is ~ 17 min, and $17/360 \approx 5$ % of the matching time interval of 6 hours.

p 2172, line 18. I thought the optimal temporal resolution is smaller (not larger) for stratiform events, which is what you get when dividing the two optimal curves.

Please see the above explanation.

Eq. 7: isn't there a root of b missing here?

Thanks for noticing, the problem in this equation is, that it should have been Xi instead of tau. We changed this in the text.

p 2178, line 14: a model resolution of 11 km does not imply that precipitation at that scale is realistically simulated as you seem to imply here.

You are correct and we did not intend to imply this; we have rewritten the sentence to make this clearer. Additionally we added more information about this subject in the discussion section and we have refrained from using the word "optimum" to avoid confusion.