Response to the referee comments on the manuscript:

Title: Developing and bounding ice particle mass- and area-dimension expressions for use in atmospheric models and remote sensing
By: Erfani, Ehsan; Mitchell, David
Article reference: acp-2015-739

We wish to thank the referees for their detailed and helpful comments on our paper. As you will see below we have responded to all of the comments with revisions designed to address the concerns of the referees. In the following response, the original referee comments appear in black and our responses appear in blue and are labeled "Author response:"

Referee comments:

Anonymous Referee #1:

Review of "Developing and bounding ice particle mass- and area-dimension expressions for use in atmospheric models and remote sensing" by Erfani and Mitchell

Recommendation: Accept after revision of manuscript.

This paper presents self-consistent ice particle mass and projected-area dimension relationships that are not power laws, but that can be easily reduced to power laws, that are valid over a much larger range of D than are power laws. This is done through analysis of data collected by a 2-dimensional stereo probe and Cloud Particle Imager in synoptic and anvil clouds at similar temperatures, and it is shown that the developed relationships are in good agreement with m-D power laws developed from recent field studies. The unique contribution of this paper is that it develops m-D and A-D relations that aren't power laws that cover larger ranges of particle sizes. Further, for implementation in schemes that require such power laws, the formalism can be converted to power law. As such, I feel that this paper is worthy and should be published in ACP subject to the suggested modifications below.

In terms of an historical perspective on the development of power laws, the authors reference the papers by Mitchell (1996, 2000, 2002) and Mitchell et al. (2006), and later on a series of papers by Heymsfield et al. (2004, 2007, 2010). It is important to note that many groups in addition to that of Mitchell and Heymsfield have been involved in the development of such power laws, and a more balanced list of references should be provided (e.g., Fontaine et al. 2014). It is also important to note that authors have derived power law relationships from measurements of bulk reflectivity in addition to measurements of bulk total mass (e.g., McFarquhar et al. 2007, MWR, and Locatelli and Hobbs 1974 should be referenced more early on in paper). I recommend that such studies also be referenced.

Author response: Thank you for introducing those papers. All of them are added to the introduction section of manuscript. Now the Introduction starts as (original manuscript, page 28518, after line 17):

"Measurements of individual ice particle mass showed that the relationships between ice particle mass and maximum dimension have the form of habit-dependent power laws (Locatelli and Hobbs, 1974; Mitchell et al., 1990; hereafter M1990). …"

And another part of Introduction has been modified (original manuscript, page 28519, after line 17):

"…Also, McFarquhar et al. (2007) used PSDs and radar reflectivities measured during spiral decents in the stratiform regions of mesoscale convective systems to determine the power law for each spiral. In addition, the recent study by Fontaine et al. (2014) employed ice particle images and radar reflectivies to derive the temperature-dependent power exponent and prefactor of power laws for tropical anvil clouds. …"

Why are the results from the Sierra Cooperative Pilot Project used to assess the aircraft results. Since these observations were obtained before the advent of the antishattering tips, there could be some contamination in the results. But, as Jackson and McFarquhar (2014) showed the shattering does not make a significant contribution to the mass measured by the probe, so perhaps the use of the SCPP data are ok. In any event, it would be good to have some discussions somewhere in the paper about the uncertainties associated with the probe measurements, and why those uncertainties do not affect the principal conclusions of the study.

Author response: The reason for using SCPP data is that it includes the direct measurements of individual ice particle mass. In Mitchell et al. (1990), ice particles were collected in the field during winter storms in a petri dish and then imaged under a microscope equipped with a camera. The maximum dimension of each ice particle was later measured in the lab. In addition, each ice particle was melted with a heat-lamp under the microscope, with a corresponding photo taken immediately after melting. This resulted in hemispheric water drops that were imaged in the lab to measure the diameter of the hemispheres and from that the volume and mass of each ice particle was calculated. Shattering can affect the aircraft measurements of ice particles due to aircraft high speed, but it has no significant effect on the ground-based measurements (including SCPP). Moreover, the smallest size that is measured during SCPP (~ 150 μm) is considerably larger than the size range associated with shattered ice artifacts ($D < 50$ μm; Jackson et al., 2012). It is in this sense that there is no uncertainty due to shattering for the SCPP measurement. All these explanations are added to the manuscript in Sect. 2.2 (original manuscript, page 28522, starting at line 24):

"SCPP was a 3-year field study on cloud seeding funded by the Bureau of Reclamation, and for one part of that project, the shapes, maximum dimensions and masses of 4869 ice particles were determined. As described in M1990, ice particles were collected during winter storms in a petri dish and then imaged under a microscope equipped with a camera. The maximum dimension of each ice particle (i.e. diameter of a circumscribed circle around the particle) was later measured in the lab. In addition, each ice particle was melted with a heat-lamp under the microscope, with a corresponding photo taken immediately after melting. This resulted in hemispheric water drops that were imaged in the lab to measure the diameter of the hemispheres and from that the volume and mass of each ice particle was calculated. Although shattering can affect the aircraft measurements of ice particles due to the high sampling speed, it has no significant effect on the ground-based measurements. Moreover, the smallest size that is measured during SCPP (~ 150 µm) is considerably larger than the size range of shattered ice artifacts (D < 50 µm; Jackson et al., 2012). Therefore, shattering during the SCPP measurements is not a concern. …"

The authors state on page 28523 that the objective of their study Is to develop m_d and A-D expressions that are representative of all ice particles for a given cloud type and temperature interval, suitable for use in climate models. I also note that the authors do talk some about measurement uncertainty. But, one question that they do not thoroughly address is variability. Past studies have shown that there is a lot of variability in derived m-D and A-D relations. Further, there is some variability even for a given cloud type of a fixed temperature. Given this, do the authors expect that representative relations can be found that are representative for all particles? Some discussion about variability and how the authors address such variability should be given.

Author response: The discussion on the variability of m-D expression had been provided at the first paragraph of Sect. 3. The natural variability associated with ice particle mass measurements was minimized in two ways. First, *m* was estimated from the BL2006 *m-A* relationship for *D* > 200 µm (which represents the mean *m-A* behavior in their dataset and thus removes much of the natural variability in *m*), and second, variability was reduced by averaging the SPARTICUS PSD within each 5 °C *T* interval, as described in Sect. 2, producing one mean PSD of number, area and mass concentration for each *T* interval.

We also added more discussions: McFarquhar et al. (2007) showed that there is considerable variability in *m-D* expression during the aircraft measurements of stratiform regions of mesoscale convective systems, and they used different *m-D* expression for each flight. However as we show further in this section, the variability in *m-D* relationship based on 13 flights in synoptic cirrus clouds during SPARTICUS does not exceed 32 % of the mean bin mass value, having a mean overall value of 13.48 %. Based on this low variability, we conclude that our *m-D* expression is representative of all ice particles for the cloud type indicated (continental midlatitude synoptic or anvil cirrus clouds) and temperature interval.

Now, the first paragraph in Sect. 3 has changed (original manuscript, page 28527, starting at line 16):

"While in principle each PSD can be used to produce an *m-D* or *A-D* expression, in practice only the mean PSDs were used to develop the *m-D* and *A-D* expressions (explained in Sect. 2.3 and in the Supplement, Fig. S6). Although the averaging process reduces scatter, the coherency of the curves is somewhat surprising. The natural variability associated with ice particle mass measurements was minimized in two ways, thus facilitating the curve-fitting process. First, *m* was estimated from the BL2006 *m-A* relationship for *D* > 200 µm (which represents the mean *m-A* behavior in a self-consistent way and thus removes much of the natural variability in *m*), and second, variability was reduced by averaging the SPARTICUS PSD within each 5 °C *T* interval, as described in Sect. 2, producing one mean PSD of number, area and mass concentration for each *T* interval. The coherency of this data makes it amenable to curve-fitting with high precision. McFarquhar et al. (2007) showed that there is considerable variability in *m-D* expression during the aircraft measurements of stratiform regions of mesoscale convective systems, and they used a different *m-D* expression for each flight. Our results differ for the reasons described above. Moreover, as we show further in this section, the variability in *m-D* relationship based on 13 flights in synoptic cirrus clouds during SPARTICUS does not exceed 32% of the mean bin mass value, having a mean overall value of 13.48 %."

A new paragraph was then added below this paragraph to address the review question:

"If ice particle morphology does not vary much within the cirrus clouds sampled, then our *m-D* expressions should be representative of all ice particles for a given cloud type (continental midlatitude synoptic or anvil cirrus clouds) and temperature interval. Ice particle images from cirrus clouds tend to support this assumption, indicating high density, blocky-shaped irregular crystals with some bullet rosettes and side planes at larger sizes (e.g. Lawson et al., 2006b; Baker and Lawson, 2006b). But if there is a radical departure from this morphology genre and planar ice crystals having low aspect ratios (i.e. c-axis to a-axis ratio where c-axis is length of the prism face) dominate, our m-D expressions could overestimate ice particle mass by a factor of ~ 3 (Lawson, 2016)."

And we added a sentence (original manuscript, page 2852,3 line 5) and referred the reader to Sec. 3 for the discussion on variability:

"… (see Sect. 3 for the discussion of variability in *m-D* and *A-D* expressions)."

As noted in specific recommendations in the detailed comments below, one other recommendation I would give for this paper is to include some more detailed error or uncertainty analysis. In particular, assigning some uncertainties to the estimated mass amounts would have been beneficial. This would go beyond the uncertainty analysis that is done for the polynomial fit expressions for the m-D and A-D relations, but rather relate more to the uncertainties in the fundamentally measured quantities.

Author response: Similar to Fig. 6, we calculated the fractional uncertainties for the mean ice particle mass in each size bin of the measured PSDs. The pattern for the mass fractional uncertainties is similar to that for area fractional uncertainties. Mass uncertainties range between 0 and 32 % of the mean bin mass, with a mean overall value of 13.48 %. This explanation has been added to the text (original manuscript, page 28529, starting at line 16):

"… Similar to Fig. 6, we calculated the fractional uncertainties for the mean ice particle mass in each size bin of the measured PSDs (figure not shown). The pattern for the mass fractional uncertainties is similar to that for area fractional uncertainties. Mass uncertainties range between 0 and 32 % of the mean bin mass, with a mean overall fractional uncertainty of 13.48 %."

Detailed comments:

Abstract line 10. The authors claim that field measurements of individual particle m are used from the 2DS/CPI. There are no measurements of mass from these probes. Further, the measurements of mass that are used to derive m-D relations come from integrated measurements of masses of several particles combined rather than from measurements of mass of single particles (this is correctly stated on page 28520, line 14).

Author response: It is right that no measurements of mass exists from 2D-S and CPI probes, but the field measurements mentioned in line 10 refer to SCPP data in which each ice particle was imaged for the measurement of maximum dimension, and then was the ice particle melted and the diameter of the resulting hemispheric water drop was measured to calculate the actual mass. We added this explanation in Sect. 2.2 regarding the description of SCPP data. Also, we mentioned in line 10 (abstract) that this ground measurement is during a cloud seeding field campaign. We also clarified in line 11 that A and D are provided by measurements whereas m is estimated. So now, this part of abstract has changed to (original manuscript, page 28518, starting at line 10):

"… This was done by combining ground measurements of individual ice particle *m* and *D* formed at temperature $T < -20$ °C during a cloud seeding field campaign with 2-dimensional stereo (2D-S) and Cloud Particle Imager (CPI) probe measurements of *D* and *A*, and estimates of *m*, in synoptic and anvil ice clouds at similar temperatures. ..."

And the first paragraph in Sect. 2.2 is modified (original manuscript, page 28522, starting at line 24):

"SCPP was a 3-year field study on cloud seeding funded by the Bureau of Reclamation, and for one part of that project, the shapes, maximum dimensions and masses of 4869 ice particles were determined. As described in M1990, ice particles were collected during winter storms in a petri dish and then imaged under a microscope equipped with a camera. The maximum dimension of each ice particle (i.e. diameter of a circumscribed circle around the particle) was later measured in the lab. In addition, each ice particle was melted with a heat-lamp under the microscope, with a corresponding photo taken immediately after melting. This resulted in hemispheric water drops that were imaged in the lab to measure the diameter of the hemispheres and from that the volume and mass of each ice particle was calculated. …"

Page 28522, line 15. It is stated that the CPI data are used for sizes less than 100 micrometers. But, is there any diameter below which the CPI data are not used? For example, McFarquhar et al. (2013) showed that it was difficult to extract any information from CPI images for particles with D smaller than 35 micrometers.

Author response: As explained in McFarquhar et al. (2013), a widely-accepted lower limit is not available for CPI, and it might not be possible to determine the shape of particles that are smaller than a threshold. They showed that it was difficult to extract useful information from CPI images for particles with $D < 35$ μm. We used CPI data for sizes between 20 μm and 100 μm, and we modified the text (original manuscript, Page 28522, line 15) to address this size range. The reason for this had been explained in the last paragraph in Sect. 2.4. We also cited McFarquhar et al.

(2013) and changed the last paragraph in Sect. 2.4 (original manuscript, page 28527, starting at line 9):

McFarquhar et al. (2013) discussed that a widely-accepted lower limit is not available for the CPI, and they found that it was difficult to extract useful shape information from CPI images for particles with *D* < 35 μm for mixed-phase arctic clouds. However, in our study, shape is not a concern for the CPI size range we are using (20 μm < D < 100 μm) since we assume hexagonal column geometry and only require length and width measurements, which are estimated for these sizes from a data processing algorithm developed at SPEC, Inc.

Page 28522, line 19. How is the CPI mass estimated?

Author response: The CPI mass is estimated from CPI projected area and aspect ratio by the method that we introduced in Appendix B. The text is modified to address the appropriate sections (original manuscript, page 28522, starting at line 17):

"… For other temperature ranges of synoptic clouds and for all temperature ranges of anvil clouds, estimated 2D-S mass (see Sect. 2.3) is used for size greater than 200 μm and estimated CPI mass (see Sect. 2.4 and Appendix B) for size less than 100 μm. ..."

Page 28522, line 25. How is maximum dimension defined? It is important to note that past studies have defined maximum dimension differently so it is important that the exact definition of maximum dimension (or methodology used to compute maximum dimension) be given.

Author response: Ice particle maximum dimension is measured as the diameter of a circumscribed circle around an ice particle. Mitchell et al. (1990) provided image of each ice particle under microscope, and measured ice particle maximum dimension as diameter of circumscribed circle. This clarification is added to the text (original manuscript, page 28522, starting at line 24):

"SCPP was a 3-year field study on cloud seeding funded by the Bureau of Reclamation, and for one part of that project, the shapes, maximum dimensions and masses of 4869 ice particles were determined. As described in M1990, ice particles were collected during winter storms in a petri dish and then imaged under a microscope equipped with a camera. The maximum dimension of each ice particle (i.e. diameter of a circumscribed circle around the particle) was later measured in the lab. …"

Page 28523, line 21: Are particles from the smallest size bin in the 2DS used in the analysis? Jensen et al. (2013) reported that the 2DS may overestimate concentrations of particles with D < 15 micrometers due to a poorly defined depth of field. Further, the sample volume for such sized particles is very small so only a few counted particles can dominate the concentrations. Further, for the larger particles that occur on the edges of the photodiode, how much does reconstruction of partially images particles affect the estimated areas (and hence impact the calculated masses)? For what fraction of particles is reconstruction used?

Author response: Since we used CPI data for the size range smaller than 100 µm, the former problem regarding the smallest size bin in the 2DS does not affect the calculations of various m-D and A-D relationships. We cited Jensen et al. (2013) and explained their findings regarding the smallest size range (original manuscript, page 28523, starting at line 22):

"… The data in the smallest size bin (5-15 µm) should be used with caution, because Jensen et al. (2013) showed that the largest uncertainty in depth of field for this size bin results in an overestimation of number concentration for particles in the smallest size bin. Since we used CPI data for the size range smaller than 100 µm, the aforementioned problem does not affect the calculations of *m-D* and *A-D* relationships. …"

Regarding the larger particles, no reconstruction was performed, and all particles that are not entirely inside the photodiode array were excluded from the data ("all-in" criteria). This process is described in the text (original manuscript, page 28524, starting at line 20):

"The original 2D-S data used in this study had been processed by the Stratton Park Engineering Company (SPEC), Inc. using the M1 technique for measuring ice particle length and area (see Appendix A in Lawson, 2011). However, the M1 method does not insure that the ice particle is completely imaged within the sample volume (i.e. that no portion is beyond the photodiode array). … To overcome these drawbacks, the 2D-S data used here were processed using the newly developed M7 method that insures that the ice particles are completely imaged within the sample volume ("all-in" criteria). …"

Page 28524, lines 6-11: It might be worthwhile also referencing the paper of Jackson et al. (2012) who found that the application of habit specific m-D relations applied to size/shape distributions measured during the ISDAC field campaign gave better agreement with the bulk measured masses than did application of the Baker et al. (2006) approach to the measured size distributions.

Author response: This reference has been added in this paragraph (original manuscript, page 28524, starting at line 9):

"…But if there is a radical departure from this morphology genre and planar ice crystals having low aspect ratios (i.e. c-axis to a-axis ratio where c-axis is length of the prism face) dominate, our m-D expressions could overestimate ice particle mass by a factor of ~ 3 (Lawson, 2016). Such may be the case for Arctic mixed phase clouds, where Jackson et al. (2012) showed that the application of habit-specific *m-D* relationships applied to size/shape distributions in arctic stratocumulus clouds during Indirect and Semi-Direct Aerosol Campaign (ISDAC) over North Slope of Alaska had better agreement with the measured IWC (mean difference is ~ 50%) than did the application of the BL2006 approach to the measured size distributions (mean difference is ~ 100%)."

Page 28524, lines 25-29: There is a limitation to this technique in that the probe sample volume will fall off rapidly with particle size for the entire in technique meaning that very few completely imaged particles will be available for the analysis. This limitation should be specifically stated. Second, what specifically is the "most accurate estimate for maximum dimension" and how is it obtained. Presumably this is very different than the length along the direction of travel which is very different than some of the definitions of maximum dimension that have been used in the literature. Some further comments on both of these points are warranted.

Author response: We added a supplement that is provided by SPEC Inc. to compare the M1 and M7 methods. Although the sample volume decreases by using M7 method, such decrease is not considerable. Figures S1 and S3 show number concentration and area concentration as functions of maximum dimension for cases of synoptic and anvil cirrus clouds, respectively. It is seen that M1 and M7 methods agree well for both number concentration and area concentration, and larger difference between M1 and M7 methods is observed for larger particles (D > 300 µm). Moreover, the comparison of M1 and M7 methods for PSD number concentration and extinction is displayed in Figs. S2 and S4. The difference in sample area between M1 and M7 methods does not exceed 5 % and 13 % for synoptic and anvil cirrus clouds, respectively. The difference for projected area is more pronounced in anvil than in synoptic cirrus clouds due to the existence of slightly larger ice particles in anvil clouds that have a greater chance of intersecting the edges of the 2D-S field of view. This explanation is added to the text (original manuscript, page 28524, starting at line 29):

"… Although the sample volume decreases by using M7 method, such decrease is not determinant. It is shown in the supplement (Figs S1 and S2) that M1 and M7 methods agree well for both number concentration and area concentration, with a larger difference between the M1 and M7 methods observed for larger particles (D > 300 µm). Moreover, the difference in PSD projected area between M1 and M7 methods does not exceed 5 % and 13 % for synoptic and anvil cirrus clouds, respectively (see Appendix A for a detailed discussion on the comparison between M1 and M7 methods). …"

Regarding the measurement method of maximum dimension, the diameter of circumscribed circle around the particle is measured as maximum dimension in M7 method. This length is different than the length along the direction of travel (L1 that is used in M1 method). This is explained in Appendix A with more details. The definition of maximum dimension and how it is different than other studies and reference to Appendix A has been added to this part of text (original manuscript, page 28524, starting at line 26):

"… To overcome these drawbacks, the 2D-S data used here were processed using the newly developed M7 method that insures that the ice particles are completely imaged within the sample volume ("all-in" criteria), and this method uses the most accurate estimate for maximum dimension (diameter of circumscribed circle around the particle, see Appendix A). …"

And Appendix A explains them with more details (original manuscript, page 28544, starting at line 15):

"There are various methods to process 2D-S data, such as M1, M2, M4, and M7 methods (Lawson, 2011). Explanation and comparison of all these methods are beyond the scope of this paper. The M1 method was originally used in this study, but the newly developed M7 method was replaced for two main reasons. First, the M1 and M7 methods differ on the measurement of particle dimensions, as is shown in Fig. A1. The horizontal direction represents the direction of particle travel into the 2D-S probe and is sometimes referred to as the time dimension. The M1 method uses maximum dimension along the direction of travel (length scale $L1$) as the maximum dimension, whereas the M7 method uses the maximum dimension of the particle 2D image as the diameter of circumscribed circle around the particle (length scale MaxLength). Therefore, M7 method provides a more realistic measurement of maximum dimension, compared to many other studies that used *L1*. Length scale L4 in Fig. A1 is determined from the maximum number of shadowed photodiodes (vertical array) at any given instant.

Second, the M1 and M7 methods are distinct in the treatment of particles that intersect the edges of the 2D-S field of view. Using the M1 method, all particles are included in the measurement of projected area and number concentration, even particles that intersect the edges of the 2D-S field of view, and in those cases their maximum dimension and projected area is approximated. When using the M7 method, only particles that are completely inside the 2D-S field of view ("all-in" particles) are included. This provides an accurate measurement of projected area and maximum dimension for all particles. Although the sample volume decreases by using M7 method, such a decrease is not significant. Figures S1 and S3 show number concentration and area concentration as functions of maximum dimension for cases of synoptic and anvil cirrus clouds, respectively. It is seen that the M1 and M7 methods agree well for both number concentration and area concentration, with a larger difference between the M1 and M7 methods observed for larger particles ($D > 300$ µm). Moreover, the comparison of the M1 and M7 methods for the PSD number concentration and extinction is displayed in Figs. S2 and S4. The difference in PSD projected area between the M1 and M7 methods does not exceed 5 % and 13 % for synoptic and anvil cirrus clouds, respectively. Such difference in projected area is more pronounced in anvil than in synoptic cirrus due to slightly larger ice particles in anvil clouds that have a greater chance of intersecting the edges of the 2D-S field of view."

Page 28525, lines 3-7: This averaging procedure over temperature intervals will mean that your contributions from PSDs with larger mass contents will dominate. In their classic study, Marshall and Palmer (1948) averaged PSDs with similar rainrates so that different mass contents would not dominate the comparisons.

Author response: Although the averaging over temperature intervals results in a larger contribution from PSDs with larger *IWC*, we show that variability in temperature-dependent *m-D* expressions does not exceed 32 % and has a mean value of 13.48 % (see Sect. 3). Therefore, the contribution from PSDs with larger *IWC* is not determinative. Moreover, the objective of this study is to develop m-D and A-D expressions for a given temperature interval and cloud type, suitable for use in climate models. Therefore, the m-D and A-D expressions for similar precipitation rates will not serve the objective of this study.

Page 28525, lines 14 and 15: Can a more quantitative description of relatively be given?

Author response: We assume that cloud extinction and median mass size are relatively invariant with time when the cloud extinction and median mass size do not exceed 2 times their mean and are not less than 0.4 times their mean in a 60-second time period. This quantitative description has been added to the manuscript (original manuscript, page 28525, starting at line 13):

"… Moreover, the PSD selection process identified cloud regions (cloud extinction > 0.1 Km-1) where cloud extinction and median mass size were relatively stable (i.e. in a 60-second time period, the cloud extinction and median mass size should not exceed 2 times their mean and should not be less than 0.4 times their mean), making it unlikely that liquid water was present. …"

Page 28526, line 26: This assumption about hexagonal column geometry seems to be very different than past studies that have assumed small particles are typically quasispheres such as droxtals, Gaussian random spheres, Chebyshev particles or budding bucky balls. Can you justify this assumption? How important is this assumption to your final results?

Author response: Um and McFarquhar (2009) studied the radiative properties of small ice particles by assuming idealized shapes of droxtals, Gaussian random spheres, Chebyshev particles and budding bucky balls. They investigated particles with $D < 50$ μm and area ratio between 0.69 and 0.85, and they calculated ice particle mass from dimension. The purpose of our study is to estimate the mass of small ice particles from processed CPI data that contains measurements of ice particle projected area, length and width. We developed a method that utilizes all three of these properties to estimate ice particle mass. For the size-range we considered (20 to 100 μm), length-to-width ratios were generally < 1.5, confirming the presence of high-density ice particles, and for such aspect ratios, hexagonal columns appear to be as good a surrogate of small particle morphology as other shapes for estimating ice particle mass. They also provide a convenient means of using the aspect ratio estimates.

This explanation has been added to the text (original manuscript, page 28526, starting at line 25):

“This new methodology assumes that ice particles with size less than 100 μm exhibit hexagonal column geometry. Such a geometrical assumption seems reasonable based on observations for sizes smaller than 100 μm (see Lawson et al., 2006, their Figs. 4 and 5). While other authors have approximated small (e.g. D < 50 μm) ice crystals as droxtals, Gaussian random spheres, Chebyshev particles and budding bucky balls (e.g. Um and McFarquhar, 2009), our study estimates the mass of small ice particles from processed CPI data that contains measurements of ice particle projected area, length and width. We developed a method that utilizes all three of these properties to estimate ice particle mass. For the size-range we considered (20 to 100 μm), the mean length-to-width ratio was 1.41±0.26, confirming the dominance of high-density ice particles, and for such aspect ratios, hexagonal columns appear to be as good a surrogate of small particle morphology as the other shapes noted above for estimating ice particle mass. They also provide a convenient means of using the aspect ratio estimates. As shown in Appendix B, for an aspect ratio of 1.0, the difference in ice mass between the spherical and hexagonal column assumption is 4%. ”

Page 28527, line 4-5: I am a bit unclear on what is meant by not using any mass estimation technique for the size range of 100-200 micrometers. Does this mean that these particles are not considered in your calculation?

Author response: Yes. Although the assumption of small ice particles as hexagonal column is reasonable for $D < 100$ μm, it overestimates the mass for ice particles with 100 μm $< D <$ 200 μm. This is due to the fact that ice crystals in this size range begin to develop branches or extensions, becoming more complex and less compact (Bailey and Hallett, 2004, 2009). In other words, ice particles in this size range have less density than particles with $D < 100$ μm. Since the BL2006 m-A expression and the assumption of small ice particles as hexagonal column are not valid for 100 μm $< D <$ 200 μm, we did not use any mass estimation for this size range. The exception is for -65 °C $< T \leq$ -55 °C, where we used the BL2006 m-A method to estimate mass from CPI projected area for *D* between 100 and 200 μm, because the number of size bins available for $D > 200$ μm is limited (See Fig. 4, where it shows that data for this coldest temperature interval is available only for $D < 600$ μm). This is the most accurate approach for this size interval for $T \leq -55$ °C, which is critical for determining *m-D* expressions for these colder temperature intervals. This is explained in the 3rd paragraph under Sec. 2.4.

Page 28529, line 9: How useful are temperature-dependent curves? In a model scheme typically a single m-D relation is adopted, so how useful is it to have curves as functions of temperature?

Author response:

Fontaine et al. (2014) found that it is not proper to employ a single temperature-independent m-D expression for all clouds, because such an expression neglects the considerable natural variability of mass as a function of dimension. We showed that it is sufficient to categorize m-D expressions into three temperature intervals for a given cloud (see Table 1, and Fig. 4). Such classification is practical for modeling purposes. This explanation has been added to the text (original manuscript, page 28528, starting at line 21):

"… Fontaine et al. (2014) found that it is not proper to employ a single temperature-independent *m-D* expression for all clouds, because such expression neglects the considerable natural variability of mass as a function of dimension. We showed that it is sufficient to categorize *m-D* expressions into three temperature intervals for a given cloud. Such classification is practical for modeling purposes. …"

Page 28532, line 17: It is worth noting that agreement will always appear good on a logarithmic scale. How good does the agreement have to be in order to be considered good?

Author response: We calculated the percent difference on the normal scale, and not on a logarithmic scale. In other words, it is calculated as $100 \times (m_{\mathrm{SCPP}} - m_{\mathrm{SPARTICUS}}) / [(m_{\mathrm{SCPP}} + m_{\mathrm{SPARTICUS}}) / 2]$ for each size bin. In this sense, the calculated mean percent difference of 28% shows a good agreement. We modified the text (original manuscript, page 28532, starting at line 12):

"Getting still more quantitative, the percent difference of the SCPP cold habit mean mass for a given size interval was compared with the corresponding ice particle mass from the SPARTICUS curve fit. In other words, the percent difference is calculated as $100 \times (m_{\mathrm{SCPP}} - m_{\mathrm{SPARTICUS}}) / [(m_{\mathrm{SCPP}} + m_{\mathrm{SPARTICUS}}) / 2]$ for each size bin (figure not shown). Percent differences are less than 53% in all size bins, and the mean percent difference for all size-bins was 28%. Note that percent difference is calculated on the normal scale, and not on the logarithmic scale. Given the natural variability observed for ice particle masses, this level of agreement is considered good. …"

Page 28536, line 24: Can you be more quantitative on what you mean by "valid over a limited range of D". This would help those wishing to apply such relationships.

Author response: There is no quantitative size range in this case since the range of D is subjective and is determined by the error that a particular user is willing to tolerate. Errors encountered when using m-D and A-D power laws relative to this new approach are shown in Figs. 12, 13 and 14 (black and blue curves) regarding the calculation of N, D_e and V_m. These figures illustrate the increased accuracy obtained by matching the power laws to the PSD moment(s) of interest. Examples illustrating how m-D power laws are valid over limited D ranges are given in Fig. 1 and are discussed in Sect. 1.

The text in Sect. 6.1 (original manuscript, page 28536, starting at line 24) has been modified:

"While these relationships are commonly used in climate models, it is sometimes not recognized that such power laws are only valid over a limited range of D (examples include Fig. 1 and also Table 1 in Mitchell 1996). …"

And the text in Sect. 1 (original manuscript, page 28519, starting at line 17) has been modified:

"… But these approaches implicitly assume that the m-D relationship conforms to a single size-independent power law, whereas Table 1 in Mitchell (1996) indicates that it often takes two or even three m-D power laws to describe a given m-D relationship over all relevant sizes. For example, Mitchell (1996) determined three power laws for hexagonal columns for three size ranges: 30 μm < D ≤ 100 μm, 100 μm < D ≤ 300 μm, and 300 μm < D. Cotton et al. (2012 ; hereafter C2012) have developed a bulk IWC approach that yields two m-D power laws that better describe the observations, assuming an exponent of 3 for the smallest ice particle sizes (D < 70 μm). …"

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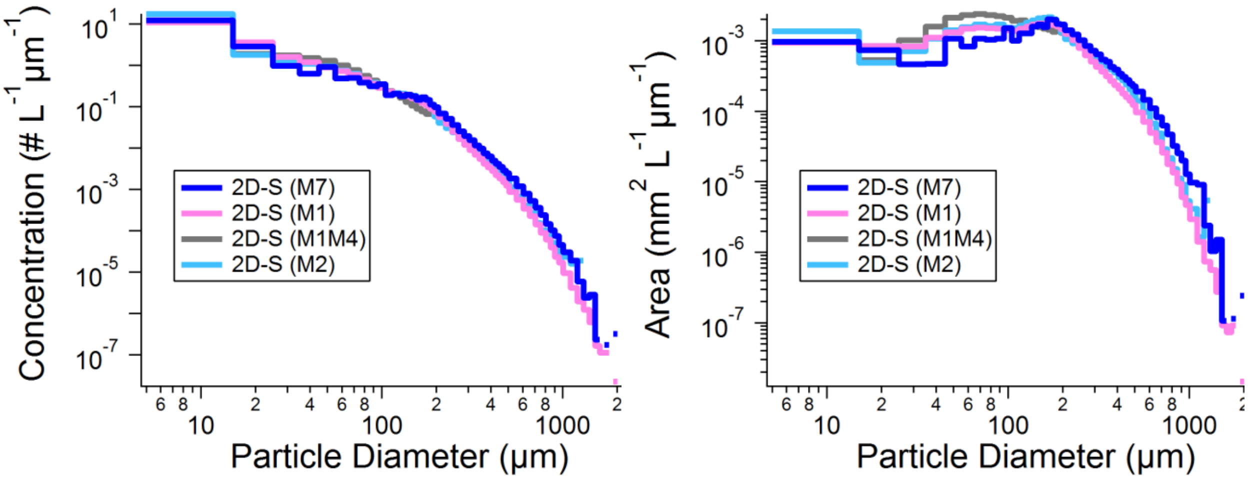
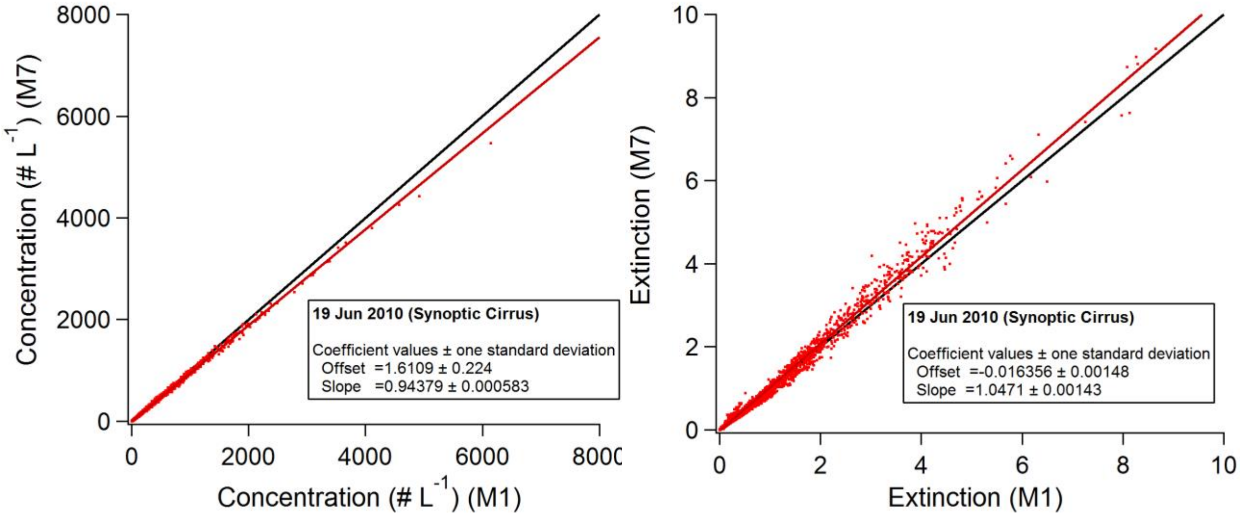
Supplement:



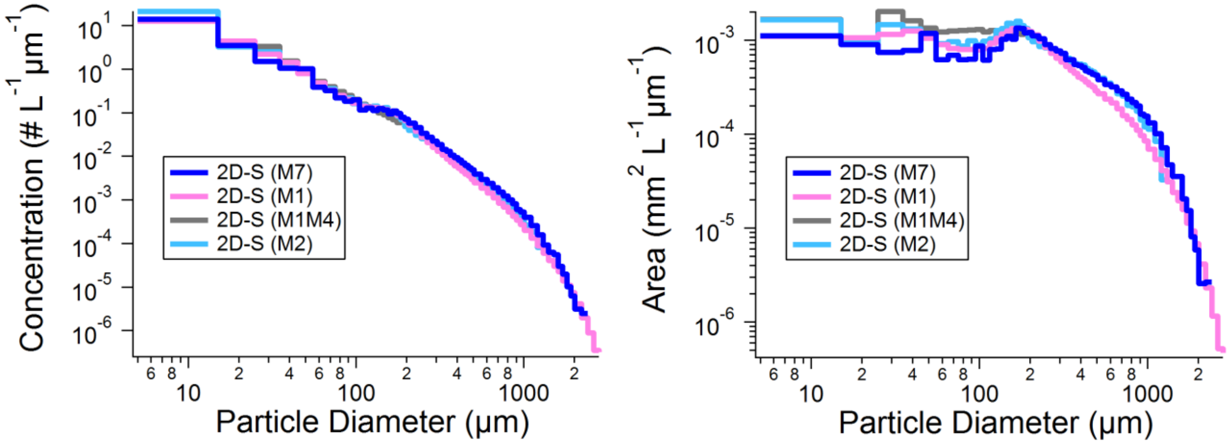
(a) (b)

Figure S1. (a) Ice particle number concentration and (b) ice particle projected area concentration as functions of maximum dimension for various processing method of 2D-S data during flight A on 19 Jan. 2010 (as example of synoptic cirrus clouds). Courtesy of Paul Lawson and Sara Lance.



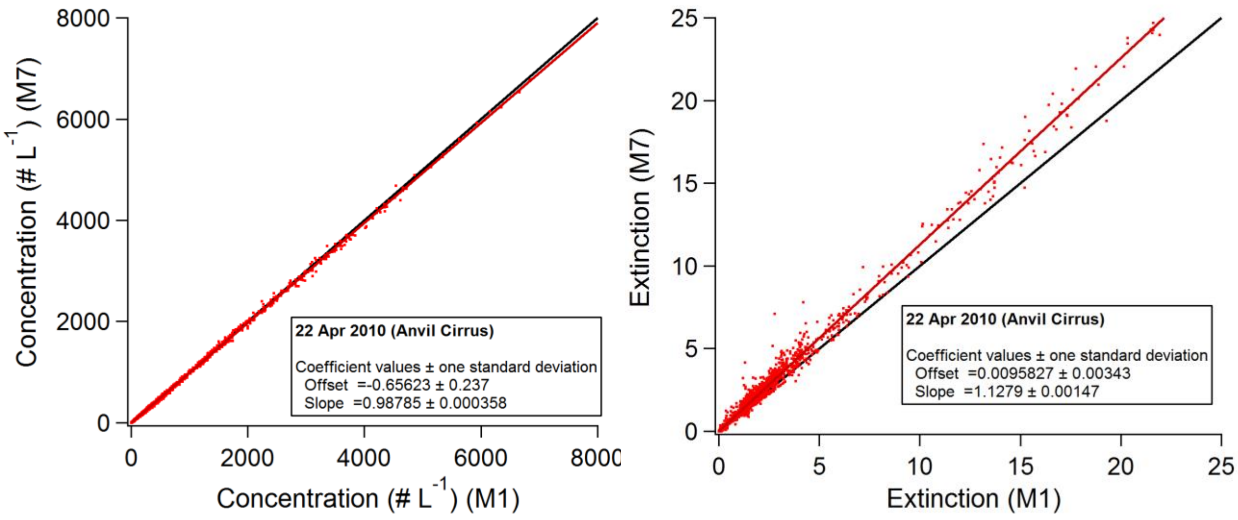
(a) (b)

Figure S2. (a) PSD number concentration from 2D-S M7 versus PSD number concentration from 2D-S M1, (b) extinction from 2D-S M7 versus extinction from 2D-S M1 during flight A on 19 Jan. 2010 (as example of synoptic cirrus clouds). Red line shows regression line to the data points, and black line displays 1:1 line. Courtesy of Paul Lawson and Sara Lance.



(a) (b)

Figure S3. Same as Fig. S1, but during flight A on 22 Apr. 2010 (as example of anvil cirrus clouds). Courtesy of Paul Lawson and Sara Lance.



(a) (b)

Figure S4. Same as Fig. S2, but during flight A on 22 Apr. 2010 (as example of anvil cirrus clouds). Courtesy of Paul Lawson and Sara Lance.

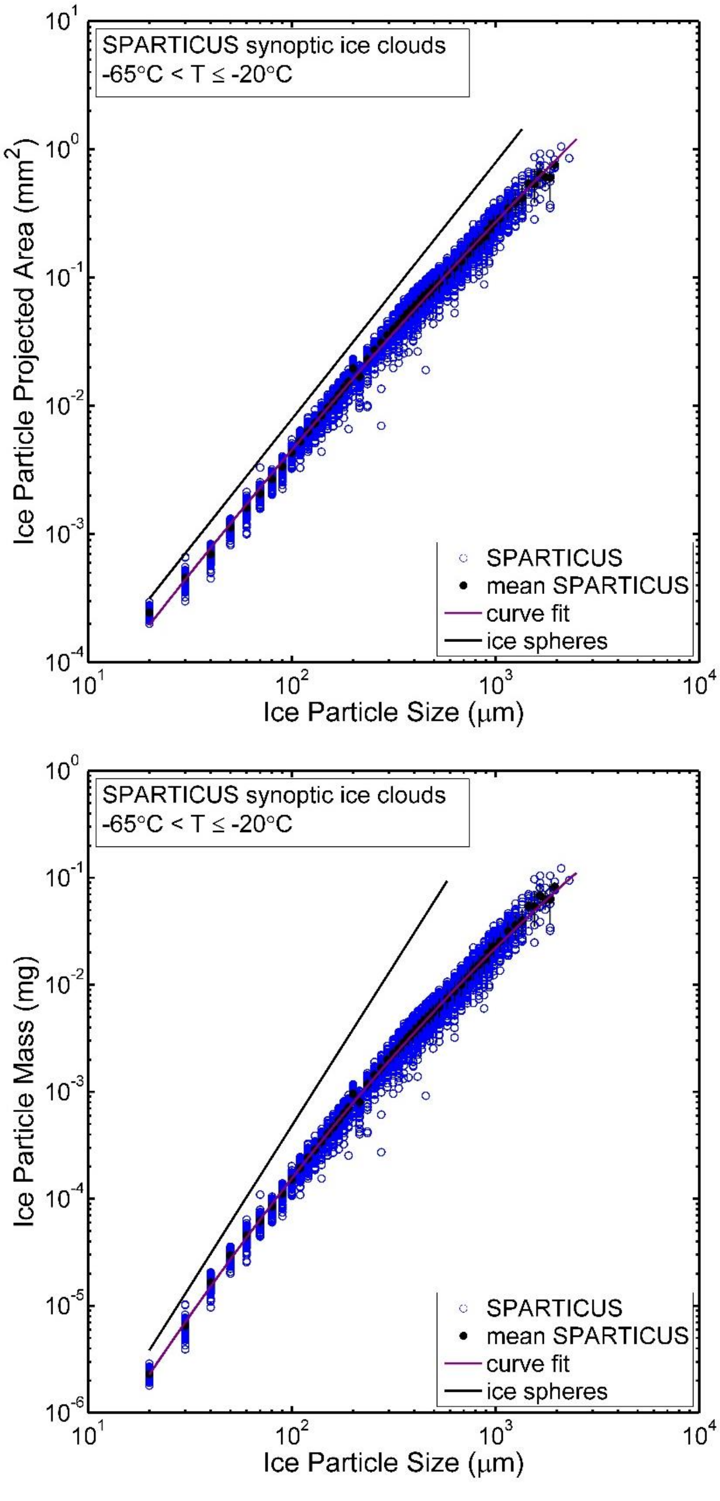


Figure S5. Dependence of (a) ice particle projected area and (b) ice particle mass on *D* based on actual PSDs regardless of temperature dependency. The SPARTICUS 2D-S data has been grouped into size-bins.

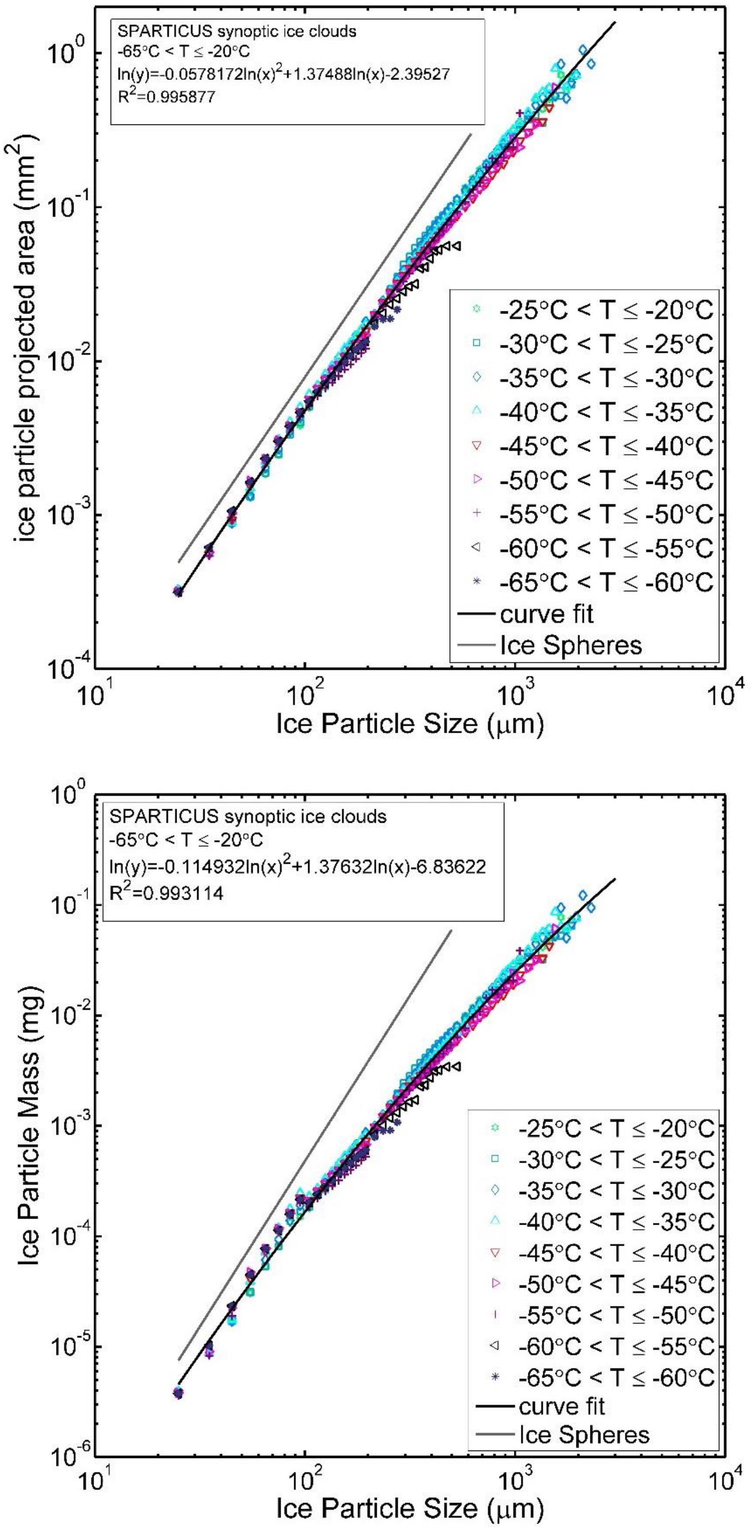


Figure S6. Dependence of (a) ice particle projected area and (b) ice particle mass on *D* based on mean PSD within the indicated temperature regime. The CPI and 2D-S data have been grouped into size-bins and 5 °C temperature intervals, and the black solid curve is a fit to these datasets.