

General comments:

According to the comments from reviewer #3 and editor Graham Feingold, we did some major changes to our manuscript to account for the main concerns that were raised:

- We discuss that the cloud droplet number concentration (CDNC) from a ground-based radar-radiometer retrieval method is very sensitive to the assumption about the droplet size distribution (DSD). We investigate its sensitivity due to uncertainties in the observations and required assumptions and compare it to sensitivities of cloud optical depth (COT). Since cloud optical depth is less sensitive to assumptions about the width of droplet size distribution, we use this quantity for comparison with passive satellite observations. This also means that we decided to omit the optimal estimation retrieval of CDNC for this study until we are able to present in-situ observations for validation.*
- We are in the view that the consistency check of ground-based with satellite-based key quantities is a main aspect of our work. We agree that it is difficult to draw conclusions from only 4 case studies. This is why we extended our investigation to a 2 year dataset at LACROS in the years of 2012 and 2013. We keep the 4 cases to highlight sources of uncertainties, which we discuss more closely in the revised version of the manuscript.*

Answers to Reviewer #3

I am reviewing the revised version of the above paper. I think this paper contains some interesting science but attempts to over-achieve by doing two things at the same time: (1) attempting to get at cloud adiabaticity from different combinations of ground-based measurements, and (2) comparing ground-based to space-borne measurements. As a result neither topics (1) nor (2) are fully developed.

As for topic (1) I think the combination of radar and passive microwave radiometers is interesting and the comparison between two approaches (combining radiometer and radar versus the method following Illingworth) forms a solid basis for a paper. I wish the authors had fully developed just this idea and addressed various remaining open issues including for example the variability in k , which will have a huge impact on radar reflectivity because k drives droplet spectrum dispersion. (There's a relatively new paper by Parol and Brenguier addressing variability in k in much detail).

We extended the discussion of the width of the droplet size distribution and its effect on both cloud droplet number concentration and cloud optical depth. While there is a major impact on CDNC, the COT is less sensitive to this assumption. We discuss the sensitivity of the retrieved CDNC and COT in terms of uncertainties of the observations in more detail. We decided to use COT quantity for cross-checking with satellite-based observations, since it is less sensitive to assumptions about the width of the DSD (i.e. the k factor).

Another interesting variable is the shape of the N profile and the impact of homogeneous versus inhomogeneous mixing. The authors are aware of the issue but dismiss it a bit too readily (around line 280). I would be interested in seeing the impact of different mixing assumptions on Z and thus on the theoretically derived N in the Illingworth approach.

If N is not considered constant with height, solving the integral over $Z(z) N(z) dz$ in the radar-radiometer retrieval approach would be complicated. Remillard et al. 2013 accounted actually for a slight vertical variability in CDNC within a radar-radiometer retrieval method using information

about vertical air motion. But their requirement is that the vertical variations remain small.

Motivated by observations of profiles of CDNC we decided to only assume that CDNC is constant with height. Boers et al. 2006 investigated the effect of the two different mixing models and came to the conclusion that retrieval results for CDNC (and also COT) agree within a few percent for both mixing models. We added this statement to our discussions.

Topic (2) is the comparison satellite against ground-based and space-borne observations. This is also interesting but the paper falls short of providing any general conclusions because of the limited nature of the case studies. The paper also completely ignores recent results by Suzuki et al., Zhibo Zheng et al., Di Girolamo et al., Maddux et al., and possibly others dealing with uncertainties and systematic errors in satellite retrievals of effective radius and other cloud parameters.

We agree that it is difficult to draw conclusions from 4 case studies only. This is why we applied the comparison of COT and LWP to a 2 year observation period at LACROS. We further use the 4 handpicked case studies to highlight specific uncertainties which complicate the comparison of large datasets.

We discussed uncertainties from satellite retrievals from COT and effective radius more detailed in the revised manuscript. As our focus in the revised manuscript is mainly on SEVIRI, we only shortly referred to systematic errors obtained for MODIS.

In summary, this is a paper with potential but as it stands it is falling short of what it could be.

My recommendation would be the authors take a step back and fully address topic (1), which could make for an excellent self-standing paper.

In general, we agree to the need to emphasize the uncertainties more, and that it is not possible to draw statistically significant conclusions about the agreement of ground-based and satellite data from just 4 cases. Nevertheless, we believe that even from some cases, one can illustrate some of the key issues affecting the reliability of both ground-based and satellite-based data, which are hopefully of interest to the scientific community. We tried to include some larger datasets where possible. For the ground-based retrieval we stronger focus on the sensitivity of CDNC and COT. We also use COT instead of CDNC as a more certain quantity for the intercomparison between satellite and ground. For the comparison to the satellite-perspective we further outline the problem of resolution effects.

We disagree with the suggestion to drop the satellite part, as in contrast to our ground-based analysis, we believe that the aspect of consistency between both satellite and ground-based datasets is one point which has not been covered extensively in the literature and is key to our research goals.

Once the ground-based approach is squared away the satellite/ground-based comparisons could be addressed.

Couple of minor issues I noted reading the paper.

Page 4, line 262 pp

Although Nd may vary vertically, it is commonly suspected that it stays nearly constant throughout the vertical column of a nonprecipitating cloud (Bennartz, 2007; Brenguier et al., 2000).

The two papers cited here are not quite relevant here. Both are dealing with remote sensing

applications. There are various papers out there showing that N is actually constant vertically through large parts of Sc clouds. Search for Brenguier, Rauber, Pawlowska, and other papers using aircraft experiments....

Changed. We refer to Pawlowska 2000, 2006 in the revised manuscript.

Page 4, line 270 pp, Eq (10) and Eq (14)

While Eq (10) does give N , it is overly sensitive to large droplets embedded in the cloud, b/c $Z \sim r_0^6$. This severely limits the use of radar data for this purpose as especially in clean clouds there are always some large droplets available. The same applies even more so to Equation (14). I see that later on the authors mention in passing that the maximum radar reflectivity did not exceed -20 dBZ (end of page 6), so that likely there was no drizzle present.

We explained the sampling method to avoid drizzle earlier in our revised manuscript. We discussed the problem that Z is very sensitive to few large droplets, but it is difficult to totally avoid this problems. We tried to filter out drizzle profiles using the Cloudnet target classification and the radar reflectivity threshold. To reduce problems due to the sensitivities of CDNC, we focus on the cross-checking with satellite observations of COT and liquid water path (LWP). Of course also these cloud properties are subject to uncertainties, which we discussed in more detail.

Answers to Editor Comments

Dear Colleagues,

I am now in receipt of a third review on your paper. The original reviewers are unavailable and I would like to move this process forward so I am raising some important issues in addition to those raised by reviewer 3.

In general I have strong concerns about the approach, and the lack of sampling statistics. Reviewer 3 has offered one option - namely to pursue either N retrieval or adiabatic retrieval but to do so more rigorously. I would like the authors to weigh their options. As it stands the manuscript is not acceptable for publication.

I offer some detailed comments and ideas below in the hopes that they will help the authors improve the paper.

Graham Feingold

Main comments:

1) I believe that caution is warranted when using a ground based radar/microwave radiometer method as reference and comparing with a passive approach (SEVIRI, MODIS). The methods differ not only in their spatiotemporal sampling but also in their sensitivity to the drop size distribution. I know the authors are well aware of this (comments to this effect are peppered along the way) but this then leads one to ask whether one should expect good comparison between the space- and surface-based approaches. Moreover, if the comparisons are good is it for the right reason. With such a small sample (4 cases) one cannot draw conclusions.

Some papers that might be helpful are

doi:10.1029/2004JD005648 and doi:10.5194/acp-12-1031-2012.

I raise these papers not to simply suggest reference to them but because I think that important

lessons can be taken from them.

doi:10.1029/2004JD005648: Aerosol indirect effect studies at Southern Great Plains during the May 2003 Intensive Operations Period

doi:10.5194/acp-12-1031-2012: The scale problem in quantifying aerosol indirect effects

We agree that ground-based retrieved CDNC and COT should not be considered as the reference, when compared to passive satellite approach, given the uncertainties. But we believe that cross-checking both perspectives can help better understanding limits and possibilities of state-of-art retrieval approaches. We discussed more detailed the sensitivities of the retrieved properties from the two perspectives. To get statistically more robust results from a comparison, we use a 2 year dataset filtered to homogeneous, liquid clouds at LACROS. We present the case studies to highlight different sources of uncertainties. Since COT is less sensitive to the assumptions about the DSD, we use this quantity instead of CDNC for cross-checking satellite and ground-based retrievals. We thereby focus more on the discussion of sources of uncertainties from both perspectives.

2) The use of radar and microwave radiometer to retrieve drop concentration is a very risky proposition given the disparate moment-weightings (0 vs 3 and 6). Frisch and colleagues have discussed how the ratio of the moments in Eq. 10 can become very unstable, which explains why you sometimes retrieve very high N (line 598). In fact in later work they resorted to using fixed (climatological N; Frisch et al. JTECH 2002). It's one thing if one retrieves Reff profiles with radar and microwave radiometer but another when one attempts N. In this regard I am in agreement with reviewer 3 that the attention to retrieval of N is not very rigorous - not withstanding the effort put into OE.

We decided to compare COT instead of CDNC, while outlining the high sensitivity of CDNC on (Z,k,fad) in the radar-radiometer-retrieval within the adiabatic cloud model. We discuss the sensitivity of COT and CDNC on assumptions and observations.

We omit the Optimal Estimation approach in this study. The high sensitivity of CDNC would require stronger constraints and would benefit from in-situ validation, which can not currently be provided.

3) Eq. 14 for the radar derived adiabatic measure also raises questions. If the authors have access to aircraft measurements - e.g., from colleagues at TROPOS they could test this sensitivity. The authors should look at the sensitivity of this value to radar thresholds; I suspect it will be quite sensitive.

This is indeed a good suggestion. Unfortunately, there is currently not yet quality-controlled data available. This is why we postpone the discussion of the Optimal Estimation approach until in situ observations can be used for validation.

Other points:

1) Lines 122-123: Can you support this claim? What are the relevant spatiotemporal scales for radiative forcing?

This statement is omitted in the revised paper.

2) "However, when the results of both approaches are in agreement, it is likely that the correspond-

ing cloud layers are well suited for the investigation of key factors determining the first indirect effect.” Could you explain why? Is this logical? What about compensating errors?

The phrasing is a bit misleading. What we meant was: for homogeneous cloud layers we suppose to find the best agreement between ground- and satellite-based observations since resolution effects will have less impact. We further suppose that retrieval errors from passive satellite observations are smallest for homogeneous clouds due to the assumptions made in the KNMI-CPP algorithm. Therefore those clouds should show the smallest uncertainties and are most suitable for investigation of aerosol-cloud interactions.

3) “Although rain might be a possible explanation for higher QL observed with the ground-based microwave radiometer, there are no signs for precipitation in both radar signal and satellite observations.” Could you explain why? Is this because of wetting of the window? Is the QL highly variable as well? (typical in rain).

Indeed, after periods of rain/drizzle, the microwave radiometer has to deal with a wetting of the radom. During inhomogeneous conditions the liquid water path shows a high temporal variability which can not be resolved by SEVIRI. The temporal variation is supposed to stem from an inhomogeneous cloud scene rather than from rain (since we filtered those cases).

4) 4.2.2: There is a great deal of discussion about differences, but given the small sample what statistical significance does this have?

We changed the structure and discussion to focus more on the uncertainties. For the comparison to the satellite we used a longer time-period and use the 4 case studies to highlight the specific problems.

5) “Due to the $N \propto R_{eff}^{-2.5}$ relationship (see Eq. 5)”
But τ in eqn 5 is also dependent on R_{eff} ...

We tried to clarify the sensitivities by discussing the relative uncertainties. We find the uncertainty in effective radius to have a large impact on the retrieved CDNC.

6) “ In our study we filtered out drizzling profiles as well as possible, but the radar reflectivity still remains very sensitive to few larger droplets in a volume, which can not totally be ruled out. Therefore also the correct radar calibration is an issue.”

This statement should have appeared in the beginning, and frankly, it might have been a point at which it which the authors decided to stick to optical measurements + microwave radiometer - particularly for the comparison. In this way they could focus on averaging issues in the comparison, rather than a mix of issues.

The reason why we initially choose radar-radiometer measurements was driven by previous studies that also applied such methods for the retrieval of cloud properties and due to its availability within the Cloudnet framework. Therefore the same approach can be applied at many other stations, for which accompanying optical measurements might not be available. In the revised manuscript we discuss the effect of averaging issues by comparing MODIS and SEVIRI products. Both apply very similar retrieval approaches, namely a combination of a visible and near-infrared channel to obtain COT and effective radius. We tried to highlight the effect of spatial homogeneity applying a homogeneous and a inhomogeneous cloud scene.

7) “captured surprisingly well for some cases. We discussed the large uncertainties that may occur depending on the observed scene and observation geometry.”

I believe that this statement should be revisited given the poor statistics. Is the comparison good because of compensating errors, pure chance, or because it really is good.

We agree that it is difficult to draw significant conclusions from 4 samples only. We tried to focus stronger on uncertainties in the revised manuscript and used longer time-periods for a comparison to get a more robust statistics as basis for our discussion. Case studies are mainly used to highlight the problems that can occur.

Minor:

1) Throughout the text there is reference to “indirect effects” when the authors really mean the microphysical response to a change in aerosol. The indirect effect is a radiative response and this is not addressed here.

We will use the term „aerosol-cloud-interactions“ instead of „indirect effects“.

2) Throughout the paper, there tends to be a lack of recognition to early work on various topics.

Line 179: Baker and Latham discussed mixing in the 1980s.

Line 250, Frisch et al. (1995) were I believe the first to perform radar/microwave radiometer

retrievals of Reff profiles and N. Line 558: Warner showed in the 1950s that shallower clouds are more adiabatic.

We added the suggested references to the early work papers.

3) Line 555-556 is an odd statement.

Is removed.

4) Rewrite “The maximum of the radar reflectivity in each profile did also not -20 dBZ...”

*The maximum of the the radar reflectivity in each profile did also not **exceed** -20 dBZ*

Investigation of the adiabatic assumption for estimating cloud micro- and macrophysical properties from satellite and ground

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Abstract. In this study the accuracy of quantities relevant for diagnosing the first indirect aerosol effect with satellite is investigated by comparing co-located cloud properties from both ground-based as well as from geostationary passive satellite observations have been used previously for diagnosing aerosol-cloud interactions and specifically the Twomey effect. In this investigation, a two year dataset together with four selected case studies are analyzed with the aim of evaluating the consistency and limitations of current ground-based and spaceborne observations. The focus is set on retrievals of cloud droplet number concentration and cloud geometrical depth. For the study we considered the sub-adiabatic cloud model which is commonly applied to retrieve cloud micro- and macrophysical quantities from passive satellite sensors like SEVIRI or MODIS. As reference we use satellite-retrieved cloud property datasets. The adiabatic cloud model is often applied and modified using a sub-adiabatic factor to account for entrainment within the cloud. Based on the adiabatic factor obtained from the combination of ground-based observations from a cloud radar, a microwave radiometer and a ceilometer from which cloud ceilometer and microwave radiometer, we demonstrate that neither the assumption of a completely adiabatic cloud nor the assumption of a constant sub-adiabatic factor is fulfilled (mean adiabatic factor 0.63 ± 0.22). As cloud adiabaticity is required to estimate the cloud droplet number concentration is derived with a newly developed optimal estimation technique. Although the ground-based observations contain detailed information about the cloud vertical structure, large uncertainties in the retrieved cloud microphysical properties were found. We investigate four different cases (27 October 2011, 1 June 2012, but is not available from passive satellite observations, 27 September 2012 and 21 April 2013) of temporally homogeneous and inhomogeneous liquid cloud layers observed over

Germany. Considering uncertainties for both ground-based an independent method to estimate the adiabatic factor, and satellite-based retrievals, we find a good agreement when temporally homogeneous single-layer clouds are considered. Overall, cloud layers were thus the influence of mixing, would be highly desirable for global-scale analyses. Considering the radiative effect of a cloud described by the sub-adiabatic with medians of the adiabatic factor around 0.65 for 3 cases and around 0.45 for one case. When model, we focus on cloud optical depth and its sensitivities. Ground-based estimates are here compared versus cloud optical depth retrieved from the Meteosat SEVIRI satellite instrument resulting in a bias of -4 and a root mean square difference of 16. While synergistic methods based on the combination of ceilometer, cloud radar and microwave radiometer enable an estimate of the cloud droplet concentration, it is highly sensitive to radar calibration and to assumptions about the moments of the droplet size distribution. Similarly, satellite-based and estimates of cloud droplet concentration are uncertain. We conclude that neither the ground-based retrievals are compared, the best agreement was found for the 21 April 2013 homogeneous case, namely a 4% relative mean difference of cloud geometrical depth and a 15% relative mean difference of cloud droplet number concentration when the sub-adiabatic factor obtained from ground-based observations is considered. For all evaluated cases, the current SEVIRI retrieval seems to underestimate the effective radius relative to ground-based and MODIS measurements for unfavourable solar zenith angles of above approximately 60° . This deviation strongly propagates to the derived cloud droplet number concentration nor satellite-based cloud retrievals applied here allow a robust estimate of cloud droplet concentration, which complicates its use for the study of aerosol-cloud interactions.

1 Introduction

Low-level liquid clouds play an important role in the energy balance of the Earth, and are found in many areas around the globe. Their microphysical and optical properties are strongly influenced by aerosol particles that act as cloud condensation nuclei (CCN). Twomey (1974) first postulated the effect of an increased aerosol number concentration in clouds, which is on the radiative budget, commonly referred to as the first indirect aerosol effect, as a climatically relevant process. The quantification of such aerosol indirect effects remains one of the main uncertainties in climate projections. If the liquid water content as well as the geometrical depth of the cloud are considered constant, a higher aerosol load results in an enhanced cloud albedo. This effect is observed in particular by means of ship tracks that form in marine stratocumulus cloud decks (e.g.). The chain of interactions of cloud microphysics and dynamics is complex and not yet fully understood. However, to quantify the effect of a change in the aerosol load on cloud albedo, it is necessary to consider both microphysics and macrophysics, which are influenced by cloud dynamical processes.

Cloud quantities that are typically used to calculate aerosol-cloud interactions, are the cloud droplet number concentration (N_d) and cloud geometrical depth (H). noted that a 15% change in the cloud geometrical depth (H_{cloud}) 15% change in H can have a similar effect on cloud albedo as a doubling of the cloud droplet number concentration (N_d). Already suggested doubling of N_d . proposed to investigate a column cloud droplet number concentration which combines H_{cloud} and N_d . These two quantities turned out to be the key parameters for quantifying the aerosol effect on cloud albedo, N_d which is a combination with H .

The aim of the current study is to gain a better understanding of the current possibilities and shortcomings when H_{cloud} and N_d of clouds are retrieved from satellite observations, by evaluating existing retrievals with ground-based observations performed over Germany. We combine observations from SEVIRI (Spinning Enhanced Visible and InfraRed Imager) onboard Meteosat Second Generation (MSG) and MODIS (Moderate-Resolution Imaging Spectroradiometer) onboard Terra and Aqua with ground-based remote sensing data obtained with ceilometer, microwave radiometer and 35-GHz cloud radar at Leipzig, Germany (51.35 N, 12.43 E) and at Krauthausen, Germany (50.897 N, 6.46 E). Those ground-based instruments are operated in the framework of Cloudnet and ACTRIS (Aerosols, Clouds and Trace gases Research Infrastructure Network).

The combination of ground-based ceilometer and cloud radar is able to provide reliable detection of cloud geometric borders. To derive N_d with this set of ground-based instruments recently suggested a radar-radiometer retrieval based on a condensational growth model taking the vertical velocity into account and allowing small variations of N_d

with height, while it is assumed vertically constant in most other studies. Since Cloudnet does not provide N_d , we developed and apply an optimal estimation technique to obtain N_d , based on the method introduced by, similarly also applied in. Given other instrument combinations such as those including lidar measurements, or solar radiation measurements would give alternative opportunities to derive N_d . Due to the under-constrained nature and assumptions made in such retrieval methods, substantial differences for the obtained microphysical parameters may occur, as pointed out by, who intercompared several ground-based retrieval methods for one case study.

While remote sensing observations from ground are always column measurements, passive satellite observations from, e.g., SEVIRI or MODIS, show a good tradeoff in terms of spatio-temporal coverage, and are therefore suitable to investigate the first indirect aerosol effect on a larger scale. Active satellite sensors on the other hand, such as the cloud profiling radar onboard CloudSat or the Cloud-Aerosol-Lidar with Orthogonal Polarization (CALIOP) onboard CALIPSO, Cloud-Aerosol Lidar and Infrared Pathfinder Satellite Observation), are able to provide vertically resolved cloud observations over larger areas that along their tracks and can be used to investigate aerosol effects on cloud properties (e.g.), but. These lack highly-resolved temporal coverage and have a smaller scanning swath than passive sensors onboard polar-orbiting satellites.

Despite their coarser spatial resolution, geostationary satellite observations benefit from the high temporal coverage of up to 5 minutes in conjunction with a high spatial coverage. This can be considered as an advantage for the determination of large-scale first indirect aerosol effects. Within this study the capabilities of geostationary satellites for cloud retrievals will be further evaluated. Validation of satellite-derived cloud parameters, such as (Q_L), with aerosol-cloud interactions, since the full daily cycle can be obtained and contrasted to ground-based observations.

If entrainment in clouds leads to a deviation from a linear increasing liquid water content, i.e. sub-adiabatic clouds, the first aerosol effect is not easily observed. To obtain key quantities from passive satellite observations, the sub-adiabatic cloud model is usually applied (e.g.). Therefore obtaining cloud adiabaticity is important for the investigation of aerosol-cloud interactions. The combination of ground-based ceilometer and cloud radar is able to provide reliable detection of cloud geometric borders. N_d from ground-based observations has only infrequently been performed. Especially the comparison of N_d and H_{cloud} from both space and ground has not yet been carried out intensively for different regions of the Earth, although pointed out that their combined retrieval of N_d and H_{cloud} would give the opportunity to derive the first indirect effect with high spatial and temporal resolution. In this

study can be retrieved from radar-radiometer measurements (?), observations including including lidar measurements (??), or solar radiation measurements (??). To derive N_d from radar-radiometer observations ? recently suggested a condensational growth model taking the vertical velocity into account and allowing small variations of N_d with height, while it is assumed vertically constant in most other studies. Due to the under-constrained nature and assumptions made in such retrieval methods, substantial differences for the microphysical properties may occur, as pointed out by ?, who intercompared several ground-based retrieval methods for one case study. ? showed that the cloud optical depth is less sensitive to the assumptions required in radar-radiometer retrieval approaches and might be considered as an alternative key quantity.

As a consistency check, we contrast satellite retrievals with the independently retrieved properties key quantities from ground-based remote sensing – using a ceilometer, a microwave radiometer and a 35-GHz cloud radar at Leipzig, Germany (51.35 N, 12.43 E) and at Krauthausen, Germany (50.897 N, 6.46 E) with observations from SEVIRI (Spinning Enhanced Visible and InfraRed Imager) onboard Meteosat Second Generation (MSG). Those ground-based instruments are operated in the framework of Cloudnet (?) and ACTRIS (Aerosols, Clouds and Trace gases Research InfraStructure Network). To our knowledge such evaluations from the SEVIRI instrument for the indirect aerosol effects’ key parameters have been rarely carried out (e. g. in ?). Previous satellite retrieval studies, retrieving N_d (e.g. in ?). Thereby, we discuss the uncertainties introduced by required assumptions when cloud microphysical properties are retrieved, and for H_{cloud} , usually apply a (sub-)adiabatic cloud model with a presumed adiabatic factor (e.g. ???). Only ? calculated this factor in advance. With that, we can assess the influence of cloud sub-adiabaticity on N_d and H_{cloud} as well as the agreement between the retrieved properties from ground and satellite. Apart from assumptions about the adiabatic factor, also uncertainties in the retrieval of optical depth and effective radius determine the accuracies of the results and will be effect of different spatio-temporal resolution. As the sub-adiabatic cloud model is a key concept for the retrievals discussed in this context study, we aim to quantify cloud adiabaticity using the available observations.

The paper is structured as follows. In Sect. 2 we introduce the adiabatic-sub-adiabatic model, relevant for the satellite-based retrieval of key parameters, as well as the retrieval methods from ground. Afterwards we describe the instruments and data processing tools used within this study in Sect. 3. In Sect. ?? these retrievals are applied to four different cases which are then used to evaluate the satellite-based observations. 4 cloud adiabaticity is investigated. Subsequently we contrast important key properties for aerosol-cloud interactions from SEVIRI and LACROS and discuss uncertainties from both perspectives

(Sect. 5). Finally, a conclusion and outlook is given in Sect. 6.

2 Cloud microphysical retrieval methods using the sub-adiabatic cloud model

In this section we present the theory of the (sub-)adiabatic sub-adiabatic cloud model and retrieval strategies for the cloud droplet number concentration from the suite of ground-based instruments as well as passive satellite observations.

2.1 Retrievals using the (sub-)adiabatic cloud model

For a moist rising air parcel we assume that the liquid water content $q_L(z)$ increases linearly with height (?) and can be related to $N_d(z)$ and the mean volume droplet radius $r_v(z)$:

$$q_L(z) = f_{ad} \Gamma_{ad}(T, p) z = \frac{4}{3} \pi \rho_w r_v^3(z) N_d(z) \quad (1)$$

Here z is the height above cloud base, ρ_w is the density of water. f_{ad} represents the sub-adiabatic fraction $\Gamma_{ad}(T, p)$ is the adiabatic rate of increase of liquid water content, in the following simply called adiabatic factor. It can be explained by the. The adiabatic factor f_{ad} can be understood as a reduction of liquid water due to evaporation influenced triggered by the entrainment of drier air masses and, which leads to $f_{ad} < 1$ (sub-adiabatic). $\Gamma_{ad} = A_{ad}(T, p) \rho_a(T, p)$ is the adiabatic rate of increase of

Integrating the liquid water content, with ρ_a the air density and A_{ad} the adiabatic increase of the liquid water content mixing ratio. In general, for with height yields the liquid water path. Aerosol-cloud interactions are usually studied as changes in cloud properties and radiative effects for a constant liquid water path (?). Therefore we will express all following physical quantities as function of given liquid water path. Observing the cloud geometrical depth in combination with the liquid water path, and knowing $\Gamma_{ad}(T, p)$, the adiabatic factor f_{ad} a range of [0.3, 0.9] is seen as common (?). From eq. (1) it is clear that either $N(z)$ or $r_v(z)$ can be affected by evaporation. ? considers two extremes: (a) homogeneous mixing, where $N_d(z)$ stays constant in the vertical layer, but the droplet radius ($r_v(z)$) is changed due to evaporation, (b) inhomogeneous mixing, where the number of droplets change (dilution of whole droplets), but the droplet radius profile is unchanged. In nature, a mixture of both processes may likely occur (?). For our study we only consider homogeneous mixing. can be calculated:

In remote sensing usually

$$f_{ad}(Q_L, H) = \frac{2Q_L}{H^2 \Gamma_{ad}(T, p)} \quad (2)$$

The geometrical depth for adiabatic clouds is obtained by resorting this equation:

$$H(Q_L, f_{ad}) = \sqrt{\frac{2Q_L}{f_{ad}\Gamma_{ad}}} \quad (3)$$

The equivalent mean volume droplet radius in a cloud depends on the cloud droplet number concentration N_d and the liquid water content:

$$r_V = \sqrt[3]{\frac{3q_L}{4\pi\rho_w N_d}} \quad (4)$$

In the following we assume homogeneous mixing and introduce the effective radius r_e . The effective radius is defined as the third over the second moment of the droplet size distribution (?) and can be linked is typically retrieved in remote sensing. The effective radius is related to the mean volume radius (r_V) with the following relationship:

$$r_e^3 = k^{-1} r_V^3$$

The factor k introducing a factor k_2 that depends on the cloud type and corresponding typical droplet size distributions. width of the droplet size distribution.

$$r_e = k_2^{-\frac{1}{3}} r_V \quad (5)$$

Typical values for marine and continental liquid water clouds k_2 are 0.67 and 0.80, respectively (?).

This leads to the following two equations for optical depth τ and effective radius r_e (compare Eq. A12, A14 in ?):

$$\tau = \frac{6}{5} \pi^{1/3} \left(\frac{4}{3} \rho_w\right)^{-2/3} (\Gamma_{ad} f_{ad})^{2/3} (k N_d)^{1/3} H^{5/3}$$

0.8 for marine and

$$r_e = \left(\frac{4}{3} \pi \rho_w\right)^{-1/3} (k N_d)^{-1/3} (\Gamma_{ad} f_{ad})^{1/3} H^{1/3}$$

Without entrainment, we find $f_{ad} = 1$ (adiabatic clouds) in all the equations above continental clouds (?), respectively. More details on the factor k_2 for the assumed gamma-size distribution can be found in the Appendix.

The typically obtained products from passive satellite remote sensing are τ and r_e . By substituting r_V with r_e using the ? retrieval method. The (sub-)adiabatic cloud model can be used to derive cloud properties such as liquid water

path (Q_L), cloud droplet number concentration (N_d) and geometrical depth (H) by inserting eq. ?? into eq. ?? and solving for the desired quantity. in eq. 4, we yield the effective radius representative for the uppermost cloud layer:

$$N_d = \frac{\sqrt{10}}{4\pi\rho_w^{0.5}k} r_e (Q_L, f_{ad}\Gamma_{ad}, N_d)^{0.5} \tau^{0.5} r_e^{-2.5} = \frac{\sqrt{18}f_{ad}\Gamma_{ad}Q_L}{\sqrt[3]{4\pi\rho_w k_2 N_d}} \quad (6)$$

$$H = \sqrt{\frac{10}{9} (f_{ad}\Gamma_{ad})^{-1} \rho_w \tau r_e}$$

$$Q_L = \frac{5}{9} \rho_w \tau r_e$$

Various different values considered for k , Γ_{ad} and f_{ad} can be found in previous studies (Table 1) due to different climatic and geographical regions on Earth (To study the microphysical response of aerosols on cloud microphysics with remote sensing techniques, together with the effective radius the optical depth is often used since both can be easily derived from e.g. continental vs. maritime). Often even adiabatic clouds are considered ($f_{ad} = 1$) (e.g. ?). In this study we take a constant value for k ($k = 0.8$), and $\Gamma_{ad}(T, p)$ using pressure and temperature obtained for cloud base height. The adiabatic factor is initially set to $f_{ad} = 1$ for the satellite-derived values of N_d and H , but is also calculated from ground-based observations in a further step. Following ? the adiabatic factor is given by the following relationship: passive satellite observations (?).

The optical depth in the sub-adiabatic model can be expressed as a function of Q_L and r_e (?):

$$f_{ad} = \frac{2Q_L}{(H_{obs}^{ground})^2 \Gamma_{ad}(T, p)}$$

$$\tau = \frac{9Q_L}{5\rho_w r_e} \quad (7)$$

We use Q_L from the ground-based microwave radiometer, H_{obs}^{ground} as the difference of cloud top height from the cloud radar and cloud base height from the ceilometer, and $\Gamma_{ad}(T_{cbr}, p_{cbr})$ using numerical weather prediction (NWP) data. Using this equation the liquid water path can be derived from passive satellite observations.

2.1 Ground-based retrieval of cloud droplet number concentration

By substituting r_e from eq. 6, we yield τ as a function of Q_L , N_d and f_{ad} :

2.0.1 Radar-radiometer based retrieval method

$$\tau(Q_L, f_{ad}, N_d) = \frac{9 \sqrt[3]{4\pi k_2 N_d^6 \sqrt[5]{Q_L^5}}}{5 \sqrt[5]{18 \rho_w^4 f_{ad} \Gamma_{ad}}} \quad (8)$$

With the given observations, the retrieval of From this equation, the cloud droplet number concentration can be based on a combination of the cloud radar and the microwave radiometer. This mainly requires an assumption about the droplet size distribution. Cloud microphysical quantities can then be described in terms of moments of this droplet size distribution. The from passive satellite observations can be calculated:

$$N_d(Q_L, f_{ad}, \tau) = \frac{20 \rho_w^2 \tau^3 \sqrt{10 f_{ad} \Gamma_{ad}}}{9 \pi k_2 \sqrt[5]{Q_L^5}} \quad (9)$$

To retrieve τ and r_e from the given ground-based observations, the cloud droplet number concentration is equivalent to the zeroth moment, the mean radius to the first moment, the liquid water content is proportional to the third moment, while the effective radius is the third over the second moment, and the radar reflectivity factor is proportional to the sixth moment. Relating these moments gives the chance to fully describe a unimodal distribution following either a gamma or lognormal shape and therefore calculating other moments of the size distribution which are not directly observed (?). Following ?, we relate the measured radar reflectivity (Z) to q_L and N_d . Thereby it is assumed that the droplet size distribution can be described by a gamma distribution with parameter β , where β is the index of the gamma function following the size distribution definition in (??):

$$N(r) \propto A r^\beta \exp(-B r)$$

Thereby B is the rate parameter and A a function of the rate parameter. A similar method has been applied in (?), but using a lognormal size distribution. Although N_d may vary vertically, it is commonly suspected that it stays nearly constant throughout the vertical column of a nonprecipitating cloud (??). To retrieve the column cloud droplet number concentration from the available single-layer observations, we integrate q_L over the cloud column and can therefore use Q_L from the microwave radiometer (compare ?):

$$N_d^{FI} = \frac{9}{2\pi^2 k \rho^2} \frac{(\beta + 6)!}{(\beta + 3)! (\beta + 3)^3} \frac{Q_L^2}{\left(\int \sqrt{Z} dz\right)^2}$$

Due to the relationship $N \propto \sqrt{Z}$, this retrieval method does not require the assumption of a linearly increasing liquid water content profile. Both, homogeneous and inhomogeneous mixing with dry air (?) can easily alter the

microphysical quantities in clouds in ways not adequately addressed within such a retrieval scheme. For example, the size distribution may become skewed and not be accurately described with a gamma shape anymore. However, ? and ? found out, that both assumptions about the mixing process result in nearly the same vertically averaged N_d .

2.0.1 Optimal Estimation method

The Optimal Estimation (OE) method, presented in the following, aims on finding the most likely state given the observations, the a-priori and the error estimates. Therefore we try to minimize a cost function following ?. The OE retrieval of cloud droplet number concentration (N_d^{OE}) and the liquid water content profile is based on the N_d is substituted in eq. 6 applying a radar-radiometer method. retrieval approach (e.g. ??, see appendix):

We further assume a vertically constant N_d , a gamma-shaped droplet size distribution with parameter β . As before, q_L , N_d , and Z are nonlinearly related. We include error estimates for the observed quantities as well as an a-priori state together with its error estimate.

$$N_d(Q_L, Z) = \frac{9 k_6 Q_L^2}{2\pi^2 \rho_w^2 \left(\int_{CBH}^{CTH} \sqrt{Z(z)} dz \right)^2} \quad (10)$$

Our observation vector (\mathbf{y}) contains the radar reflectivity Z and the microwave radiometer Q_L . Our state vector (\mathbf{x}) contains the vertically constant N_d and the natural logarithm of the vertical q_L profile. The logarithm is used to avoid the occurrence of unphysical negative liquid water contents in the minimization process.

$$\mathbf{y} = (Z, Q_L)^T; \mathbf{x} = (N_d, \ln(q_L))^T$$

The forward model ($F(\mathbf{x})$) for OE consists of two separate parts: a model (Eq. (??)) for the calculation of Q_L , and a model (Eq. (??)) for the calculation of N_d given the state vector \mathbf{x} . Then we find the optical depth and effective radius for given liquid water path to depend on the width of the droplet size distribution (k_2, k_6), the sub-adiabatic factor (f_{ad}) and the integrated radar reflectivity profile ($\int Z(z) dz$).

$$Q_L = \int \exp(\ln(q_L(z))) dz$$

It follows that $\tau \propto (k_2 k_6)^{\frac{1}{3}}$ and $r_e \propto (k_2 k_6)^{-\frac{1}{3}}$ (?).

The Jacobians are calculated numerically using finite differences for both methods as follows:

$$H(x) = \frac{\delta y_i}{\delta x_j} = \frac{F(x_i + dx_i) - F(x_i)}{dx_i}$$

We apply the Levenberg-Marquardt minimization method until convergence is reached (?). Only profiles with all required input data are processed. Only 0.1 of all the valid input profiles failed convergence within 30 iteration steps.

For the a-priori state vector, we assume that the liquid water profile follows the adiabatic scaled profile. For the a-priori N_d we set a value of which is a typical value for continental sites (?). We assume that there are no correlations between the elements in the covariance matrix, implying no correlations of the q_L uncertainties at different height levels and no correlations between q_L and N_d uncertainties. This is a rather simplistic assumption, but the variances are set reasonably large. The standard deviation for N_d is set to and for $\ln(q_L)$ to 2.5. While in this study homogeneous mixing is assumed, in general two extremes of mixing processes can be considered (??): (a) homogeneous mixing, where $N_d(z)$ stays constant in the vertical layer, but the droplet radius ($r_v(z)$) is changed due to evaporation, (b) inhomogeneous mixing, where the number of droplets change (dilution of whole droplets), but the droplet radius profile is unchanged. In nature, a mixture of both processes may likely occur (?). Without entrainment, we find $f_{ad} = 1$ (adiabatic clouds). The assumption of homogeneous mixing is supported by observations from ???. The adiabatic factor in this study is considered as representative for the full vertical cloud depth. For such an adiabatic factor f_{ad} a range of [0.3, 0.9] is seen as common (?).

Just as for the background error covariance matrix, we assume for the observation error covariance matrix that there is no cross-correlation, and that all off-diagonal terms are thus zero. Different values for k_z , Γ_{ad} and f_{ad} in eq. 9 have been considered in previous studies using passive satellites (Table 1) due to various reasons (e.g. different cloud regimes, continental vs. maritime). Often even adiabatic clouds are assumed ($f_{ad} = 1$) in the retrieval process (e.g. ?).

The observation error covariance can be split up into individual contributing parts such as forward model error, radiometric noise error, and representativeness error. In this study the representativeness error is neglected, since observations and state variables are on the same grid. Radiometric noise errors are given by the Cloudnet algorithm. The forward model error is estimated by applying values of β in the range of 1 to 6 to the radar forward model and taking the variance of the resulting reflectivity values for a sample cloud profile with a geometrical extent of 700 and linearly increasing q_L in steps of 0.1 per 100.

Given the retrieved N_d^{OE} and the theoretical adiabatic liquid water content for the observed cloud geometrical depth, we are able to calculate an adiabatic radar profile applying the relationship of q_L , Z and N_d of ?. If we relate Z_{ad} to the Z_{obs} from the cloud radar we obtain a second method to calculate the adiabatic factor (f_{ad}^{OE}):

$$f_{ad}^{OE} = \frac{\int Z_{obs} dz}{\int Z_{ad} dz}$$

3 Data

3.1 Instruments and retrievals

Data Satellite data from SEVIRI (?) are used for the geostationary satellite perspective. SEVIRI is used, which provides 12 spectral channels covering the visible, the near infrared, and the infrared spectrum. The channels used here have a nadir resolution of 3 km x 3 km. The spatial resolution, which decreases towards the poles and is about 4 km x 6 km over our region of interest (Central Europe). In this study we use the 5-min temporal resolution data from the Rapid Scan Service (RSS). The SEVIRI radiances in the different channels are used as input for the Nowcasting Satellite Application Facility (NWCSAF/NWC SAF) algorithm (?) which provides a cloud mask, cloud top height, and cloud classification.

The NWCSAF cloud mask is used for deriving cloud phase, cloud optical depth, and effective radius with the KNMI (Royal Netherlands Meteorological Institute) cloud physical properties (CPP) algorithm (?), developed in the context of the satellite application facility on climate monitoring (CMSAF, ?). To derive To obtain the cloud mask, different multispectral tests using SEVIRI channels are applied in order to discriminate cloudy from cloud-free pixels. The cloud top height for low, liquid clouds is obtained by using a best fit between measured brightness temperatures in the 10.8 μm channel and simulated values using the RTTOV radiative transfer model (?) applied to atmospheric profiles from the ECMWF NWP model (European Centre for Medium-Range Weather Forecasts) numerical weather prediction (NWP) model.

The NWC SAF cloud mask is used in order to derive cloud phase, cloud optical depth, and effective radius with the KNMI (Royal Netherlands Meteorological Institute) cloud physical properties (CPP) algorithm (?), developed in the context of the satellite application facility on climate monitoring (CM SAF, ?). Using a channel in the visible spectrum (0.6 μm) together with an absorbing channel in the near infrared (1.6 μm) (?), the CPP algorithm retrieves cloud optical depth as well as effective radius which are the effective radius representative for the uppermost cloud part. As this method relies on solar channels it works reflectance channels, it is applied only during daytime.

MODIS is Also data from MODIS is used within this study. MODIS is an imaging spectrometer onboard the satellites Terra (descending node) and Aqua (ascending node) which probe the Earth's atmosphere from a polar orbit that results in one daytime overpass per satellite per day over the region of interest. MODIS measures in 36 bands in the visible, near-infrared, and infrared spectrum, with some bands having a spatial resolution of up to 250 m. The cloud physical properties (?) are retrieved in a similar manner as for SEVIRI, but at 1 km spatial resolution using the channels 0.6 μm (band 1, over land) and 2.1 μm (band 7, over land

and sea). In addition, effective radius retrievals are available using the channels at $1.6 \mu\text{m}$ (band 6) and $3.7 \mu\text{m}$ (band 20) together with band 1. Note that band 6 on the Aqua satellite suffers from a stripe-problem (?). In this study MODIS collection 5.1.6 is used for the retrieved cloud optical depth and effective radius.

The ~~ground-ground-based~~ remote sensing instruments of the Leipzig Aerosol and Cloud Remote Observations System (LACROS) comprise a 35-GHz MIRA-35 cloud radar, a HATPRO (Humidity And Temperature PROfiler) microwave radiometer, and a CHM15X ceilometer, which are used also for field campaigns. All instruments are operated in a vertically pointing mode. The raw measurements are processed with the Cloudnet algorithm package (?). The output data is available in an unified temporal resolution of 30 s and a vertical grid of 30 m. Cloudnet uses further information from a NWP model (here: COSMO-DE). In this study we use the attenuation-corrected radar reflectivity from the cloud radar, ~~together with its error estimate~~, the liquid water path obtained from the microwave radiometer, as well as the cloud base and top height retrieved from ceilometer and cloud radar, respectively. The vertical Doppler velocity from the cloud radar is also utilized. Furthermore Cloudnet provides a target classification applying a series of tests to discriminate cloud phase, drizzle or rain, and aerosols or insects.

3.2 ~~Cases~~Data selection

For this study, we ~~focus on four~~ use a 2 year period covering 2012 and 2013. We focus on ideal cases to gain a better understanding of the microphysical processes within the cloud ~~by ruling out side-effects accompanying complicated cloud scenes~~. In order to avoid uncertainties caused by inhomogeneous cloud scenes, such as multi-layer clouds as well as possible. We, we consider single-layer cloud systems which are entirely liquid and non-drizzling as ideal. We chose cases in a way that cloud layers are well-observed by all.

Cloud profiles as observed from the ground are filtered according to the following conditions:

- No occurrence of drizzle/rain in Cloudnets target classification (and no drizzle/rain in the 2 nearest neighbour profiles allowed.)
- Values of LWP are between 25 gm^{-2} and 400 gm^{-2} . The lower limit is due to typical instrument uncertainty of the microwave radiometer and the upper limit due to typical thresholds for drizzle occurrence (?).
- The liquid cloud layer must be situated between 300 m and 4000 m above ground.
- The cloud geometrical depth is between 100 m and 2000 m.
- No ice cloud layer within the first 4000 m above ground is present. Thin ice cloud layers above are excluded

from calculation of cloud geometrical depth. The microwave radiometer is not sensitive to ice, so that the LWP should not be affected.

- No vertical gaps in the cloud layer are present.
- $Z_{\text{max}} < -20 \text{ dBZ}$ within the cloud profile to avoid occurrence of drizzle (?).

The comparison of optical and microphysical properties between ground-based instruments and by and MODIS and SEVIRI. In this study, we present, selected from the LACROS observationsm, two temporally rather homogeneous cases (27 October 2011 observed at Leipzig, and 21 April observed at Krauthausen), and two more inhomogeneous cases (1 June 2012 is only applicable under daytime conditions. Thereby, we have to consider the different spatial and temporal resolution, as well as the different viewing zenith angle on the cloudy scene. For SEVIRI a parallax shift occurs at higher latitudes. The satellite viewing zenith angle for Leipzig is 58.8° . Within this study the average cloud top height is between 1 km and 3 km (see Table 2). This would result in a horizontal displacement of max. 5 km. ? did find a significant difference only for inhomogeneous clouds considering parallax correction. Taking also into account the spatial resolution of SEVIRI over Central Europe of $4 \text{ km} \times 6 \text{ km}$, we decided to neglect the parallax correction for our study, instead we consider surrounding pixels. For SEVIRI a field of 3×3 pixels (case studies), and 5×5 pixels (longer-term statistics) centered on the ground site is used and spatially averaged.

We will furthermore present four hand-selected cases to highlight specific problems more closely. For the four case days, 27 September 2012, both observed at Leipzig). In the following the terms homogeneous and inhomogeneous clouds always refer to the temporal homogeneity unless stated otherwise. For the ± 15 surrounding SEVIRI pixels of the ground observations, we calculate the spatial inhomogeneity parameter following ~~?, ?~~, using the 3×3 SEVIRI pixel field, which can be interpreted also in terms of temporal inhomogeneity (χ) if ~~the frozen turbulence hypothesis is applied~~ advection of clouds over a fixed location is considered:

$$\chi = \frac{\exp(\overline{\ln \tau})}{\overline{\tau}} \frac{\exp(\overline{\ln \tau})}{\overline{\tau}} \quad (11)$$

A short overview of the ~~cloud-layer case~~ characteristics is given in ~~Table table~~ 2. The cloud boundaries are shown along with the cloud radar reflectivity profile in Fig. 1. ~~Although we do not focus on the satellite cloud tops in this study we included these in Fig. 1. While for some time periods a good agreement can be seen, also periods with large discrepancies are found. Differences may~~

result from semitransparent cirrus cloud layers (21 April 2013), inversion layers (27 October 2011) or broken cloud conditions (1 June 2012 and 27 September 2012). In the following we sum up the synoptic conditions for each case.

The cases are as follows. A high pressure system dominates the synoptic weather pattern on 27 October 2011 (Fig. 1a). The temperature at the 850 hPa pressure level over Leipzig is around 5 °C. Therefore the stratocumulus cloud layer that is observed between 10:30 and 13:00 UTC consists entirely of water droplets. Its geometrical depth increases in the beginning of the observation period. The Cloudnet classification indicates a cloud deck even before (not shown); although the radar is not sensitive enough to detect the thin cloud layer between 10:00 and 10:30 UTC.

The weather pattern on 21 April 2013 (Fig. 1b) is quite similar compared to the first case with the high pressure influence being stronger. The temperatures at the 850 hPa pressure level are slightly positive. During the whole observation period at Krauthausen a closed cloud deck is visible. The ground-based observation of ground-obtained cloud top height shows only small variability, while the cloud base is more inhomogeneous during the beginning of the observation period. A thin overlying cirrus cloud deck can be observed around 10:00 UTC and between 11:00 and 12:00 UTC.

00Z. An upper-level ridge covers Central Europe on 1 June 2012 (Fig. 1c), but the area around Leipzig is also influenced by a surface low. Temperatures at 850 hPa lie around 10 °C. The stratocumulus cloud deck with the cloud tops slightly below 2000 m between 12:00 and 16:00 UTC is broken with some cloudy periods in the early afternoon that are not well detected by the cloud radar.

00Z and 14:00Z is broken. The weather pattern for the 27 September 2012 (Fig. 1d) shows Leipzig directly in front of a well pronounced trough. Temperatures at 850 hPa lie again around 10 °C and the cloud types vary between stratocumulus and shallow cumulus. The cloud base height increases throughout the day. After 16:00 UTC also some precipitation can be observed for a short time.

4 Results Cloud adiabaticity

The following investigation is built on the observations from ground (cloud base height from ceilometer, cloud top height and Z from cloud radar, Q_L from the microwave radiometer) and from passive satellites (τ , r_e).

We will first focus on ground-based retrievals and evaluate Entrainment of dry air into the clouds leads to evaporation of cloud water and therefore to a deviation from the adiabatic liquid water content profile. Knowledge of the adiabatic factor f_{ad} , followed by a comparison of is required to calculate key quantities for investigating aerosol-cloud interactions from passive satellite observations. Therefore we first study

cloud adiabaticity, before conducting a intercomparison of ground-based and satellite key properties as well as discuss sources of its uncertainties. The adiabatic factor can be calculated from the ground-based N_d retrieval results using the FI and OE method. Afterwards the key quantities H , N_d , Q_L obtained from satellite observations of SEVIRI and MODIS will be evaluated against the respective ground-based observations. We calculate the cloud droplet number concentration and cloud geometrical depth from the passive satellite-derived τ , r_e , assuming in the first step $f_{ad} = 1$ and in a second step the f_{ad} calculated from the ground-based will further investigate possibilities to estimate it from passive satellite observations.

4.1 Retrieval of cloud properties Adiabatic factor from ground-based observations

4.1.1 Cloud adiabatic factor

Entrainment of dry air leads to deviations from the linearly increasing q_L profile. The cloud adiabatic factor as calculated from Eq. (2) The ground-based adiabatic factor (f_{ad}) is calculated using Q_L from the microwave radiometer and H_{obs}^{ground} can quantify such deviations, H as the difference of cloud top height from the cloud radar and cloud base height from the ceilometer, and $\Gamma_{ad}(T_{cbh}, p_{cbh})$ using NWP data in Eq. (2).

The time series of the adiabatic factor calculated for the two homogeneous cases is shown in Fig. 2a,b. The adiabatic factor at 27 October 2011 lies in the range from 0.4 to 0.9. Short time periods with $f_{ad} > 1$ occur. These points are likely to be artefacts? suggests a range of typical values of [0.3, 0.9]. We omitted adiabatic factors with $f_{ad} > 1.0$ since those are most likely affected by the measurement uncertainties, since the occurrence of “super-adiabatic” cloud profiles in nature is physically implausible. Such artefacts may easily especially arise due to uncertainties in Q_L and $H_{cloud} - H$ for thin clouds. In contrast to the original Cloudnet code, our calculation of the adiabatic factor allows for values greater than one $f_{ad} > 1.0$. Within Cloudnet “superadiabatic” profiles are avoided by increasing the cloud top height if the adiabatic integrated integrated adiabatic q_L is smaller than Q_L measured by the microwave radiometer. We omitted adiabatic factors with $f_{ad} > 1.0$ since we believe that those are most likely affected by the measurement uncertainties. This can be seen when considering the uncertainties that influence the adiabatic factor. For example, consider a cloud with $Q_L = 100$ and $H_{obs}^{ground} = 324$ that is adiabatic ($f_{ad} = 1$). The Q_L retrieval uncertainty (microwave radiometer instrument error + retrieval error) is approximately 20 and the H_{obs}^{ground} uncertainty of the ceilometer and the cloud radar is at least ± 60 due to the vertical resolution. Accounting for the maximum uncertainty ($Q_L = 120$, and $H_{obs}^{ground} = 64$) or ($Q_L = 80$ and $H_{obs}^{ground} = 384$), the resulting adiabatic factor

would be 1.81 or 0.57, respectively. This shows that with the current uncertainty limits of the ground-based observations the adiabatic factor is still prone to large uncertainties especially for geometrically thin clouds.

For cross-checking with an independent approach, we also calculate the adiabatic factor using the information of the radar reflectivity profile. We see an example time-series for one case (21 April 2013) is shown in Fig. 7 that the mean adiabatic factor calculated from the radar profiles is generally a bit lower, and that the correlation for all four cases is quite good with 62 to 95, and root mean square differences between 0.14 and 0.24. This difference is likely explained by uncertainties in $H_{\text{obs}}^{\text{ground}}$ and Q_L , but also in Z obtained from the cloud radar and the retrieved N_d . In the following we will use the adiabatic factor calculated from Q_L and $H_{\text{obs}}^{\text{ground}}$.

On 21 April 2013 we 2 (see the supplements for more cases). For this we find values of the adiabatic factor f_{ad} between 0.2 and 0.6 before 09:00 UTC. The radar reflectivity measurements (Fig. 1b) reveal that the cloud base is more inhomogeneous during this time period than later on. After 09:00 UTC the adiabatic factor oscillates varies between 0.5 and 1.0. Overall, the adiabatic factor also found for the other homogeneous case agrees-

From Fig. 3a we find a mean of $f_{\text{ad}} = 0.63$ and the IQR as [0.46, 0.81] for the entire dataset covering 2012 and 2013. This corresponds well with the range of values of [0.3, 0.9] suggested typical value of 0.6 given by ?.

For the two inhomogeneous cases, Overall, there is a large spread of values covering the full physical meaningful range from 0 to 1 (mean values for individual cases as presented in Fig. 1 are listed in Table 4). The adiabatic factor is not only changing from case to case, but also varying with time for individual days, reflecting the natural variability of entrainment processes. The variability of the variability of the adiabatic factor (Fig. 2c,d) is larger is larger for the inhomogeneous cases than for the homogeneous cases considered before ones (Table 4), but the range of values is similar. This shows that independent from temporal cloud homogeneity the majority of clouds seems to be sub-adiabatic. Therefore considering a constant adiabatic factor like in previous studies (Table 1) is problematic.

Figure 7 reveals When looking for proxies for the adiabatic factor, we find a tendency that geometrically thicker clouds are less adiabatic, while mainly the thin clouds ($H_{\text{obs}}^{\text{ground}} < 400 \text{ m}$) are responsible for the cloud profiles. This (Figure 3b). Already ? found a decrease in the adiabatic factor with height. It also supports the findings of ?, who observed the tendency that thicker clouds are less adiabatic in the Southeast Pacific. The investigation of Mainly the thin clouds ($H < 400 \text{ m}$) result in $f_{\text{ad}} > 1$, as also found by ?, and therefore the investigation of such thin clouds remains challenging. We therefore neglect cloud profiles with $f_{\text{ad}} > 1$ in the following.

? used observations of two cases with temporally homogeneous stratocumulus clouds over Leipzig, Germany, and

found that in case of occurrence of updrafts in clouds, the q_L profile is tends to be more adiabatic. To investigate if such a behaviour also occurs for our cases we apply the cloud radar Doppler velocity at the cloud base. The average vertical velocity of each cloud profile is found at at cloud base for all samples in 2012 and 2013 is found to be -0.1 ms^{-1} with the majority of points (93%) in the range $[-1, 1] \text{ ms}^{-1}$. Considering this the vertical velocity as function of cloud adiabacity (Fig 3c) we find a large spread, which makes it difficult to detect a clear dependence of cloud adiabacity on updraft speed. However if we distinct influence of updraft speed on cloud adiabacity. However, the notch around the median in the box-whisker-plot does not overlap for updraft and downdraft regimes. According to ? the median can be judged to differ significantly on the 95% confidence interval if there is no overlay in the notches. We further calculate the median adiabatic factor for the updraft and downdraft regimes individually, we find for each of our for the four selected cases, and find for three out of four cases that clouds are slightly more adiabatic in the updraft regime (Table 4). This behaviour is expected from adiabaticity and also supported by the findings of ?. They report that this effect is strongest at the cloud base and blurs when the data points are averaged over the whole cloud profile.

4.1.1 Cloud droplet number concentration from radar-radiometer retrievals

4.2 Adiabatic factor from satellite observations

N_d is used as the main parameter in many investigations of the first indirect aerosol effect. Advances have been made over the last two decades to apply retrievals for N_d combining ground-based cloud radar and microwave radiometer. We applied such a method following ? (hereafter: FI, see Sect. ??). Furthermore we compare those results with the newly developed Optimal Estimation approach (see Sect. ??). From ground-based observations we can show that the adiabatic factor is highly variable even for one location. Therefore we can also expect strong variability for cloud regimes over different regions observed by satellite (e.g. maritime vs. continental). To obtain ACI key quantities from passive satellite observations the adiabatic factor is required over a larger domain. The DWD operates a ceilometer network in Germany (?) which can be used to obtain the cloud base height (CBH). The question remains if Q_L and CTH from SEVIRI are accurate enough to allow for an estimate of the adiabatic factor using Eq. 2. To adress this question, we contrast Q_L and CTH obtained from SEVIRI with LACROS.

Contrasting the N_d from OE with the FI method, we find that the absolute mean difference of N_d^{OE} . We investigate liquid clouds in a two-year period covering 2012 and N_d^{FI} considering all cases is 1642013. Since the estimate of the adiabatic factor from passive satellite observations is

expected to be applied over a larger domain, it should be independent from ground-based information. Therefore the sampling is now done in terms of satellite observed quantities. An area of 5×5 pixels (total of 25 pixels) centered at the location of LACROS is considered for each available SEVIRI observation. For this pixel field we obtain average, standard deviation of CTH and the liquid cloud fraction. The liquid fraction is determined by the cloud type classification for each pixel from CPP. We require 90% of the pixel field (23 out of 25 pixels) to be classified as pure liquid clouds. As additional constraint, the standard deviation of CTH for the 25 pixels has to be smaller than 400 (19). Overall, the FI method tends to yield lower values than the OE method, even though some outliers with unreasonably large values can be found ($N_d^{OE} > 2000 \text{ cm}^{-3}$). In contrast to the FI method the OE method is also able to give information about the remaining uncertainty by considering measurement uncertainties as well as the uncertainty of the background state. With a quite large background uncertainty assumed to be 300, we can see that the information (measurement and uncertainties) from the ground observation is able to reduce the final analysis error for N_d , but more constraints are required to obtain N_d with even higher accuracy. This would be desirable to better evaluate satellite observations. For LACROS we use the observation averaged using a window of 10 minutes around the SEVIRI observation time. No requirements regarding the cloud phase are made for LACROS.

4.3 Comparison of cloud properties from satellite and ground

Cloud microphysical retrievals that are based on either satellite or ground-based remote sensing both have their advantages and shortcomings. However, when the results of both approaches are in agreement, it is likely that the corresponding cloud layers are well suited for the investigation of key factors determining the first indirect effect.

By comparing ground-based and satellite observations, we have to consider the different spatial and temporal resolution, different error sources of the instruments as well as the different viewing zenith angle on the cloudy scene. For SEVIRI we have to consider a parallax shift at higher latitudes. The satellite viewing zenith angle for Leipzig is 58.8° . Instruments give the actual geometrical CTH while from passive satellites a radiative CTH is obtained. Ignoring this physical difference we can see that the SEVIRI CTH is positively biased (Fig. 4a). ? reports a very similar overestimation (320 m) with a large standard deviation of 1030 m for low, opaque clouds. Considering the central pixel of the field does not change the result significantly, showing that the cloud fields are rather homogeneous and should therefore be suitable for such a comparison. The observed

bias is not explained by the limited vertical step size of 200 m in the SEVIRI CTH product. A likely explanation of this bias is found in the representation of inversions. Splitting the sample by model inversions did not provide significantly better results, but the actual inversions might not be well represented by the model. Such a case can be seen for 27 October 2011. There, the CTH is roughly 1000 m lower than for the other 3 (see Table 2). This would result in a horizontal displacement of max. 5. cases presented here, but the retrieved satellite CTH lies at 2000 m. Considering the spatial resolution of SEVIRI over Central Europe of 4 closest radiosounding of Lindenberg (Germany), we find two inversion layers on top of each other between $900 \times 6 \text{ m}$ and $3000 \times 6 \text{ m}$, we decided to neglect the parallax correction for our study. To address the uncertainty of the satellite observations from SEVIRI and also MODIS we calculated the standard deviation of the surrounding pixels. For SEVIRI ± 1 pixel around the m, which results in ambiguities in finding the correct cloud height. Differences may also result from semitransparent cirrus cloud layers (21 April 2013), or broken cloud conditions (1 June 2012 and 27 September 2012).

For the comparison of Q_L we impose the condition that the values are between 20 gm^{-2} and 400 gm^{-2} . The comparison can only be applied during daytime. Both requirements reduce the number of samples by 56% compared to the CTH sample. The difference of Q_L has a distribution with a distinct peak close to zero (Fig. 4c). There is a small negative bias of -21 gm^{-2} , which is within the uncertainty range of the ground-based measurements, not even considering the uncertainty of the satellite-based estimate. Similar to the CTH comparison we see that the distribution of the central pixel is added, resulting in a field of 9 satellite pixels. To cover a comparable area for MODIS, we add ± 9 pixel around the central pixel. For the comparison of the time series obtained from space and ground we applied data averaging only if mentioned. As pointed out in the following discussion for inhomogeneous scenes, omitting temporal averaging not significantly different from the field average, although the spread is larger. The distribution and the standard deviation are consistent with the observations in the validation study of ? for the Cloudnet stations of Chilbolton and Palaiseau. Similar to their study we see a slight negative skewness, which stems from larger Q_L values seen from the ground-based MWR. ? also reported that accuracy is reduced for higher Q_L values. Further possible explanations for differences in Q_L observed from ground and SEVIRI can be found in remaining cloud inhomogeneities and sampling differences. Generally, unfavorable viewing angles that occur especially in winter conditions can lead to considerable differences of ground and satellite quantities.

4.2.1 Liquid water path

Considering the uncertainty of $20 \text{ in } Q_L$ for the ground-based microwave radiometer, large uncertainties in the satellite retrieval. In our sample the majority of the absolute mean difference cases occur in summer months (April to September, 80%). Looking at specific case days, we find the mean difference of Q_L for two homogeneous cases between SEVIRI and the ground-based MWR is in good agreement. We find mean differences (relative mean difference) of H in reasonable agreement ($8 \text{ (} 14 \text{ gm}^{-2} \text{ (} 10\% \text{))}$ for 21 -April 2013, $1625 \text{ (} 28 \text{ gm}^{-2} \text{ (} 32\% \text{))}$ for 27 -October 2011, 27), while there are larger differences for two inhomogeneous cases ($50 \text{ (} 62 \text{ gm}^{-2} \text{ (} 87\% \text{))}$ for 1 -June 2012 and $2233 \text{ (} 42 \text{ gm}^{-2} \text{ (} 80\% \text{))}$ for 27 -September 2012, September 2012).

On 27 -October 2011 we find larger differences in a similar study by ? found a standard deviation of 369 m between satellite-based adiabatic CBH and ceilometer CBH. They applied CTH and Q_L mainly after 12:00 from AVHRR (Advanced Very High Resolution Radiometer) and assumed adiabatic clouds to compare the spatially and temporally averaged satellite product. The same comparison between SEVIRI and radiosonde observations resulted in a standard deviation of $\pm 290 \text{ UTC with up to m (?)}$. They suggest that this method can be applied for convective clouds in their early growth stage, which are located near the condensation level. Their sample is focused on relatively thin water clouds ($\sim 250 \text{ m}$), which are more likely close to adiabaticity according to our Fig. 3b. As we will discuss in the following the adiabatic factor for such thin clouds is very sensitive to errors in cloud geometrical depth, so that an instantaneous retrieval of the adiabatic factor is not feasible.

4.3 Uncertainty estimate of the adiabatic factor

To investigate the uncertainties that influence the calculation of the adiabatic factor, we consider an adiabatic cloud ($f_{ad} = 1$) with $Q_L = 100$ (Fig. 3). Although rain might be a possible explanation for higher gm^{-2} and $H = 324 \text{ m}$ and $\Gamma_{ad} = 1.9 \cdot 10^{-3} \text{ gm}^{-4}$. The Q_L observed with the ground-based microwave radiometer, there are no signs for precipitation in both radar signal and satellite observations. The effective radius observed from satellite near cloud top lies clearly below the value of 14 retrieval uncertainty (microwave radiometer instrument error + retrieval error) is approximately 25 which was suggested by ? as the threshold for drizzle/rain forming clouds. The maximum of the radar reflectivity in each profile did also not 20 gm^{-2} and the vertical resolution of the ceilometer and the cloud radar results in at least ± 60 , which is commonly taken as a drizzle threshold (??). The observed difference might well be attributed to the satellite retrieved Q_L . For the same time period we also find disagreement in N_d from SEVIRI and ground and will discuss possible reasons in this context later.

For the inhomogeneous cases, the Q_L obtained from the ground-based microwave radiometer is highly variable.

Especially the Cloudnet observations on 27 -September 2012 show rapid changes of Q_L with peaks around 400m uncertainty of H . Accounting for the maximum uncertainty ($Q_L = 125$ and cloud-free periods. The SEVIRI temporal pattern is more smooth, because the satellite signal represents an average over different sub-pixel clouds within the field of view due to gm^{-2} , and $H_{\text{obs}}^{\text{ground}} = 264 \text{ m}$) or ($Q_L = 75 \text{ gm}^{-2}$ and $H_{\text{obs}}^{\text{ground}} = 384 \text{ m}$), the resulting adiabatic factor would be 1.89 or 0.54, respectively. This shows that with the current uncertainty limits of the ground-based observations the adiabatic factor is still prone to large uncertainties especially for geometrically thin clouds.

If we consider the root mean square differences (RMSD) of the comparison of ground and satellite-based values with $\Delta Q_L = 67 \text{ gm}^{-2}$ and $\Delta \text{CTH} = 1174 \text{ m}$, we can clearly see that especially the observed bias in CTH can result in large uncertainties of an instantaneous estimate of the adiabatic factor especially for thin clouds. For the lower spatial resolution adiabatic cloud considered above, this RMSDs result in a relative uncertainty for the adiabatic factor of 727%, neglecting uncertainties at the CBH. Even considering a cloud that is twice thick, the relative uncertainty is still 362%. This shows that subsampling the SEVIRI observations to homogeneous, liquid clouds does still show differences when compared to a ground-based reference that are too large to estimate the adiabatic factor with sufficient reliability, mainly due to uncertainties in the CTH product. With this approach using Q_L and H we cannot determine the adiabaticity of clouds with a reasonable accuracy. Therefore we will have a look on the microphysical quantities.

5 Microphysical key quantities for aerosol-cloud interactions

The cloud geometrical depth H and cloud droplet number concentration N_d are used as the main parameters in many investigations of aerosol cloud interactions (and therefore the first indirect aerosol effect) as both cloud properties have a direct effect on cloud albedo. Due to the required assumptions about the droplet size distribution a retrieval of cloud droplet number concentration from a radar-radiometer approach remains highly uncertain. ? follows an alternative approach to retrieve τ instead of N_d and demonstrated it to be less sensitive to the assumption of the width of the droplet size distribution.

5.0.1 Cloud geometrical depth

In the following, we will cross-check key quantities H and τ from ground and satellite. We will also discuss the effect of uncertainties in our observations for the sub-adiabatic cloud model on N_d , τ and H .

Contrasting $H_{\text{ad}}^{\text{SEVIRI}}$ with the $H_{\text{obs}}^{\text{ground}}$ (Fig. 5)

5.1 Cloud geometrical depth H intercomparison from space and ground

Contrasting SEVIRI H (eq. 3, using f_{ad} from ground-based observations) with the LACROS H , we are able to investigate the same quantity obtained with two independent physical retrieval approaches.

The correlation coefficient is 0.47–0.89 for 21 April 2013 after 09:00 UTC, 0.59, 0.70 for 27 October 2011, 0.41–0.38 for 1 June 2012, and 0.12–0.45 for 27 September 2012. The correlation increases when temporally averaging is applied (2012 and increases by 10%, 39%, 118% and 71% for 30 min temporal averaging, respectively (see Table 5).

The improvement of correlation is not surprising when comparing averaged data as also pointed out in other studies (e.g. ??). However, a longer averaging period could remove the original variability of the data. The correlations for temporally averaged data are within the range of values that were obtained by ?, ? and ?. ? found correlations of 0.71 between SEVIRI and Cloudnet for a homogeneous stratocumulus cloud layer. ? found correlations of 0.62 between in-situ and MODIS retrieved H , and could show a better agreement of $H-H$ when the adiabatic factor is explicitly calculated and considered. ? found correlations of 0.54 (0.7 for $H < 400\text{m}$ with cloud fraction $> 90\%$) comparing radiosonde-derived cloud geometrical depth to respective MODIS observations. In their study ? reported that satellite values were higher compared to the ground-based ones. The reason for this can potentially be explained by a bias of MODIS-retrieved r_e but also in the choice of the adiabatic factor in the retrieval of H (Eq. ??). Satellite-derived H increases if we choose $f_{ad} < 1$ instead of $f_{ad} = 1$.

If the adiabatic factor obtained from ground is applied to Eq. ?? instead of $f_{ad} = 1$, we find that the mean difference (relative mean difference) for the two homogeneous cases reduces from 87(31) to 45(16).

5.2 Cloud optical depth τ intercomparison from space and ground

The intercomparison of SEVIRI with LACROS retrieved τ results in differences of 2.3 (8%) for 21 April 2013, 3.6 (21%) for 27 October 2011, and from 87(23) to 14(9.3 (76%) for 1 June 2012 and 8.0 (61%) for 27 September 2012. The higher resolution of the ground-based observations leads to larger variability also for the homogeneous cases. The median conditions result in a good fit to the satellite (τ, Q_L)-pairs (Fig. 5) for the homogeneous case on 21 April 2013. The same holds true for this case the satellite pairs are also within the ground-based temporal interquartile range (IQR). The situation is similar even for the inhomogeneous case at on 1 June 2012. The situation turns out to be more complicated when looking at the inhomogeneous case on 27 September 2012. Overall satellite τ and Q_L

show lower values, which result likely due to broken-cloud effects in the SEVIRI retrieval. For broken clouds within the SEVIRI pixel the satellite receives a combined signal from the clouds but also from the surface. Such moving, broken cloud fields result in a smoother temporal pattern from the satellite perspective. From the time-height radar reflectivity cross-section on 27 September 2012 with a reduction from 149(47) to (29), but not for between 11:00 UTC and 15:00 UTC a larger number of cloud gaps can be seen, which could explain why the subpixel surface contamination plays a larger role than on 1 June 2012. The Cloudnet observations on 27 September 2012 where the mean difference increases from 86 show rapid changes of Q_L with peaks around 400 (24gm^{-2}) and cloud free periods. The observed larger deviations of SEVIRI found on 27 October 2011 are likely due to low values (< 5) to 216 ($60\mu\text{m}$) of effective radius in the KNMI-CPP retrieval. These are likely a result of the unfavourable viewing conditions with a large solar zenith angle ($> 60^\circ$) under relative azimuth angles close to 180° around noon for this case, for which ? pointed out the low precision of the retrieval. These values are filtered out following ?, but the remaining points might likely also be affected by the same issue.

To highlight the importance of considering the actual adiabatic factor for the retrieval process, we calculated the optical depth (Eq. 7) from the ground-based observations following the radar-radiometer approach with an adiabatic factor $f_{ad} = 1$ and with the ground-obtained adiabatic factor. Afterwards we compare it to the satellite-retrieved values. Applying $f_{ad} = 1$ the mean difference in optical depth is increased from 2.3 to 8.5 on 21 April 2013, and is also higher for the other cases (see Table 6).

For the cases investigated here, we saw a better agreement in H for available MODIS retrievals compared to SEVIRI if $f_{ad} = 1$ is chosen. Indeed, clouds are actually sub-adiabatic while the retrieval assumes adiabatic clouds. This could counteract a high bias in MODIS r_e that is reported in previous studies (?). For the four cases considered in this study, the distribution of differences between SEVIRI and ground-based τ for the 2012 and 2013 sample of low-level, homogeneous, liquid clouds is presented in Fig. 4b. As for Q_L there is a distinct peak around zero with negligible bias, but a considerable standard deviation of 16. This shows that on average the agreement between satellite and ground-based τ is reasonable, considering the number of collocated observations with MODIS is not sufficient in order to determine which effect is predominant for the bias. Therefore a larger dataset would be desirable for a more in-depth investigation uncertainties in the retrieval as well as uncertainties due to parallax, collocation and spatial resolution. Those uncertainties will be discussed in more detail in the following sections.

5.2.1 Cloud droplet number concentration

5.3 Ground-based uncertainties

The retrieval of radar-radiometer retrieval depends upon the observations of Q_L , H and $Z(z)$. Also the choice of the mixing model is able to change the retrieved quantities, but ? comes to the conclusion that this effect is small. N_d from passive satellite observations relies on the (sub-)adiabatic cloud model. In the following we contrast depends further on k_6 , which only depends on the width of the droplet size distribution (see Eq. 10 in the Appendix).

We take two typical cloud profiles from our observations. For those cloud profiles we evaluate the sensitivity of the retrieved N_d retrieved from ground with the OE method and the adiabatic ($f_{ad} = 1$) retrieved values from MODIS and SEVIRI. The retrieved N_d are shown in Fig. 7. At 21 April 2013 the values agree within the uncertainty range with a mean difference (relative mean difference) of 29(10%) between SEVIRI and OE retrievals for the whole time period. In Table 3 we list the sensitivities to each input parameter when the other parameters are kept constant.

For 27 September 2012 and 1 June 2012 we find mean differences (relative mean differences) of (7%) and 103% $Z(z)$ we follow ? and assume an uncertainty range of ± 2 (43dBZ), which would represent a calibration bias constant with height. ? points out the strong influence of drizzle on the cloud reflectivity. Errors of 30-60% , respectively. At 27 October 2011 we find larger differences between SEVIRI and the ground-based N_d . At the beginning of the observation period (before 10:30) have to be anticipated for q_L profile retrievals. Those retrieval approaches are based on very similar principles as the radar-radiometer retrieval method (?). In our study we filtered out drizzling profiles as well as possible. For the four case days the effective radius observed from satellite near cloud top lies clearly below the value of 14 UTC) the N_d^{SEVIRI} values are much lower than the N_d^{OE} ones. After 10:30 μm which was suggested by ? as the threshold for drizzle/rain forming clouds. The maximum of the radar reflectivity in each profile also did not exceed -20 UTC N_d^{SEVIRI} shows twice as large values as N_d^{OE} , resulting in a mean difference of 488dBZ, which is commonly taken as a drizzle threshold (?). We cannot totally rule out the possibility that few larger droplets were present, for which the radar reflectivity is very sensitive. For the uncertainty of H , we assume ± 60 (154m. For Q_L we assume a typical uncertainty of ± 25) for the whole day.

To find explanations for the large deviations found on 27 October 2011, we calculated optical depth and effective radius from N_d^{OE} and H_{obs}^{ground} , respectively, using the adiabatic model (Eq. (??) and Eq. gm^{-2} given microwave radiometer observations. The width of the droplet size distribution for continental clouds exhibits a large spread of values in literature as can be seen in ?. If we

consider the maximum range of observations, the effective variance ν of the gamma size distribution could take values between 0.2 up to 0.043 (?). By comparing these to the satellite-retrieved values we are able to attribute the observed differences mainly to differences in effective radius, for which SEVIRI gives lower values (Fig. 3c). Before 10:30UTC the mean difference in the effective radius is 2.5 compared to 3.4 afterwards. $k_2 = 0.48$ and $k_2 = 0.87$, respectively). For the standard retrieval we assume $\nu = 0.1$ ($k_2 = 0.72$).

N_d is most sensitive to the assumption about the width of the droplet size distribution, especially to changes in the range of smaller values of the effective variance. This can be understood as $N \propto k_6$ and k_6 is a monotonically decreasing function of the effective variance. For higher values of ν the other uncertainty contributions are equally or even more important. Since the real droplet size distribution is usually unknown, it is difficult to estimate the actual uncertainty when assuming $\nu = 0.1$. From our cases we find that the uncertainty in Q_L differences (Fig. 3a) can be attributed mainly to optical depth differences (Fig. 3b), which follows the same temporal pattern. Comparing the two satellite observations of might be more important than the uncertainty in radar reflectivity. Both can result in more than 50% relative uncertainty for the retrieval of N_d .

As can be seen from Eq. 7, the same cloud scene in the area of around \pm optical depth τ is sensitive to the same input parameters as N_d , but also depends on f_{ad} . Therein the combined uncertainty of Q_L and H is reflected. From Table 3 we find that τ is most sensitive to uncertainties in Q_L , especially for observed low values of Q_L . In contrast to N_d it is not as sensitive to the assumption about the width of the droplet size distribution. While for N_d the uncertainties in the low-range of ν is above 100%, it is below 20% for τ . Since the natural variability of droplet size distributions is large and difficult to constrain without in-situ observations, τ turns out to be a more stable quantity for contrasting to other observation, as already suggested by ?.

In Fig. 5 we present the uncertainty of τ as a function of Q_L , based on the median observations from the ground-based time-series. We use a representative average of N_d over the whole time-period and investigate the effect of its temporal variability on the retrieved τ . ? used a climatological mean value for N_d in order to retrieve r_e and reported an average N_d of $212 \pm 107 cm^{-3}$ at the Southern Great Plains site for continental clouds, which is similar to the median value found for our example cases in Fig. 5. We see that assuming a 50% uncertainty for both, N_d and τ , results in an increasing uncertainty of τ with Q_L , with the uncertainty due to ΔN_d being slightly larger, although Δf_{ad} cannot be neglected.

5.4 Satellite uncertainties

5.4.1 Uncertainties of cloud droplet number concentration and cloud geometrical depth

Since cloud droplet number concentration N_d is obtained with the sub-adiabatic model using Eq. 9, it depends on the uncertainties of τ and r_e , but also on f_{ad} , k_2 and Γ_{ad} .

? reported a 150 around Leipzig (not shown), we find spatial inhomogeneities of cloud microphysics that can not be resolved in the same way by SEVIRI as it is possible for MODIS. Furthermore SEVIRI has to deal with a large solar zenith angle ($> 60^\circ$) under relative azimuth angles, close to 180° around noon, for which ? pointed out the lower precision of the retrieval cm^{-3} error for optically thick clouds ($\tau > 20$) resulting from a 10% error in τ . The absolute error of N_d increases with increasing τ assuming a constant error in r_e . N_d is also very uncertain for values of $r_e < 8 \mu m$. ? found that cases with $r_e < 5 \mu m$ are rare compared to typical value of $10 \mu m$ for liquid clouds. ? argue that those should not be considered due to the large uncertainty.

Another influencing factor is the difference of the effective radius retrieval due to the different channels used by MODIS (if the individual errors are assumed to be normally distributed, the relative errors of N_d and H are given by:

$$\left(\frac{\Delta N_d}{N_d}\right)^2 = \left(\frac{\Delta k_2}{k_2}\right)^2 + \left(\frac{\Delta \Gamma_{ad}}{2\Gamma_{ad}}\right)^2 + \left(\frac{\Delta f_{ad}}{2f_{ad}}\right)^2 + \left(\frac{\Delta \tau}{2\tau}\right)^2 + \left(\frac{5\Delta r_e}{2r_e}\right)^2 \quad (12)$$

and

$$\left(\frac{\Delta H}{H}\right)^2 = \left(\frac{\Delta \Gamma_{ad}}{2\Gamma_{ad}}\right)^2 + \left(\frac{\Delta f_{ad}}{2f_{ad}}\right)^2 + \left(\frac{\Delta \tau}{2\tau}\right)^2 + \left(\frac{\Delta r_e}{2r_e}\right)^2 \quad (13)$$

Uncertainties of τ and r_e stem from the assumption of plane-parallel vertical-uniform cloud layers, partially covered cloud pixels (?), 3D effects (?), and large solar zenith angles (?). Uncertainties of effective radius further arise from its vertical profile. The use of different channels results in discrepancies in r_e . MODIS uses a channel centered at $2.1 \mu m$ and SEVIRI (, while SEVIRI uses $1.6 \mu m$) for the standard retrieval products. From MODIS, additional effective radius retrievals from channels at $1.6 \mu m$

and $3.7 \mu m$ are available. Theoretically, the $3.7 \mu m$ channel should represent the effective radius closer to the cloud top for adiabatic clouds, while the $2.1 \mu m$ and $1.6 \mu m$ channels receive the main signal from deeper layers within the cloud. Cloud observations do not always show an increase of effective radius from channel $1.6 \mu m$ over $2.1 \mu m$ to $3.7 \mu m$ as is expected for plane-parallel, adiabatic clouds (?). Comparing mean differences of effective radius from SEVIRI and each of the three available MODIS channels, we find the smallest difference in In this study we estimate the uncertainties in passive satellite τ and r_e considering the MODIS channel at 1.6 . The mean difference in this case is 0.86 . This is not surprising as both channels cover more or less the same wavelength range. The difference increases when MODIS channels 2.1 and 3.7 are used. Intercomparing the effective radii retrieved from the three MODIS channels results in slightly smaller differences. The difference of MODIS channels at 2.1 and 1.6 is 0.68 , while the difference of the retrieval at MODIS channels at 2.1 and at 3.7 is 0.51 , with 10% following ? (SEVIRI) and following ? (MODIS), although uncertainties are probably larger for unfavourable conditions (large solar zenith angles, broken clouds).

Due to the $N \propto r_e^{-2.5}$ relationship (see Eq. 9) even small differences of effective radius result in large uncertainties of N_d . Explicitly considering this error propagation, we find for 27 October 2011 at 11:45 UTC that the observed difference in effective radius of 1.33 between MODIS and SEVIRI results in an uncertainty of 306 . The uncertainty due to differences in effective radius of 0.34 between MODIS channels 2.1 and 1.6 is 57 . For the adiabatic factor we assume a relative error of 35% considering a constant adiabatic factor (0.6) and its variability (0.22) as obtained from two-year LACROS observations. For comparison ? assumed an uncertainty in the adiabatic factor of 0.3 . This resulted in a numerically evaluated error of around 26% considering typical values of effective radius and optical depth.

The importance of r_e for the retrieval of N_d from passive satellite imagers has already been pointed out by previous studies. Those which were mainly based on MODIS (????). ? report a high bias of MODIS-derived r_e , but also state that the choice of the other parameters in the retrieval (namely k_2 , Γ_{ad}) is able to compensate for this effect so that still a good agreement between MODIS retrieved and in-situ values could be achieved. A high bias of r_e occurs for broken cloud conditions (?). ? also saw a good agreement for MODIS derived N_d (using $f_{ad} = 0.8$) with CALIOP (Cloud Aerosol Lidar with Orthogonal Polarization), although they found a high bias in r_e compared to POLDER (Polarization and Directionality of the Earth Reflectance). ? also points out the importance of the effective radius for the N_d retrieval. As mentioned before, for our study only few MODIS observation points are available, but we already see that discrepancies in r_e in comparison to SEVIRI are a major source of uncertainty. ? estimated the uncertainty of k_2 to be negligible (around 3%) for $N_d \cdot N_d < 100 cm^{-3}$, following ? . ? used a

variability of $k_2 = 0.8 \pm 0.1$ in a global study, which results in a relative uncertainty of 12.5%. ? found a similar mean value for 33 cases of stratocumulus and cumulus clouds with a even smaller variability, even slightly lower than the variability in ?. Therefore 12.5% might be seen as an upper uncertainty limit for k_2 .

? also state for satellite retrievals of N_d (and also H_{ad}) that f_{ad} and Γ_{ad} are the most important uncertainty factors. They estimated the uncertainty of k to be negligible (around 3%). By considering the whole seasonal variability of cloud base temperature, they ? obtained an error of 24% for the adiabatic lapse rate of liquid water mixing ratio ($\Gamma_{ad}(T, p)$). In our study Γ_{ad} has a smaller contribution to those uncertainties due to the fact that we are using model data to gain more reliable information about cloud base temperature and pressure instead of considering one constant value like a constant value of Γ_{ad} as in e.g. ?. If we compare Γ_{ad} calculated from satellite cloud top temperature and pressure with the one calculated from cloud base values observed from ground-ground we find an uncertainty of 15% considering all 4 cases the four case days. As we see some deviations in the cloud top height, we believe that this uncertainty can be mainly attributed to wrong incorrect satellite estimates of cloud top temperature and pressure. ? further assumed an uncertainty in the adiabatic factor of 0.3. This resulted in a numerically evaluated error of around 26% considering typical values of effective radius and optical depth. To highlight the importance of considering the actual adiabatic factor for the retrieval process, we calculated the optical depth (Eq. (??))

? state for satellite retrievals of N_d (and also H_{ad}) that f_{ad} and effective radius (Eq. (??)) from the ground-based observations using N_d^{OE} and H_{obs}^{ground} with adiabatic factor $f_{ad} = 1$ or the ground-obtained adiabatic factor. Afterwards we compare it to the satellite-retrieved values obtained with the CPP algorithm. When the adiabatic factor is assumed constant of $f_{ad} = 1$ the mean difference in optical depth is 9.95 on 21 April 2013. When the adiabatic factor obtained from the ground-based measurements is considered, this mean difference is drastically reduced to 2.90. The mean difference of effective radius is reduced from 1.15 to 0.12.

Therefore, we aim to adjust N_d^{SEVIRI} Eq. 9 for the homogeneous cases by setting the adiabatic factor to the value obtained from the ground-based observation. The results can be seen in Fig. ?. On 2013-04-21 the adjusted N_d^{SEVIRI} is generally slightly lower due to the observed sub-adiabaticity. Only before 09:00UTC the adjustments lead to a better comparison to ground-obtained values. This case still shows the smallest relative mean difference of SEVIRI and ground-retrieved Γ_{ad} are the most important uncertainty factors. Considering our uncertainty estimates, the largest contribution to the uncertainty of N_d is given by the relative uncertainty of effective radius (25%), followed

by f_{ad} (18%), k_2 (12.5%), Γ_{ad} (7.5%) and τ (5%). Considering the error propagation of H , assuming the same errors as for N_d , we find the largest uncertainty due to the adiabatic factor with 15. For 27 October 2011 the retrieved N_d^{SEVIRI} is also generally reduced, diminishing also the mean difference to the ground-retrieved values in this case (relative mean difference is reduced from 154 to 114). The reason that including the adiabatic factor does not always lead to a better agreement can likely be attributed to the uncertainties of ground observations (discussed in Sect. ??). Although we were not able to see always an improvement in agreement of 17.5%, followed by Γ_{ad} (7.5%) and τ (5%) and r_e (5%).

The importance of r_e for the retrieval of N_d from passive satellite imagers has already been pointed out by previous studies. Those were mainly based on observations from MODIS (???) and report a high bias of MODIS r_e , especially for broken clouds (?). ? also state that the choice of the other parameters in the retrieval (namely k_2 , Γ_{ad}) is able to compensate for this effect so that still a good agreement between MODIS retrieved and in-situ values could be achieved. As mentioned before, for our study we focused on the intercomparison of τ instead of N_d by considering, since the ground-based calculated f_{ad} , ? found a better agreement in N_d when considering it in their study. Since clouds are clearly sub-adiabatic in all our 4 cases independent of season, we believe that applying an adiabatic factor smaller than one is advantageous over considering adiabatic clouds in the retrieval retrieval of τ is less sensitive to the required assumptions.

For the inhomogeneous cases shown in Fig. 7c, d, a high temporal variability in N_d^{OE} can be seen. N_d^{MODIS} and the N_d^{OE} agree well within the uncertainty range. For the comparison of N_d^{SEVIRI} and N_d^{OE} we find good agreement in the beginning and end of the observation period

5.4.2 Uncertainties due to spatial resolution

To investigate the effect of spatial resolution, we use collocated MODIS and SEVIRI observations. We use the products of MODIS at 1 km spatial resolution. We reproject all MODIS pixels to the 3x3 SEVIRI pixels so that both instruments cover the same area. We then average the MODIS 1 km resolution data to SEVIRI's spatial resolution (4 km x 6 km). In a further step we average a 3x3 pixel field from SEVIRI and the MODIS pixels at original resolution and calculate their standard deviation. In this way we tried to use MODIS to account for SEVIRI's subpixel variability, while neglecting deviations due to the differences of both instruments and retrievals. In Fig. 7 the results for (a) the inhomogeneous case at 1 June 2012, when the clouds are more homogeneous.

The underestimation of N_d^{SEVIRI} compared to N_d^{OE} can likely be attributed to broken cloud effects on the SEVIRI retrieval. For broken clouds within the SEVIRI pixel the satellite receives a combined signal from the clouds but

also from the surface. The same explanation can also be applied to the second inhomogeneous case (27 September 2012). It remains open to which extent the subpixel surface contamination leads to a bias in the retrieved cloud parameters especially for inhomogeneous cloud scenes when the brightness temperature actually does not represent the cloud radiative temperature.

While some of the differences between satellite and ground-based retrievals of N_d can be attributed to the invalidity of and (b) the homogeneous case at 21 April 2013 are shown. For the inhomogeneous case we can clearly see the large spread of MODIS τ values, which is reduced to a similar range as for SEVIRI τ when averaged to the same spatial resolution. The spread of the optical depth is found larger than for the effective radius. For the homogeneous case the spread is smaller. Differences between MODIS and SEVIRI after averaging are in a similar range for both cases. When comparing averaged data, MODIS and SEVIRI show similar results for both cases. However, the differences, especially in terms of r_e can be of the same magnitude than those to ground-retrieved values. There is considerable difference when taking either the closest pixel to the adiabatic assumption and coarse spatial resolution of the satellites, it has to be mentioned that the ground-based retrieval strongly relies on the accuracy of the radar reflectivity and therefore also on the radar calibration and attenuation corrections for atmospheric gases and liquid water that are made within the Cloudnet algorithm. ? points out the strong influence of drizzle on the cloud reflectivity. Errors of 30–60 have to be anticipated for q_T profile retrievals. Those retrieval approaches are based on very similar principles as our OE method (?). In our study we filtered out drizzling profiles as well as possible, but the radar reflectivity still remains very sensitive to few larger droplets in a volume, which can not totally be ruled out. Therefore also the correct radar calibration is an issue location or the spatially averaged value, while the closest pixel does not necessarily result in a better agreement with the ground-based value (Fig. 7). Therefore we can conclude that especially for inhomogeneous cases, the sub-pixel variability introduces an important additional uncertainty factor.

6 Summary and Conclusions

To investigate the accuracy of satellite-based estimates of aerosol indirect effects, we have studied the validity of the (sub-)adiabatic cloud model as a conceptual tool commonly applied in previous studies (e.g. ??). The (sub-)adiabatic cloud model allows indirectly to estimate cloud geometrical depth (H_{cloud}) and cloud droplet number concentration (N_d) from In this work, we aimed to evaluate the consistency and limitation of current ground-based and satellite cloud retrieval products that are used to quantify

aerosol-cloud interactions. We used a two year dataset with four selected case studies.

Cloud properties have been used previously for diagnosing aerosol-cloud interactions and specifically the Twomey effect from both ground-based supersites (e.g. ?) as well as geostationary passive satellite observations (e.g. ?). The sub-adiabatic cloud model as a conceptual tool is commonly applied and modified using an adiabatic factor to account for entrainment within the cloud.

As reference, we used a Based on cloud geometric depths obtained from the combination of ground-based active and passive remote sensing instruments with high temporal and vertical resolution to provide detailed information of the cloud vertical structure. We could, however, demonstrate that such retrievals also have considerable uncertainties. cloud radar and ceilometer, and liquid water path from a microwave radiometer, we demonstrated that for a two year dataset, neither the assumption of an adiabatic cloud nor the assumption of a temporally constant sub-adiabatic factor is fulfilled (mean adiabatic factor 0.63 ± 0.22).

Considering the number of uncertainties for both the satellite and ground perspective, and those originating from the issue of representativity of the two perspectives, our comparison showed that the temporal evolution of cloud micro- and macrophysical quantities is captured surprisingly well for some cases. We discussed the large uncertainties that may occur depending on the observed scene and observation geometry. As the adiabatic factor is required to estimate key quantities for aerosol-cloud interaction studies, but cannot be obtained from passive satellite observations within a sufficient uncertainty range, an independent method to estimate the adiabatic factor, and thus the influence of mixing, would be highly desirable for global-scale analyses. We were able to support previous findings which reported that thinner clouds are closer to adiabaticity (?) as well are clouds that show upwind motion at the cloud base (?).

The cloud geometrical depth can be obtained with To investigate aerosol-cloud interactions from passive satellites the cloud droplet number concentration is widely used as a key parameter. An intercomparison with ground-retrieved values is complicated as it turns out that its retrieval from a ground-based remote sensing directly from ceilometer cloud base and radar cloud top heights. The mean difference of SEVIRI and ground-based cloud geometrical depth is lowest for the two presented homogeneous cases when the radar-radiometer approach is very sensitive to assumptions about the width of the droplet size distribution and the radar calibration. Retrieved values of cloud droplet number concentration can change by more than 135% just due to wrong assumptions made for the width of the droplet size distribution. From passive satellite we find the main sensitivity to uncertainties in the effective radius. We conclude that neither the ground-based adiabatic factor is considered with values down to 14(4). Overall we found sub-adiabatic cloud layers. The adiabatic factor varied in

time and attained values similar to those reported by ? . For 3 out of 4 cases we obtained similar median values around 0.65 ± 0.2 at different seasons. Although larger datasets are required to draw robust conclusions about a typical adiabatic factor, this value could be a first guess for homogeneous stratocumulus clouds as they occur over Central Europe. For thin clouds the uncertainty remains large due to the high relative uncertainties of liquid water path and cloud geometrical depth. This also leads to superadiabatic artefacts in the retrieval. With increasing geometrical depth, the clouds become less adiabatic. We also found that clouds are slightly more adiabatic when the cloud profile is dominated by positive vertical velocity (updrafts). Although a larger dataset would be desirable to draw more robust conclusions, our results support those from ? and ? nor satellite-based cloud retrieved properties used here allow to obtain a robust instantaneous estimate of cloud droplet concentration, which complicates their use for the study of aerosol-cloud interactions.

We developed an Optimal Estimation (OE) retrieval to estimate N_d demonstrated that cloud optical depth from ground-based radar and microwave radiometer observations, which does not require the assumption of a linear increasing liquid water content profile. While the mean difference of N_d retrieved from SEVIRI and the ground-based OE was 29 radar-radiometer retrievals is less sensitive to the assumptions about the droplet size distribution and is therefore better suited to investigate indirect aerosol effects, consistent with the conclusions of ? . It is most sensitive to uncertainties in the liquid water path (changes of up to 50% for an uncertainty of 25 (10) for one of the two homogeneous cases, for the second one we saw a large bias of 488(154). In these the MODIS retrieval was closer to the ground-retrieved values. We were able to attribute this large bias mainly to an underestimation of the effective radius within the current SEVIRI retrieval. Even small differences in effective radius result in large uncertainties of gm^{-2} are possible).

Given an independent retrieval of cloud optical depth, e.g. from shadowband radiometer retrievals (?), and information such as radar Doppler velocity (?), should give further options for validation. Applying such additional observations in an optimal estimation scheme might give the opportunity to better constrain the retrieved cloud droplet number concentration. Also the application of cloud radar scanning capabilities together with radiance zenith measurements might improve the retrieval (?). For validation of those cloud droplet number concentration due to the $N_d \propto r_e^{-2.5}$ relationship. Further research about the influence of observation geometry and spatial resolution effects on effective radius and optical depth differences between MODIS and SEVIRI is required. The OE approach to retrieve cloud droplet number concentration from ground could be further improved by including more independent observations, e.g. from solar radiation observations (e.g. ?), which are available at several ground-based supersites as for

LACROS, retrievals accompanying in-situ measurements are required.

Indications have been detected throughout this study that assumptions about cloud subadiabaticity may help to explain differences between satellite and instantaneous comparisons of optical depth between space and ground may result in large differences, especially for broken cloud conditions and unfavourable viewing conditions. Applying spatial and temporal averaging and subsampling to rather homogeneous, liquid clouds leads to a reasonable agreement in cloud optical depth for a majority of observations during a two year period at LACROS, especially considering the large number of assumptions and uncertainties.

Besides the the retrieval uncertainties, differences in spatial resolution affect the comparison not only between space and ground observations, but also between space-based instruments of different resolution and viewing angles (i.e. SEVIRI, MODIS). We highlighted, that especially for inhomogeneous cases, sub-pixel variability is an important uncertainty factor, but that averaging does not necessarily result in a better agreement to ground-based retrievals. Therefore, satellite retrievals should take into account that liquid water clouds are mostly subadiabatic observations than taking the closest pixel to the location. To generalize such results more collocated MODIS, SEVIRI and ground-based observations need to be examined.

So far only four cases were analyzed, but given the network of Cloudnet/ACTRIS in Central Europe this offers the opportunity to investigate the climatology of the adiabatic factor and investigate its regional, seasonal or synoptical dependency. Using more data from a greater network would give statistically more robust insights in further studies.

With the upcoming Meteosat Third Generation (MTG) satellite (?) a higher spatial resolution of cloud products will be available and should therefore mitigate issues due to spatial resolution for the geostationary perspective. Also the sounder capabilities of MTG should give new opportunities, e.g. to overcome problems of cloud geometrical depth retrievals from passive satellites by using additional information from the oxygen A-band following the method as outlined by (e.g. ??). And therefore might give the possibility to obtain the adiabatic factor over a larger domain.

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7 Appendix

To obtain the factors k_2 and k_6 in the sub-adiabatic cloud model a gamma size distribution is assumed in the form of (?):

$$\begin{aligned} \eta(r) &\equiv A r^\beta \exp(-\Lambda r) \\ &\equiv \frac{\eta_0}{\Gamma(\frac{1-2\nu}{\nu}) r_e \nu^{\frac{1-2\nu}{\nu}}} \left(\frac{r}{r_e}\right)^{\frac{(1-3\nu)}{\nu}} \exp\left(-\frac{r}{r_e \nu}\right) \end{aligned} \quad (14)$$

with

$$\begin{aligned} \beta &\equiv \frac{1-3\nu}{\nu} \\ \Lambda &\equiv \frac{1}{r_e \nu} \\ A &\equiv \eta_0 \frac{\Lambda^{\beta+1}}{\Gamma(\beta+1)}. \end{aligned} \quad (15)$$

Hereby the effective radius r_e , its effective variance ν and the total number density of droplets η_0 is used. The effective radius is defined as the third over the second moment of the droplet size distribution (?) and can be linked to the mean volume radius (r_v) with the following relationship:

$$r_e^3 = k_2^{-1} r_v^3 \quad (16)$$

From the gamma size distributions its n-th moments can be derived by (?):

$$\begin{aligned} M_{\eta,n} &\equiv A \int r^{n+\beta} \exp(-\Lambda r) dr \\ &\equiv A \frac{\Gamma(\beta+n+1)}{\Lambda^{(\beta+n+1)}}. \end{aligned} \quad (17)$$

The factor k_2 is then only a function of the width of the droplet size distribution:

$$k_2 = \frac{M_2(\eta)^3}{M_3(\eta)^2} = (1-2\nu)(1-\nu) \quad (18)$$

The radar reflectivity as proportional to the sixth moment of the droplet size distribution can be expressed as a function of the cloud droplet number concentration N_d , the liquid water content q_L and factors that depend on the width of the droplet size distribution (k_6) (?):

$$Z = \frac{9}{2\pi^2 \rho_w^2} k_6 \frac{q_L^2}{N_d}. \quad (19)$$

Similar to k_2 , the factor k_6 is defined:

$$k_6 = \frac{M_6(\eta)}{M_3(\eta)^2} = \frac{(\nu+1)(2\nu+1)(3\nu+1)}{(1-2\nu)(1-\nu)} \quad (20)$$

Integrating over the cloud geometrical depth, we can solve the equation for the liquid water path:

$$Q_L = \left(\frac{9}{2\pi^2 \rho_w^2}\right)^{-\frac{1}{2}} \int \frac{1}{k_6(\nu(z))} \sqrt{N_d(z)} \sqrt{Z(z)} dz \quad (21)$$

In the homogeneous mixing model $N_d(z)$ and $\nu(z)$ are assumed constant with height. ? considers a column-averaged N_d by weighting with the square-root of radar-reflectivity:

$$\int \sqrt{N_d(z)} dz = \frac{\int \sqrt{N_d(z)} \sqrt{Z(z)} dz}{\int \sqrt{Z(z)} dz} = \sqrt{\overline{N_d}} \quad (22)$$

Using the latter relationship, we yield a retrieval method for the column-averaged $\overline{N_d}$:

$$\overline{N_d}(Q_L, Z, k_6) = \frac{9k_6 Q_L^2}{2\pi^2 \rho_w^2 \left(\int \sqrt{Z(z)} dz\right)^2} \quad (23)$$

Eq. 23 can be substituted into eq. 6 and 7 to eliminate N_d and to obtain a ground-based estimate of τ and r_e .