Manuscript prepared for Atmos. Chem. Phys.
with version 2014/07/29 7.12 Copernicus papers of the LaTeX class copernicus.cls.
Date: 19 May 2016

On the climatological probability of the vertical propagation of stationary planetary waves

Khalil Karami[1], **Peter Braesicke**[1], **Miriam Sinnhuber**[1], **and Stefan Versick**[1,2]

[1]Institute for Meteorology and Climate Research, Karlsruhe Institute of Technology, Karlsruhe, Germany
[2]Steinbuch Centre for Computing, Karlsruhe Institute of Technology, Karlsruhe, Germany

Correspondence to: Kh. Karami (khalil.karami@kit.edu)

Abstract. We introduce a diagnostic tool to assess a climatological framework of the optimal propagation conditions for stationary planetary waves. Analyzing 50 winters using NCEP/NCAR reanalysis data we derive probability density functions (PDFs) of positive vertical wavenumber as a function of zonal and meridional wave numbers. We contrast this quantity with classical climatological means of the vertical wavenumber. Introducing a Membership Value Function (MVF) based on fuzzy logic, we objectively generate a modified set of PDFs (mPDFs) and demonstrate their superior performance compared to the climatological mean of vertical wavenumber and the original PDFs. We argue that mPDFs allow an even better understanding of how background conditions impact wave propagation in a climatological sense. As expected, probabilities are decreasing with increasing zonal wave numbers. In addition we discuss the meridional wave number dependency of the PDFs which is usually neglected, highlighting the contribution of meridional wave numbers 2 and 3 in the stratosphere. We also describe how mPDFs change in response to strong vortex regime (SVR) and weak vortex regime (WVR) conditions, with increased probabilities of the wave propagation during WVR than SVR in the stratosphere. We conclude that the mPDFs are a convenient way to summarize climatological information about planetary wave propagation in reanalysis and climate model data.

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1 Introduction

The impact of the background atmospheric state on planetary wave propagation was first investigated by Charney and Drazin (1961) based on linear wave theory. They showed the importance of the background zonal wind for the vertical propagation of large scale waves from the troposphere into the stratosphere. They found that vertical propagation of stationary planetary waves can only occur when the zonal mean zonal wind is positive. In addition, a strong stratospheric polar night jet of the Southern hemisphere during winter will block and possibly reflect large scale waves. This implies that the zonal mean zonal wind should be smaller than a critical value for vertical propagation. This theory also suggest that large scale waves (zonal wave number=1, 2, 3) are more likely to propagate upwards because their associated critical wind speeds are higher. Studies by (Matsuno, 1970; Lin, 1982; Huang and Gambo, 2002; Limpasuvan and Hartmann, 2000; Hu and Tung, 2002; Dickinson, 1969) not only confirmed this theory but also stressed the importance of vertical shear of the zonal mean zonal wind as well as the vertical gradient of the buoyancy frequency for vertical propagation of large scale waves.

Matsuno (1970) introduced the refractive index for stationary planetary waves (or alternatively vertical wavenumber) as a diagnostic tool for studying the influence of the background zonal flow on planetary wave propagation. According to linear wave theory planetary waves, away from the source regions, tend to propagate toward the region of large positive vertical wavenumber squared. The existence of Rossby waves are prohibited where the vertical wavenumber squared is small or negative, which can happen if the zonal mean zonal wind is easterly, or westerly exceeding the critical wind speed.

The refractive index of Rossby waves as a diagnostic tool provides a framework in which the dynamical forcing of the stratosphere by tropospheric waves can be investigated. However, as shown by Li et al. (2007) the traditional analysis of the refractive index squared makes it difficult, if not impossible, to study the climatological state of the background flow for propagation of planetary waves. In calculating the climatology of the refractive index squared, the problem arises from averaging a time series that could consist of positive and negative values that may cancel each other and hence makes the interpretation of climatologies of this quantity difficult. Another weakness of the vertical wavenumber is that it is somewhat vague. Randel (1988) pointed out that, while using the vertical wavenumber as a diagnostic tool one should not overemphasize the details, since it is a qualitative guide. For instance Smith (1983) found that planetary waves can only propagate when and where the vertical wavenumber squared is positive and very large or avoid the region of large negative values of the vertical wavenumber. The vagueness arises from vague expressions such as "very large positive" and "very large negative" values of the vertical wavenumber which demonstrates the arbitrariness of the classic time mean diagnostic.

Here we attempt to address the modeling of such vagueness which has not been previously addressed. We present an algorithm based on fuzzy logic theory which addresses the above-mentioned vagueness and provides an estimate of the favorability of atmospheric background condition for planetary wave propagation as a function of latitude and altitude. Any diagnostic tool should be consistent with the general knowledge about stationary Rossby wave propagation condition (Table 1). The first and second criterion of the Table 1 are the most important findings of the seminal papers of Charney and Drazin (1961) and Matsuno (1970). They made a great contribution on the understanding of the propagation of planetary scale disturbances from the troposphere into the stratosphere. Eliassen and Palm (1961) based on the wave-mean flow interaction theorem showed that the planetary waves also have a strong influence on the zonal mean zonal wind. Matsuno (1970) and Charney and Drazin (1961) argue that only ultra-long waves (wave numbers 1-3) have the capability to propagate from the troposphere into the middle atmosphere. The criterion 3 expresses that the jet maxima blocks the planetary wave propagation and penetration through the jet maxima is prohibited Karoly and Hoskins (1982). The study of Chen and Robinson (1992) shows that the key parameter that controls the planetary wave propagation is the properties of the tropopause which acts like a valve for the vertical wave propagation from the troposphere into the stratosphere. Furthermore the study of Hu and Tung (2002) and Li et al. (2007) indicated that the large positive vertical shear of zonal wind at the tropopause height tends to enhance wave propagation (criterion 4).

Chen and Robinson (1992) and Hu and Tung (2002) have discussed the importance of vertical shear of zonal mean zonal wind on the vertical propagation of Rossby waves. Chen and Robinson (1992) showed that penetration of planetary waves from the troposphere into the stratosphere is sensitive to small changes in the vertical shear of zonal wind near the tropopause height. Hu and Tung (2002) identified that a positive vertical shear of zonal wind enhances wave propagation across the tropopause. Similarly large negative shear of zonal wind tends to trap the planetary waves in the troposphere and hence less is left to penetrate into the stratosphere. Any diagnostic tool that attempts to provide a climatology of stationary Rossby wave propagation conditions should reflect this theory. In fact, we try to develop an algorithm that is capable of demonstrating the enhancing influence of positive vertical shear of zonal wind and impeding influence of negative vertical shear of zonal wind on stationary Rossby wave propagation from the troposphere to the stratosphere.

Figure 1 shows the climatology of the zonal mean zonal wind and the vertical shear of zonal mean zonal wind ($\mathrm{ms}^{-1}\mathrm{km}^{-1}$) for the Northern hemisphere winter months. Northern hemisphere winter months include December, January and February (DJF) and Southern hemisphere winter months include June, July and August (JJA). Due to the larger meridional temperature gradient between the tropics and mid latitudes, the magnitude of the wind shear between 20°N-40°N is about four times stronger than the vertical shear at higher latitudes. Regardless of magnitude, it is evident that it is positive in the troposphere and negative in the stratosphere in this latitude band. At tropopause heights of these regions, where the sign changes, we expect to see a discontinuity in the Rossby wave propagation as discussed by Hu and Tung (2002). We will show that our new diagnostic is consistent with this theory while both the time mean of vertical wavenumber squared and the probability of positive vertical wavenumber introduced by Li et al. (2007) cannot capture this characteristic. In this study, we focus on the vertical propagation of the planetary wave, as there are also many studies using vertical wavenumber studying the horizontal propagation of the planetary waves.

2 Data and method

In the current study we used daily mean zonal wind and temperature from the National Center for Environmental Prediction-National Centre for Atmospheric Research (NCEP-NCAR) (Kalnay et al., 1996) to calculate the vertical wavenumber of Rossby waves for 50 winters (1961-2010) of both Northern and Southern hemispheres. The vertical wavenumber for stationary planetary waves is defined as:

$$m^2_{k,l}(y,z) = \left(\frac{N^2}{f^2\cos^2(\phi)}\right)\left[\frac{\overline{q_\phi}}{\overline{u}} - \left(\frac{k}{a}\right)^2 - \left(\frac{\pi l}{2a}\right)^2 - \left(\frac{f cos(\phi)}{2NH}\right)^2\right] \tag{1}$$

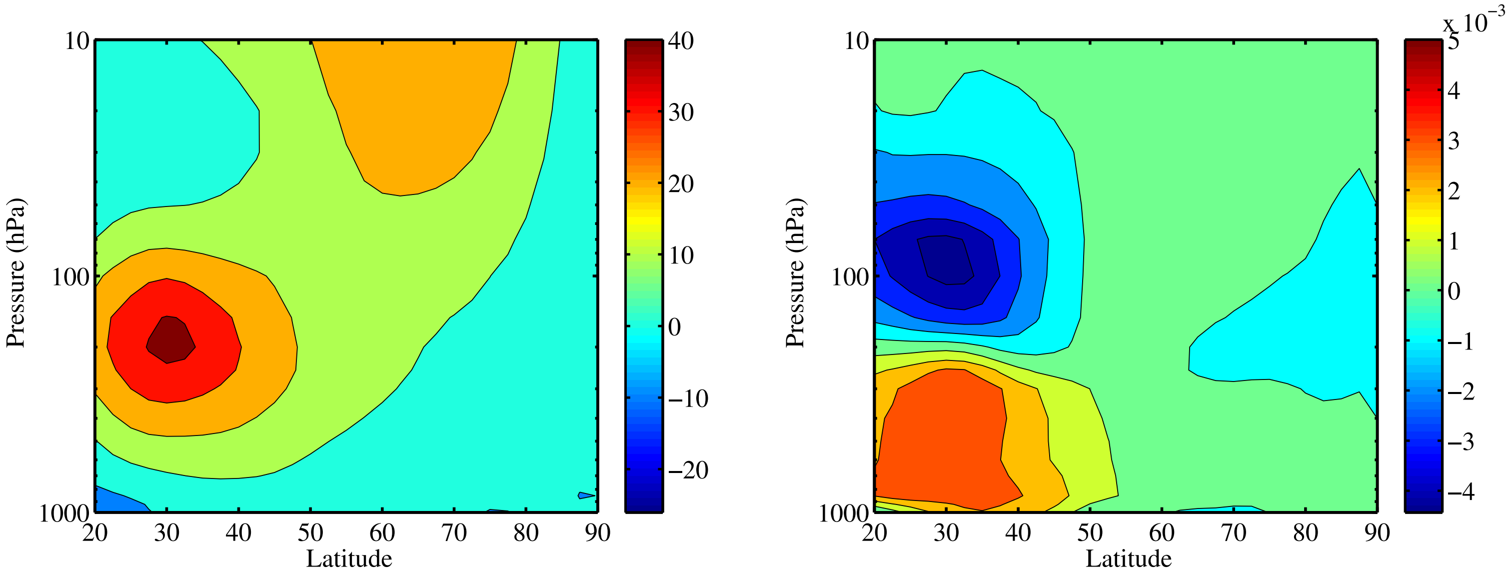


Figure 1. Climatology of the zonal mean zonal wind (left) in and the vertical shear of zonal mean zonal wind (right) for the Northern hemisphere during DJF. The units are ms^{-1} for zonal mean zonal wind and $\mathrm{ms}^{-1}\mathrm{km}^{-1}$ for the vertical shear of zonal mean zonal wind respectively.

Table 1. A summary of known facts about stationary Rossby wave propagation. Any diagnostic tool that attempts to provide a climatology of stationary Rossby wave propagation conditions should be consistent with these criteria. These criteria refer only to the linear waves.

1	For all stationary Rossby waves the most favorable propagation conditions are in the lower troposphere of the mid-latitude region. Upper troposphere and lowermost stratosphere of mid-latitude regions are also favorable for Rossby wave propagation.	Matsuno (1970) and Charney and Drazin (1961)
2	For large scale waves (horizontal and meridional wave numbers 1 to 3) the probability to propagate vertically is highest.	Matsuno (1970) and Charney and Drazin (1961)
3	Rossby waves tend to propagate on the edges of strong westerly winds and avoid penetrating through the jet maxima. Therefore, the strong stratospheric polar night jet of the Southern hemisphere in the winter will block and reflect large scale waves.	Karoly and Hoskins (1982)
4	Strong vertical shear (positive) is likely to enhance the vertical propagation of waves.	Chen and Robinson (1992)

where

$$\overline{q_\phi} = cos(\phi)[\frac{2\Omega}{a}cos(\phi) - \frac{1}{a^2}\frac{\partial}{\partial\phi}[\frac{\frac{\partial}{\partial\phi}(\overline{u}cos(\phi))}{cos(\phi)}] - \frac{f^2}{\rho_0}[\frac{\partial}{\partial z}\frac{(\rho_0\frac{\partial}{\partial z}\overline{u})}{N^2}]] \qquad (2)$$

is the meridional gradient of the zonal mean potential vorticity which is a fundamental quantity in Planetary wave dynamics and the stability of the zonal mean flow (Andrews et al., 1987). Here H, k, l, ρ_0, f, N^2, a, Ω and ϕ are the scale height, zonal and meridional wavenumbers, air density, Coriolis parameter, buoyancy frequency, the Earth's radius and rotation frequency and latitude respectively (Andrews et al., 1987; Matsuno, 1970). The definition of the current version of the vertical wavenumber of Rossby waves that depends on the two-dimensional wavenumbers (zonal and meridional wavenumbers) can be found in (Sun et al., 2014; Sun and Li, 2012).

Figures 2 and 3 show the time Mean Refractive Index Squared (MRIS, in the plots weighted with the Earth radius squared) of 50 winters for Northern and Southern hemispheres respectively. The dependence of the MRIS on the zonal (k=1,2,3) and meridional wavenumbers (l=1,2,3) is visible in both figures. It can be seen that the multi-year average of MRIS gives unsatisfactory results. For instance, for (k,l)=(1,1) very high values of the vertical wavenumber squared are found in high latitudes of the troposphere and the lower stratosphere. Moreover, in most areas of mid and high latitudes of the troposphere alternating positive and negative values of the vertical wavenumber squared leads to a noisy structure and makes the interpretation very difficult. The problem originates from overlapping of positive and negative values in the time-series and results in a reduction of climatological information. Such features of the time mean are also discussed by others (Mukougawa and Hirooka, 2004; Li et al., 2007). Too high values of MRIS northward of 75°N in the lower stratosphere are not consistent with criterion 3 in Table 1, because the strong jet is expected to block wave penetration from the troposphere to the stratosphere. The MRIS is also not able to capture the meridional wavenumber dependency on the wave propagation conditions (criterion 2 in Table 1). For example in the Southern hemisphere, the difference between time mean of for wave (2,1), (2,2) and (2,3) in the stratosphere (above 100 hPa) is small, suggesting no considerable influence from the meridional wavenumbers on the vertical propagation of planetary waves from the troposphere to the stratosphere. In the current study, the time mean vertical wavenumber squared is calculated by the time mean of the instantaneous vertical wavenumber derived from the daily zonal mean field. As shown in Fig. 2 and Fig. 3 the time mean vertical wavenumber has a noisy structure. One possibility to reduce the noise level is to calculate the vertical wavenumber of the time-mean zonal mean fields instead (Fig. A3). However time-dependent Rossby waves propagate on the instantaneous atmospheric state and not on the time-averaged fields. Therefore we focus on an approach to reduce the level of noise in the time-averaged instantaneous vertical wavenumber.

3 Probability of positive vertical wavenumber squared

Li et al. (2007) introduced the frequency distribution of days with negative vertical wavenumber squared as an alternative metric to describe how planetary waves can propagate. Figure 4 shows the probabilities of positive vertical wavenumber squared for Northern hemisphere winter time expressed as the percentage of days with positive $m^2_{k,l}(y,z)$ for wave (1,1), (1,2) and (1,3). By comparing to the time mean of the same waves we conclude that this quantity is capable of describing the required wave properties better than the time mean of $m^2_{k,l}(y,z)$. However, it results in high values of probability between 20°N-40°N in the lower and middle stratosphere. This might be an over-optimistic result, because it is due to small positive values at these locations that exist throughout the winter season. In this respect the climatology of probability of positive refraction index squared does not meet the criterion 4 in Table 1.

Further evidence to show the importance of $\frac{\partial}{\partial z}\overline{u}$ for vertical propagation of Rossby waves can be provided by calculating the normalized vertical component of the Eliassen-Palm (EP) flux. Figure 5 shows that the normalized vertical component of EP flux has a minimum at the tropopause, indicating that upward penetration of waves is suppressed by the negative values above tropopause heights as suggested by Hu and Tung (2002). Sensitivity of $m^2_{k,l}(y,z)$ to $\overline{u}$ can be studied by comparing the values of $a^2\frac{\overline{q_\phi}}{\overline{u}}$ and $a^2\frac{\overline{q_\phi}}{10ms^{-1}}$. Figure 6 shows the climatology of $a^2\frac{\overline{q_\phi}}{\overline{u}}$ and $a^2\frac{\overline{q_\phi}}{10ms^{-1}}$ for DJF in the Northern Hemisphere. The subpolar maxima of $a^2\frac{\overline{q_\phi}}{\overline{u}}$ in the troposphere are not related to small values of the zonal wind at these regions, since by taking away the $\overline{u}$, the maxima are shifted to subtropics (25°N-40°N). This implies that small values of $\overline{u}$ rather than $\frac{\partial}{\partial z}\overline{u}$ at subpolar regions cause the maxima of $m^2_{k,l}(y,z)$ at these regions.

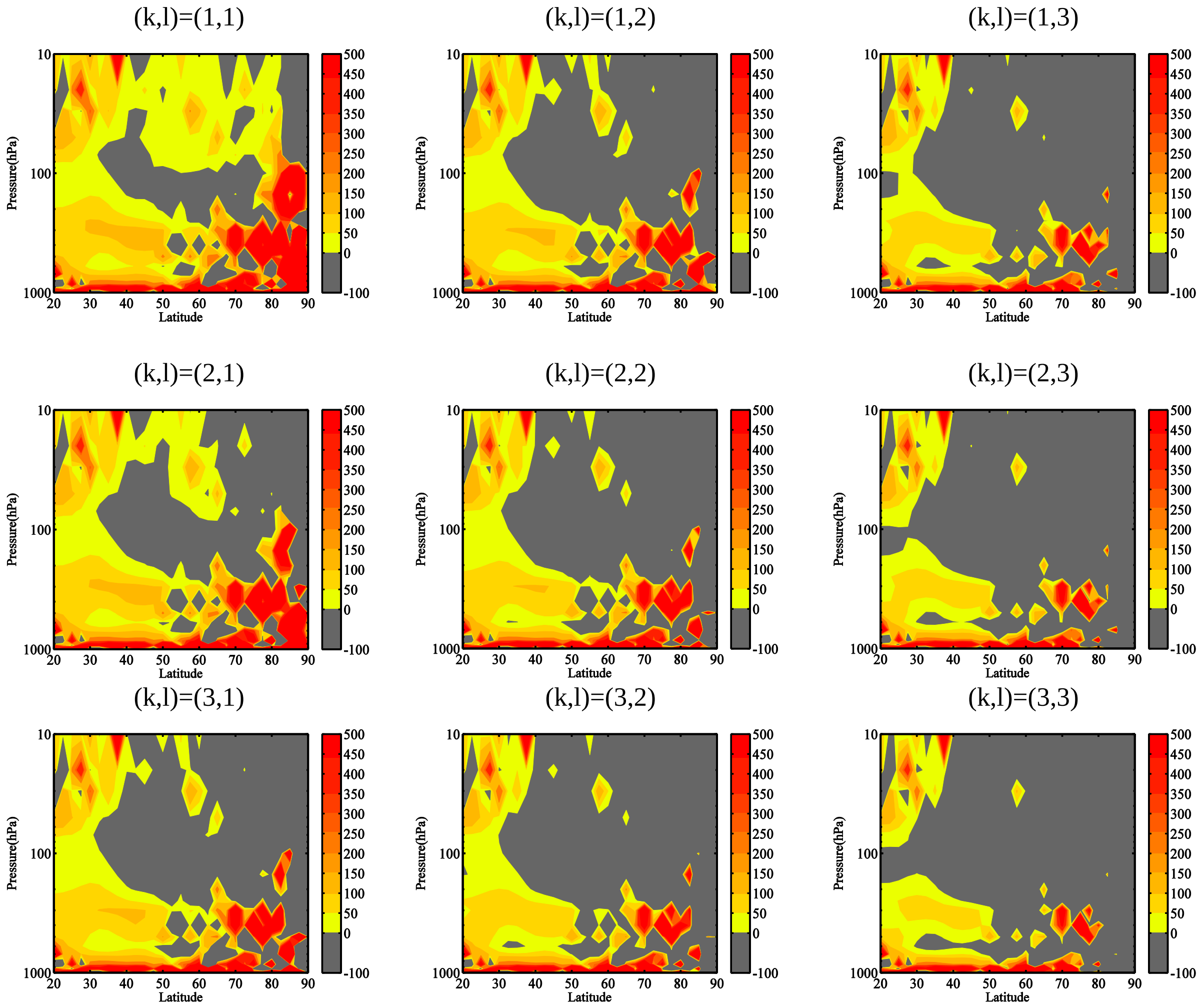
4 Probability of Favorable Propagation Condition for Rossby waves

A long standing issue in the interpretation of $m^2_{k,l}(y,z)$ is its vagueness. As suggested by Matsuno (1970), large waves tend to propagate in regions of positive vertical wavenumber $m^2_{k,l}(y,z)$ while they may be refracted or absorbed where $m^2_{k,l}(y,z) < 0$. Here (in the light of fuzzy sets and logic), we attempt to address the modeling of such vagueness. Fuzzy logic is a mathematical method for answering questions with imprecise information (such as very large or very small vertical wavenumber)., it deals with reasoning that is approximate rather than fixed and precise. The basic approach is to assign a value between zero and one to describe the area between the upper and lower limit.The upper and lower limits are referring to the maximum and minimum values of any variable that fuzzy logics tries to set various MVF for them. In classical logic everything is either true or false. However, in fuzzy logic truth is a matter of degree (Zadeh, 1965; Novak et al., 1999).

Here we assume that instead of each of the individual $m^2_{k,l}(y,z,t)$ contributing equally to the time-mean $m^2_{k,l}(y,z)$, some $m^2_{k,l}(y,z,t)$ contribute more than others. In this way, we distinguish between small positive and very large positive values to let very large positive values influence the final result more than small positive values. In this way classes or sets whose boundaries are not sharp will be introduced. We introduce $\mu_{Ro}(y,z,t)$ as the Rossby wave MVF which provides mPDF and estimate the probability of favorable propagation condition of Rossby wave $Pr_{Ro}(y,z)$, as a function of latitude and height. We also provide the physical basis of the proposed method. For a detailed discussion of Membership Value Function (MVF), see the Appendix.

The advantage of our analysis over the traditional analysis of the vertical wavenumber is that without any reduction in the information due to cancellation of negative and positive values of the vertical wavenumber squared, we estimate the likeliness for planetary waves to propagate from one region to another at any time, altitude and latitude.

In the Fig. 7 the black curve shows the MVF used in the calculation of favorable propagation condition of Rossby waves. For the negative $m^2_{k,l}(y,z,t)$ region (part a) this function suggests that the rate of attenuation is very high and therefore wave propagation is prohibited in this region.


(k,l)=(1,1) (k,l)=(1,2) (k,l)=(1,3)

(k,l)=(2,1) (k,l)=(2,2) (k,l)=(2,3)

(k,l)=(3,1) (k,l)=(3,2) (k,l)=(3,3)

Figure 2. Climatology of vertical wavenumber squared ($a^2 m^2_{k,l}(y,z)$) of 50 winters (1961-2010) in the Northern hemisphere. Regions with negative $a^2 m^2_{k,l}(y,z)$ are shaded with gray color.

Since our method is still based upon the linear wave theory, we assume a linear relationship between the magnitude of the $m^2_{k,l}(y,z,t)$ and the probability of favorable propagation conditions for positive $m^2_{k,l}(y,z,t)$ in a way that the higher the values of the $m^2_{k,l}(y,z,t)$ the chances of propagation for the Rossby waves increases linearly (part b). Large values of the $m^2_{k,l}(y,z,t)$ occur near the critical line where zonal mean zonal wind approaches zero ($\overline{u} < 0.5\,ms^{-1}$ in this study). This region is also not favorable for Rossby wave propagation since at this region the linear wave theory breaks down and waves start to break and the waves are absorbed (part c). The region where vertical wavenumber squared is larger than 600 is not favorable for wave propagation. At these regions the zonal mean zonal wind approaches zero. This condition often happens in the upper troposphere/lower stratosphere where westerlies become weak in the winter season near the Arctic. Therefore most of the differences between Fig. 4 and Fig. 8 for Rossby wave (1,1) at the above-mentioned regions can be associated with setting μ_{Ro} to zero for $m^2_{k,l} > 600$. In the study of Li et al. (2007) the effect of the critical line on Rossby wave propagation is neglected since all the positive values of the $m^2_{k,l}(y,z,t)$ are regarded as though small and very large positive values of the $m^2_{k,l}(y,z,t)$ are equally favorable places for wave propagation. In fact very high values of the $m^2_{k,l}(y,z,t)$ are not necessarily favorable conditions for the Rossby wave propagation. In this study the $m^2_{k,l}(y,z,t)$ higher than 600 is considered as the critical line region, obtained from the climatology of the vertical wavenumber when $\overline{u} < 0.5\,ms^{-1}$. As we will show, this function gives us an improved picture of planetary

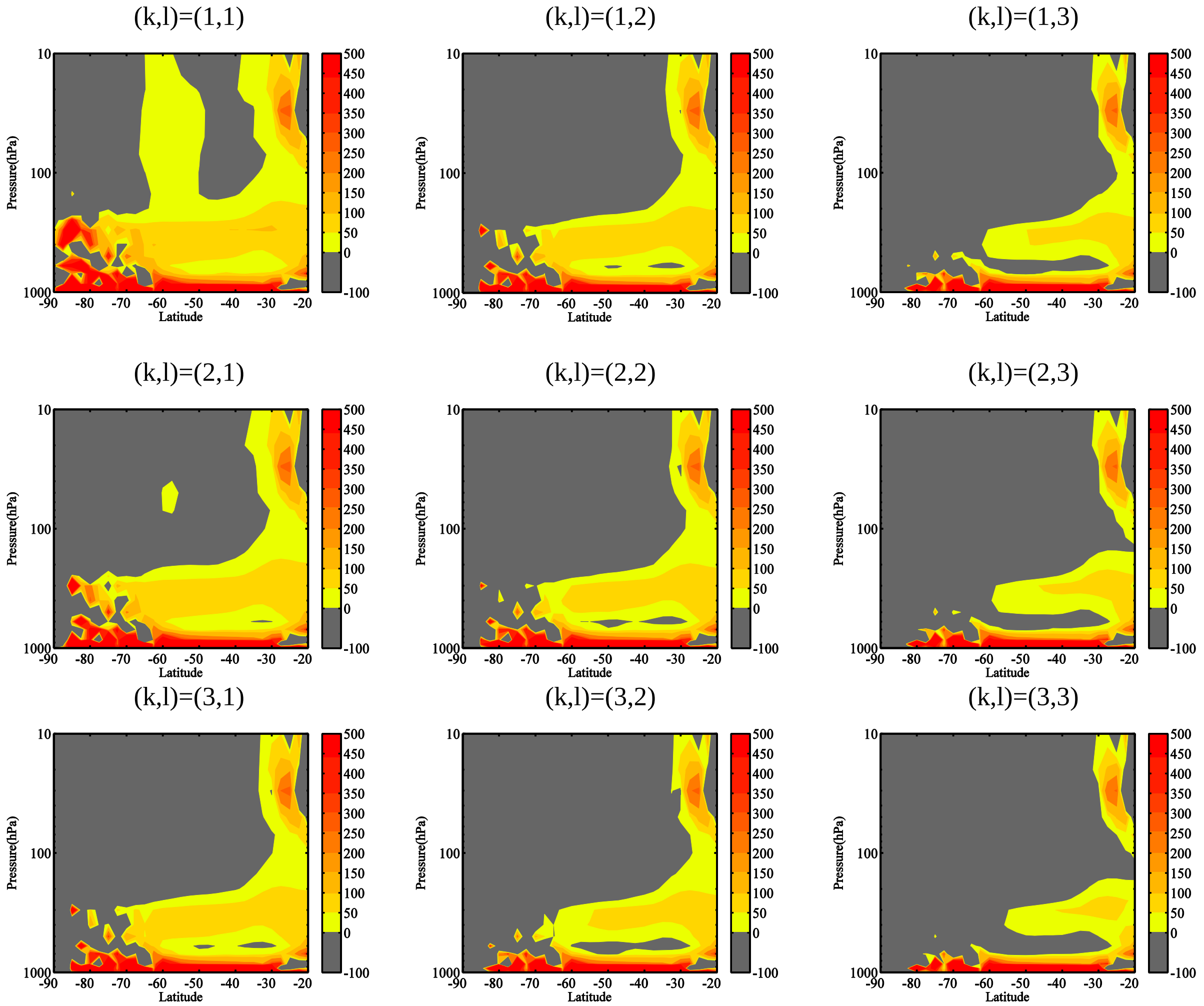


Figure 3. Climatology of vertical wavenumber squared ($a^2m^2_{k,l}(y,z)$) of 50 winters (1961-2010) in the southern hemisphere. Regions with negative $a^2m^2_{k,l}(y,z)$ are shaded with gray color.

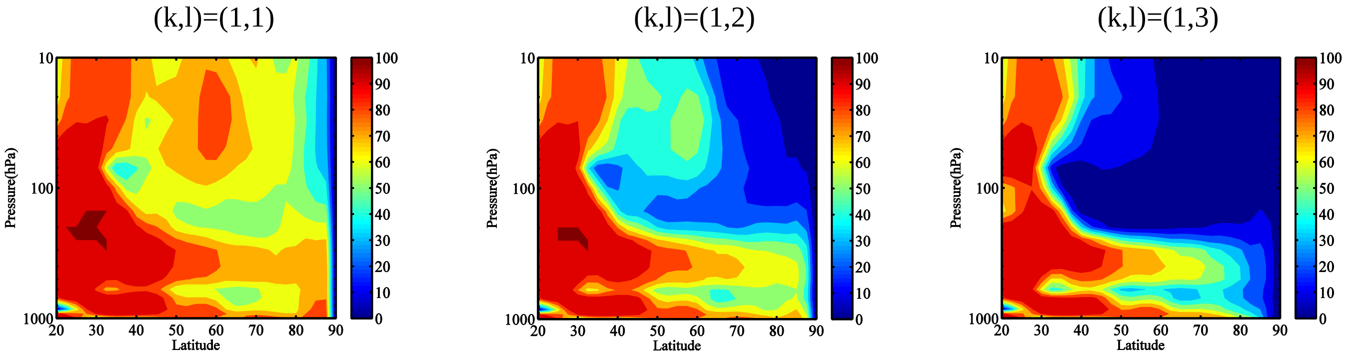


Figure 4. Probability of positive vertical wavenumber squared for Northern hemisphere wintertime for wave (1,1), (1,2) and (1,3).

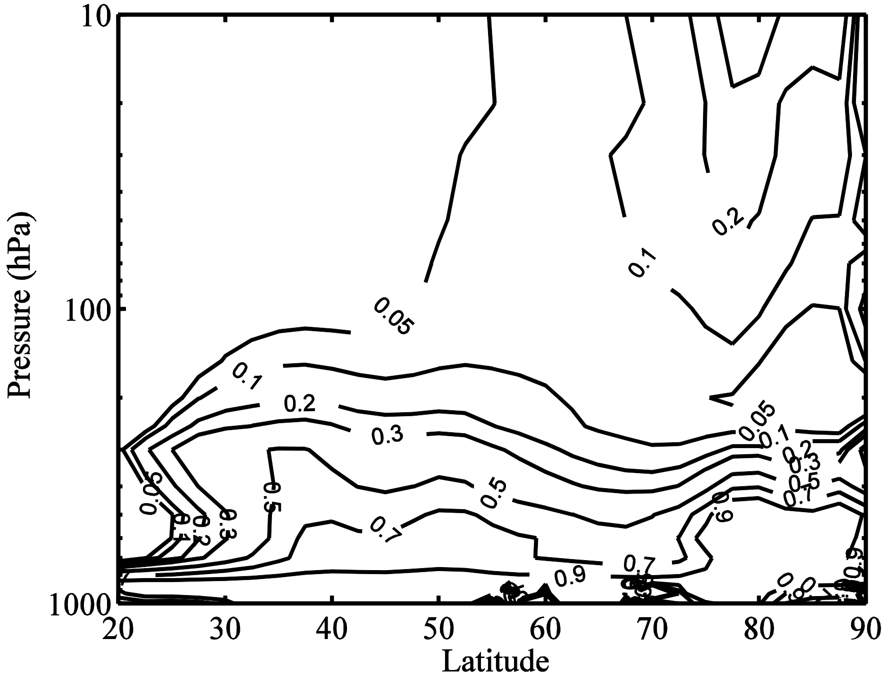


Figure 5. Climatology of vertical component of EP flux normalized by vertical component of EP flux at 850 hPa for DJF at Northern hemisphere. Discontinuity of this quantity at the tropopause heights indicates the strong suppression of wave penetration from troposphere into the stratosphere at lower stratosphere.

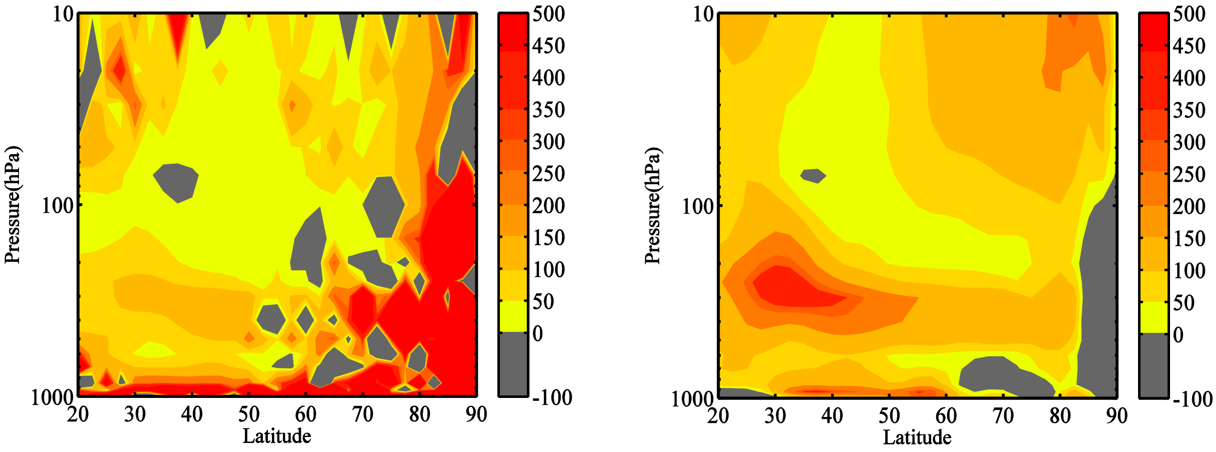


Figure 6. climatology of $a^2\frac{\overline{q_\phi}}{\overline{u}}$ (left) and $a^2\frac{\overline{q_\phi}}{10}$ (right) for DJF in the Northern hemisphere.

wave propagation condition in climatologies. Higher values of $Pr_{Ro}(y,z)$ provide a window of opportunity for planetary waves to propagate at any latitude and height. Likewise, smaller values of $Pr_{Ro}(y,z)$ demonstrate the places where Rossby waves propagate away from these regions. The sensitivity of $Pr_{Ro}(y,z)$ values to the shape of the MVF function is discussed in Appendix A.

5 Results and Discussions:

Figure 8 demonstrates the climatology of probability of favorable propagation condition of Rossby waves for zonal wavenumbers (k=1, 2, 3) and meridional wavenumbers (l=1, 2, 3) for the Northern hemisphere winter season. The most common feature for all waves are their rather large probability to propagate in the troposphere (below 200 hPa) in winter season. It is also evident that the most favorable propagation condition is in the lower troposphere of the mid-latitude region. The values of Fig. 8 are independent of Rossby wave generation and explain how the waves, when generated, would propagate given the structure of the mean flow. However the regions of highly favorable Rossby wave propagation and source region for wave generation (asymmetries at the surface, land-sea contrasts, and sea surface temperature asymmetries) are coincident. It is also clear that longer waves have more opportunity to penetrate to the stratosphere.

Karoly and Hoskins (1982) by using ray tracing technique from geometrical optics and wave propagation in a slowly varying medium, showed that wave rays which are parallel to the group velocity vector tend to refract toward large vertical wavenumber squared. They also found that Rossby waves have a tendency to propagate along great circles and most of the upward propagation of Rossby waves will be refracted toward the equator (even if the vertical wavenumber squared were positive at all height in their study). Similar to this theory, we also found a channel or waveguide of large probability of favorable propagation condition for Rossby waves. The strong westerlies act as a waveguide of Rossby waves and direct them vertically through the tropopause and allow them to penetrate to higher altitudes from their source region (troposphere). These areas are south of 40°N in winter of the Northern hemisphere for large waves and are indicated by $Pr_{Ro}(y,z) > 50\%$.

The study of Karoly and Hoskins (1982) also revealed that Rossby waves tend to propagate on the edges of strong westerlies and avoid penetrating through the jet. This fact is also clear in our results, where north of 60°N and above 200 hPa, the probability of favorable condition for Rossby waves show relatively smaller values, comparing to similar altitude ranges between 30°N and 50°N. The maxima south of 40°N at 100 hPa in the mPDF shows that the region is favorable for wave propagation. At the same region, the vertical component of the EP fluxes have small magnitudes. However as shown in Li et al. (2007) the horizontal component of EP fluxes has a large values at this region (Fig. 5 (e) in the study of Li et al. (2007)). Since the current study concentrates only on the vertical wave propagation, not all aspects of the Fig. 5 can be directly compared with the Fig. 8. The same climatologies as Fig. 8 are presented in Fig. 9 for the Southern hemisphere. Similar to the Northern hemisphere, all large scale waves have a rather large chance to propagate in the troposphere in winter. It can be seen that the larger the waves, the probability of favorable condition for them to propagate upward are larger.

Figure 10 demonstrates the differences between probability of positive vertical wavenumber (calculated by PDFs) and probability of favorable propagation condition of Rossby waves (calculated by mPDFs) for Northern hemisphere wintertime for wave (1,1), (1,2) and (1,3). The maximum difference is found at 20°N-40°N of the middle and upper troposphere which can reach to 50%. This unsatisfactory result of the probability of positive vertical wavenumber is due to small positive values at these places which is consistent throughout the winter season. The area of maximum difference between $Pr_{Ro}(y,z)$ and probability of positive vertical wavenumber remains the same for all wavenumbers at both Northern and Southern hemispheres (not shown).

As Fig. 8 and Fig. 9 show the most important difference between the Northern and Southern hemisphere occurs in the high latitudes of the stratosphere, where in the Northern hemisphere, zonal wavenumber=1 has a good opportunity to propagate ($Pr_{Ro}(y,z)$>40%), while in the Southern hemisphere it has a rather poorer chance to propagate. This is consistent with the theoretical explanation of the vertical propagation of Rossby waves from the troposphere to the stratosphere by Charney and Drazin (1961).The zonal mean zonal wind should be weaker than a critical strength for upward propagation of Rossby waves. The strong stratospheric winter polar vortex of the Southern hemisphere will block and reflect wave activity. The critical strength depends on the scale of the wave and is not a function of the background zonal regime.

A significant piece of information which is lost from the time mean of $m^2_{k,l}(y,z)$ is the role of meridional wavenumbers on the wave propagation conditions. For instance in the Southern hemisphere, the difference between the time mean of $m^2_{k,l}(y,z)$ for wave (2,1), (2,2) and (2,3) in the stratosphere (above 100 hPa) is not large which is one of the unsatisfactory results of time mean of $m^2_{k,l}(y,z)$. It is only in the light of $Pr_{Ro}(y,z)$ values that we can understand the impact of meridional wavenumbers on the wave propagation in the stratosphere. Note that, at the same latitude range of the Southern hemisphere, $Pr_{Ro}(y,z)$ values are as high as 45% for wave (2,1) in mid-latitudes of stratosphere, while

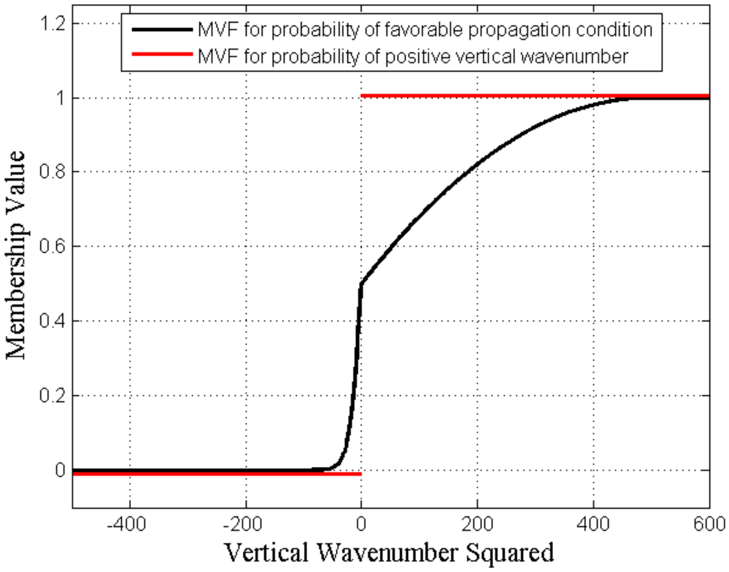


Figure 7. MVF used in the calculation of favorable propagation condition of Rossby waves (black curve). Red lines show MVF for calculating probability of positive vertical wavenumber which are used by Li et al. (2007). In their study the effect of the critical layer (part c) is not considered.

the $Pr_{Ro}(y,z)$ values reach to less than 5% for wave (2,3).

6 Usefullness and appropriateness of $Pr_{Ro}(y,z)$

In order to test the appropriateness of the $Pr_{Ro}(y,z)$ in climatological studies of stationary planetary wave propagation, we further investigate the sensitivity of the $Pr_{Ro}(y,z)$ to different zonal flow regimes in the stratosphere. Following Castanheira and Graf (2003), we constructed two data sets based upon the strength of the westerlies in the lower stratosphere (50 hPa) at 65°N. According to the Charney and Drazin (1961) criterion, if the background flow is westerly and smaller than the latitude and wave number dependent critical Rossby velocity, the planetary waves can penetrate from the troposphere into the stratosphere, otherwise wave reflection occurs and tropospheric flow may be modified. Strong Vortex Regime (SVR) is identified when $\overline{u}_{50}(65N) > 20ms^{-1}$ and Weak Vortex Regime (WVR) is considered when $0 < \overline{u}_{50}(65N) < 10ms^{-1}$, where $\overline{u}_{50}(65N)$ is the 50 hPa zonal mean zonal wind at 65°N. The $20ms^{-1}$ threshold reflects the critical Rossby velocities ($20ms^{-1}$) for ZWN=1 for a climatological Northern hemisphere zonal wind profile. The WVR events do not correspond to the Sudden Stratospheric Warmings (SSWs) in the current study. Since during SSWs the linear wave theory breaks down and waves start to break and the waves are absorbed, the vertical wavenumber and probability of the favorable wave propagation (both are based on the linear wave theory) have limitations for studying the wave propagation druing SSWs.

Table 2 demonstrates the periods of different polar vortex regimes that last for at least 30 consecutive days in DJF. Since in DJF the stratospheric flow consists of strong westerlies (in the absence of vertical wave propagation), the number of SVR events are more than WVR events. The results of $m^2_{k,l}(y,z)$ and $Pr_{Ro}(y,z)$ for WVR and SVR for wave (1,1) are presented in Fig. 11. It is found that in comparison to climatologies (Fig. 8) both WVR and SVR show similar patterns. However, the waveguide at mid latitudes is much narrower in SVR than WVR. In addition, the average values of $Pr_{Ro}(y,z)$ in the stratosphere are greater in WVR than SVR. These results show that planetary waves have more chance to penetrate and force the stratosphere in WVR than SVR. In other words, values of $Pr_{Ro}(y,z)$ are sensitive to stratospheric westerlies and are consistent with the general knowledge about planetary wave propagation from the troposphere to the stratosphere. An enhancement of wave propagation northward of 70°N in the lower stratosphere and a

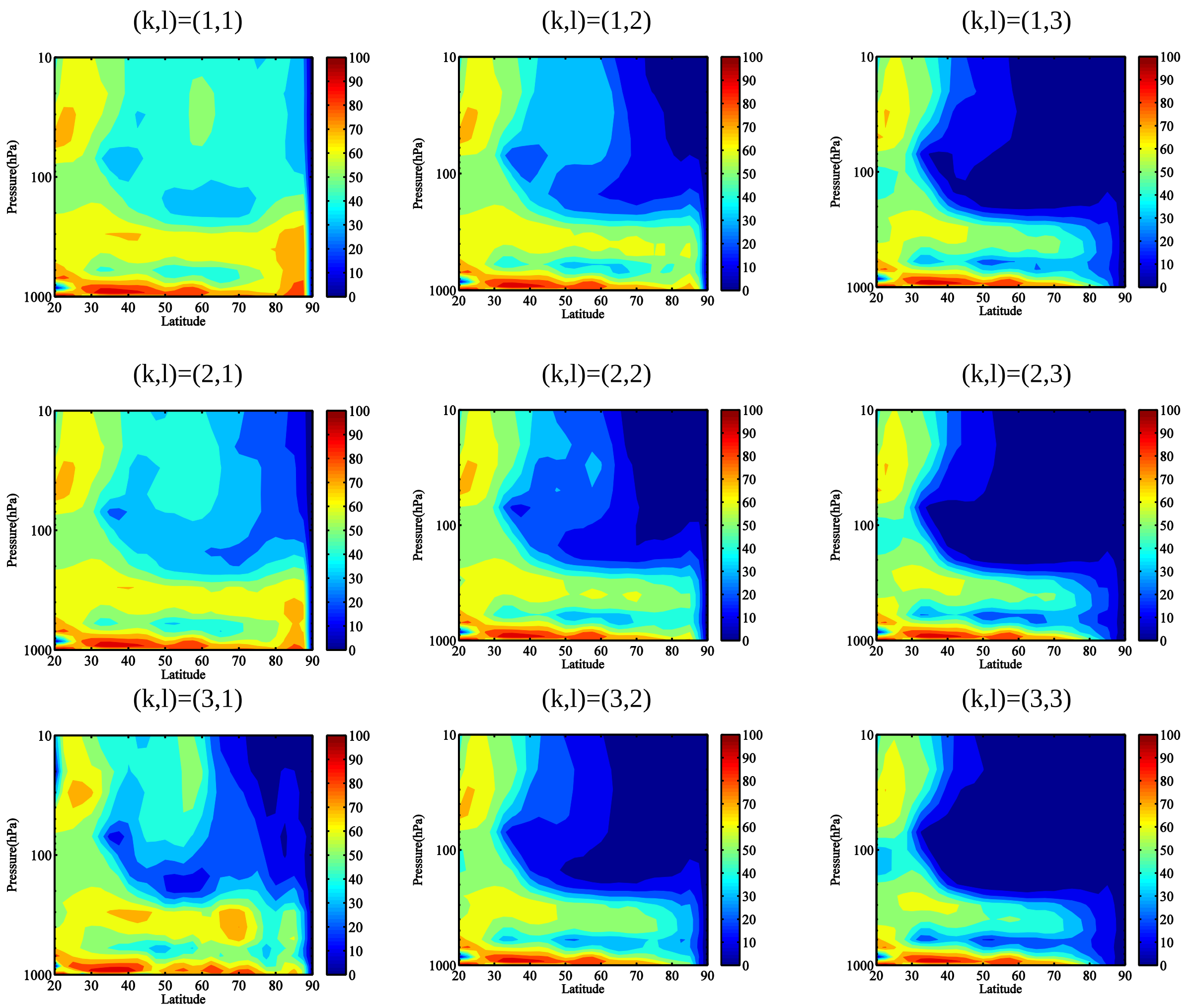


Figure 8. Probability of favorable propagation condition for Rossby waves derived from 50 winters (1961-2010) in the Northern hemisphere. The higher the values, it is convenient for planetary waves to propagate to that regions. In contrast, planetary waves tend to propagate away from regions of low values of this quantity.

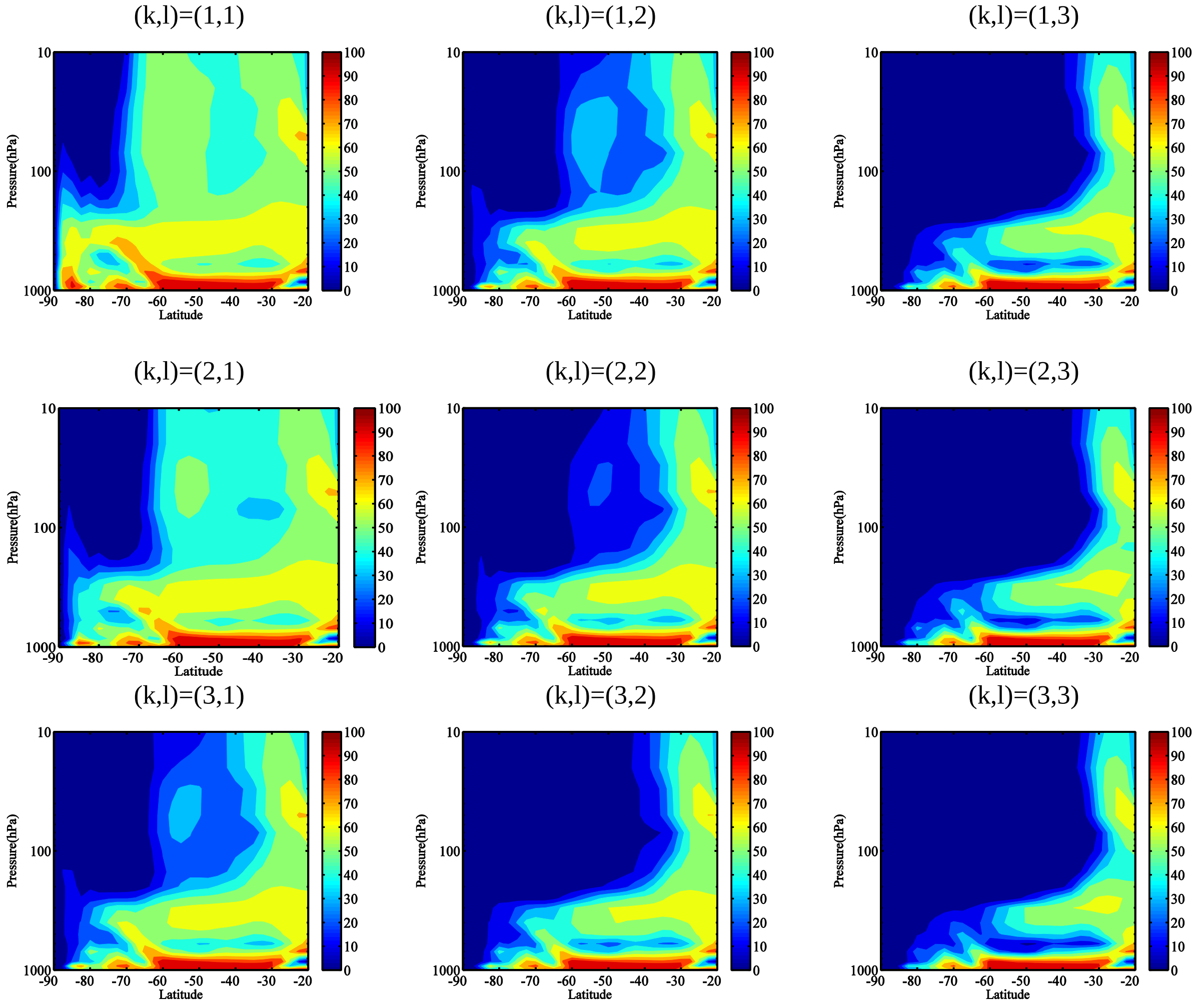


Figure 9. The same as Fig. 8 but for Southern hemisphere wintertime.

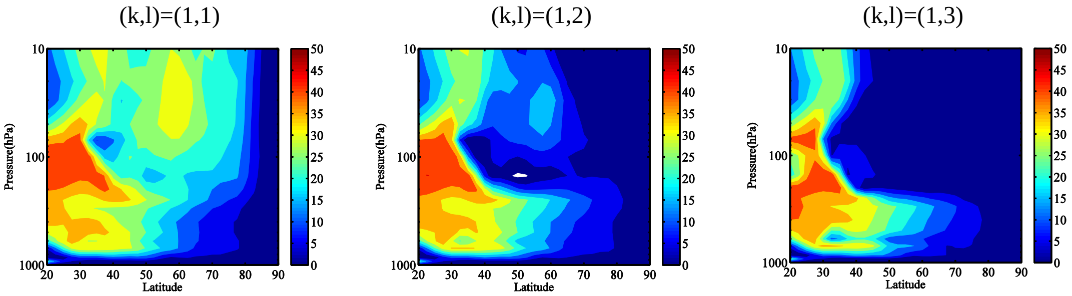


Figure 10. The differences between the probability of positive vertical wavenumber squared and the probability of favorable propagation condition of stationary Rossby waves.

Table 2. Periods of polar vortex regimes lasting for at least 30 consecutive days in DJF; left: Strong Vortex Regime. Right: Weak Vortex Regime.

Strong Vortex Regime (SVR)		Weak Vortex Regime (WVR)	
Starting date	Ending date	Starting date	Ending date
20 Dec 1961	20 Feb 1962	20 Dec 1968	27 Jan 1969
24 Dec 1963	28 Feb 1964	28 Dec 1984	13 Feb 1985
03 Jan 1967	28 Feb 1967	09 Dec 1998	11 Jan 1999
01 Dec 1975	28 Feb 1976	02 Jan 2004	28 Feb 2004
01 Dec 1987	14 Jan 1988		
16 Dec 1988	17 Feb 1989		
17 Dec 1989	28 Feb 1990		
01 Dec 1991	18 Jan 1992		
05 Dec 1992	11 Feb 1993		
01 Dec 1994	18 Jan 1995		
07 Dec 2004	21 Feb 2005		
30 Dec 2006	26 Feb 2007		
23 Dec 2007	13 Feb 2008		

slight reduction in the favorability of wave propagation between 50°N-70°N in the stratosphere are found for WVR. On the other hand it can be seen that due to the high level of noisiness the interpretation of the difference of $m^2_{k,l}(y,z)$ between WVR and SVR is very difficult. Since the highest difference in the favorability of wave propagation between WVR and SVR occurs northward of 50°N in the stratosphere, we further calculate the difference in the vertical component of EP flux between WVR and SVR in this region (Fig. 12). An enhancement of vertical EP flux is obtained northward of 65°N in the lower stratosphere during WVR while a decrease in this quantity is obtained southward of this region in the middle and upper stratosphere. By comparing the differences of $m^2_{k,l}(y,z)$, $Pr_{Ro}(y,z)$ and vertical component of EP flux during WVR and SVR, it can be seen that the pattern of differences between $Pr_{Ro}(y,z)$ and vertical component of EP flux are similar. Therefore, based upon these analyses, we suggest that this diagnostic tool can be useful for studying the propagating properties of the planetary waves.

7 Conclusions

Climatological values of the time mean of the vertical wavenumber squared derived from 50 winters (1961-2010) of both Northern and Southern hemispheres are calculated to show several problematic features of this important quantity in climatologies. In order to improve these unsatisfactory results, we introduced probability density functions (PDFs) of positive refractive indices as a function of zonal and meridional wave numbers. We also compared this quantity with a modified set of PDFs (mPDFs) and demonstrate their superior performance compared to the climatological mean of refractive indices and the original PDFs. Without any reduction in the information, $Pr_{Ro}(y,z)$ estimates the likeliness for stationary Rossby waves to propagate from one region to another at any time, altitude and latitude in a climatological sense. The higher the $Pr_{Ro}(y,z)$ the easier it is for planetary waves to propagate. Smaller values of $Pr_{Ro}(y,z)$ demonstrate the places where Rossby waves are absorbed or reflected from these regions. It is also found that by using this quantity one can easily study the difference in stationary Rossby wave propagation between different meridional wavenumbers without the difficulty of the interpretation of the noisy structure of the time mean vertical wavenumber. Our diagnostic tool is also capable of demonstrating the enhancing influence of positive vertical shear of zonal wind and impeding influence of negative vertical shear of zonal wind on stationary Rossby wave propagation from the troposphere to the stratosphere. The better performance of the mPDF suggests that relatively small but positive numbers of the vertical wavenumber squared play an important role to offer an favorite propagating condition for planetary waves in the stratosphere. This diagnostic tool successfully shows that for WVR there is more space for the vertical propagation of Rossby waves from the troposphere to the stratosphere. In contrast, SVR tend to block and reflect vertical propagation of stationary Rossby waves. It is also worthwhile mentioning that both the vertical wavenumber and probability of the favorable wave propagation are still qualitative tools to study the vertical propagation of Rossby waves from the troposphere to the stratosphere. Since our diagnostic tool is consistent with the theoretical understanding of vertical propagation of Rossby waves from the troposphere to the stratosphere, we suggest that this diagnostic tool has the capacity to be used in assessing planetary wave propagation conditions in climate models.

Acknowledgements. Khalil Karami and Miriam Sinnhuber gratefully acknowledge funding by the Helmholtz Society within the Helmholtz Young Investigators group: Solar variability, cli-

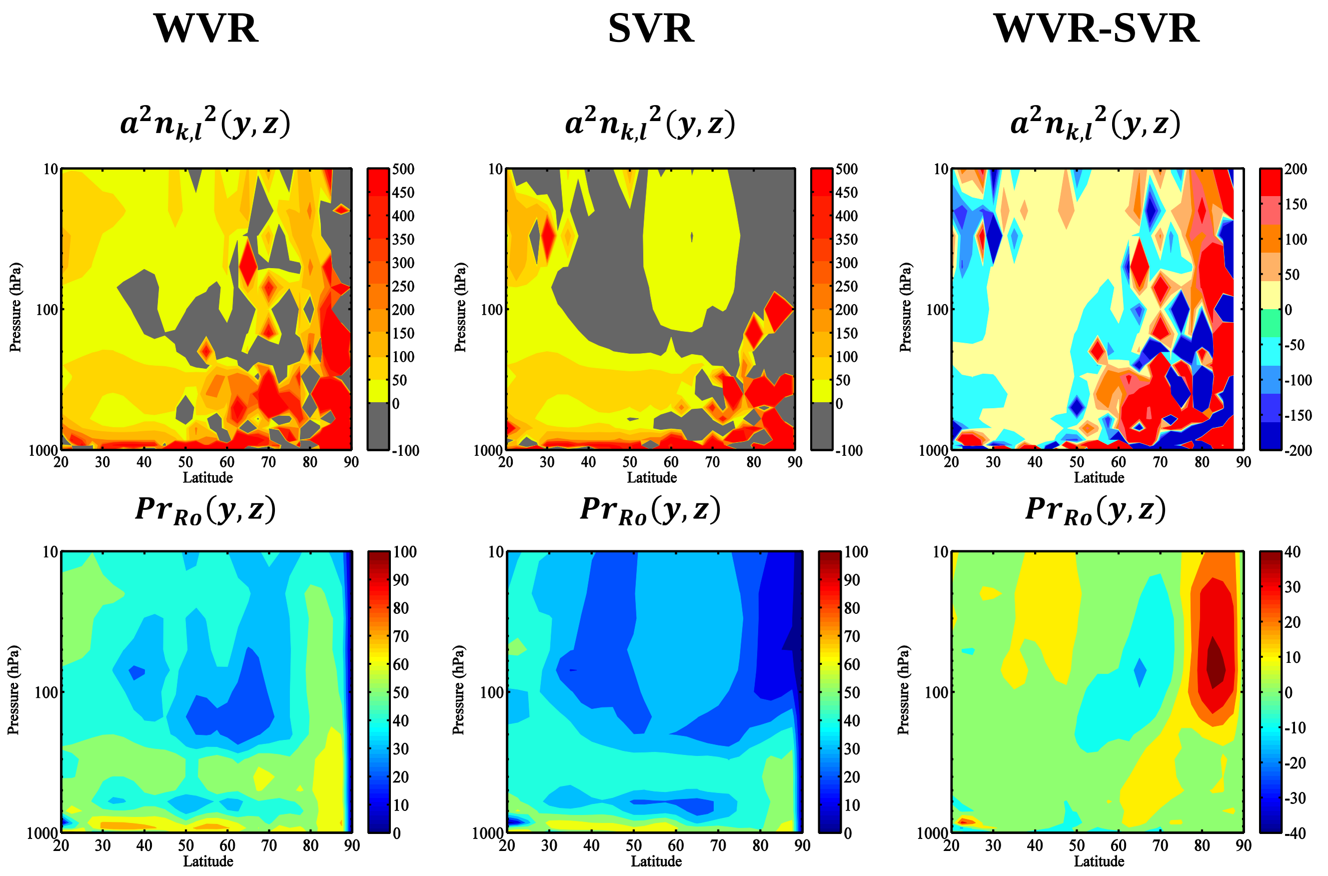


Figure 11. $a^2 m_{k,l}^2(y,z)$ (first row) and $Pr_{Ro}(y,z)$ (second row) during WVR and SVR.

mate, and the role of the mesosphere/lower thermosphere, project NWG-642. NCEP Reanalysis data provided by the NOAA/OAR/ESRL PSD, Boulder, Colorado, USA, from their Web site at http://www.esrl.noaa.gov/psd.

Appendix A

The probability of favorable propagation condition of Rossby waves $Pr_{Ro}(y,z)$ can be written as:

$$Pr_{Ro}(y,z) = \frac{\sum_{t=1}^{n} \mu_{Ro}(y,z,t)}{\sum_{t=1}^{n} t} \times 100 \tag{A1}$$

where $\mu_{Ro}(y,z,t)$ as modified set of PDFs (mPDFs) is defined as:

$$\mu_{Ro} = \begin{cases} 0 \quad if \quad m_{k,l}^2 \le 0, \\ (8.3\times 10^{-4} \times m_{k,l}^2(y,z)) + 0.5 \quad if \quad 0 < m_{k,l}^2 < 600, \\ 0 \quad if \quad m_{k,l}^2 \ge 600 \end{cases} \tag{A2}$$

Here 8.3×10^{-4} is the slope of line b in the Fig. 7. The variable t is the time step and in the current study the daily mean values of the temperature and zonal wind are used in the calculations. In the study of Li et al. (2007) PDFs (red lines in the Fig. 7) are defined as:

$$\mu_{Ro} = \begin{cases} 0 \quad if \quad m_{k,l}^2 < 0, \\ 1 \quad if \quad m_{k,l}^2 > 0, \end{cases} \tag{A3}$$

In order to test the sensitivity of $Pr_{Ro}(y,z)$ to the shape of MVF, we evaluated the values of $Pr_{Ro}(y,z)$ for several potential MVFs. Figure A1 demonstrates the shapes of three MVFs that are used to calculate the values of $Pr_{Ro}(y,z)$. It can be seen from Fig. A2 (first row) that MVF1 gives unsatisfactory results above 200 hPa, where for wave (3,3) we expect very low values of $Pr_{Ro}(y,z)$ poleward of 40°N. This function (MVF1) neglects the fact that Rossby waves tend to quickly attenuate in low values of vertical wavenumber squared. The values of $Pr_{Ro}(y,z)$ can reach as high as 50% at these latitudes and altitudes. MVF2 and MVF3 also give unrealistic results where the values of $Pr_{Ro}(y,z)$ are

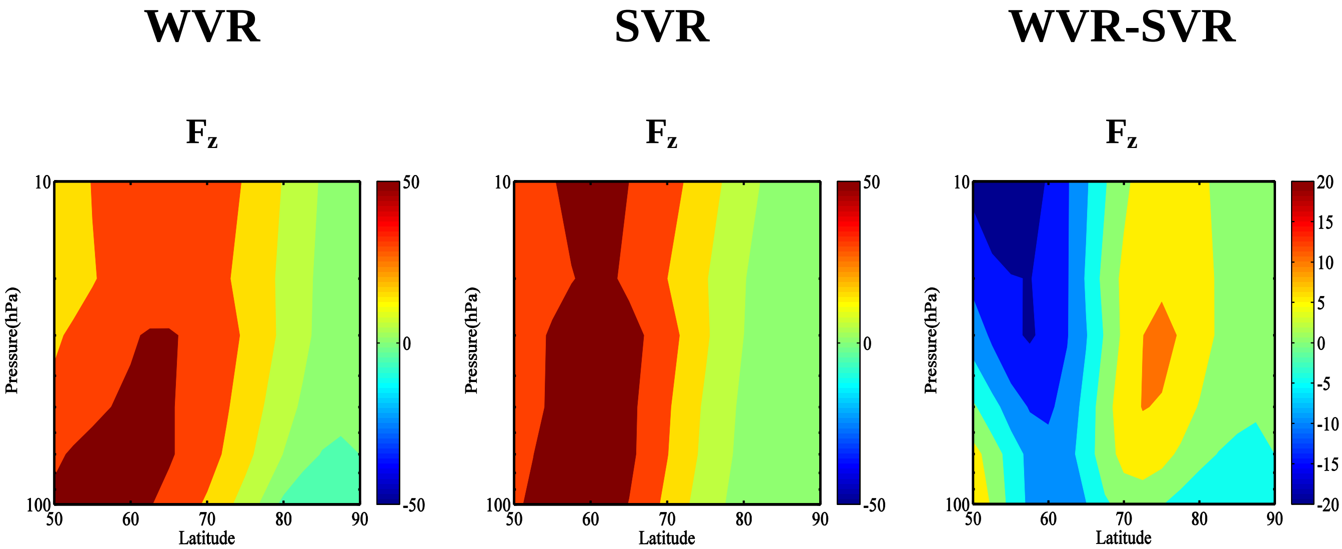


Figure 12. Same as Fig. 11 but for the vertical component of EP flux. The values are divided by 10^5. Since the highest differences in the $m^2_{k,l}(y,z)$ and $Pr_{Ro}(y,z)$ between WVR and SVR are in the high latitude stratosphere the vertical component of EP fluxes are shown in this region.

too low in the stratosphere for all waves. These MVFs block all waves in the troposphere. Furthermore, they do not provide any waveguides in which Rossby waves can penetrate from troposphere to the stratosphere.

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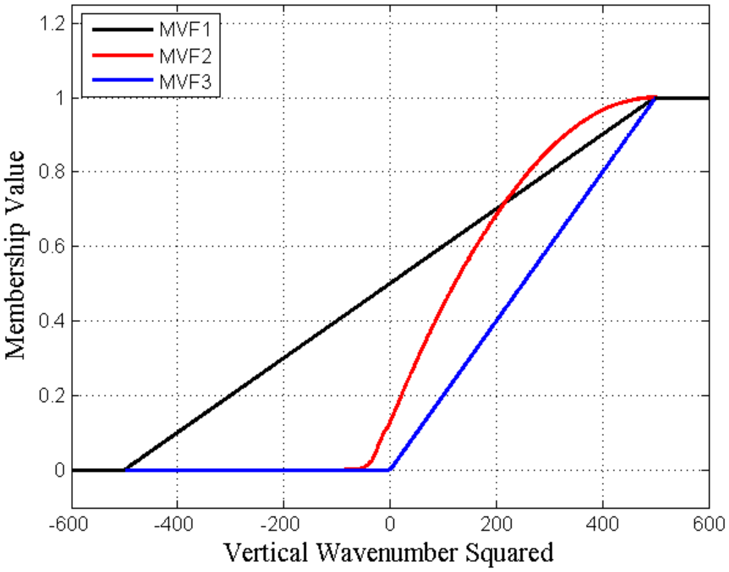

Figure A1. Shape of three MVFs that are used to calculate the values of $Pr_{Ro}(y,z)$.

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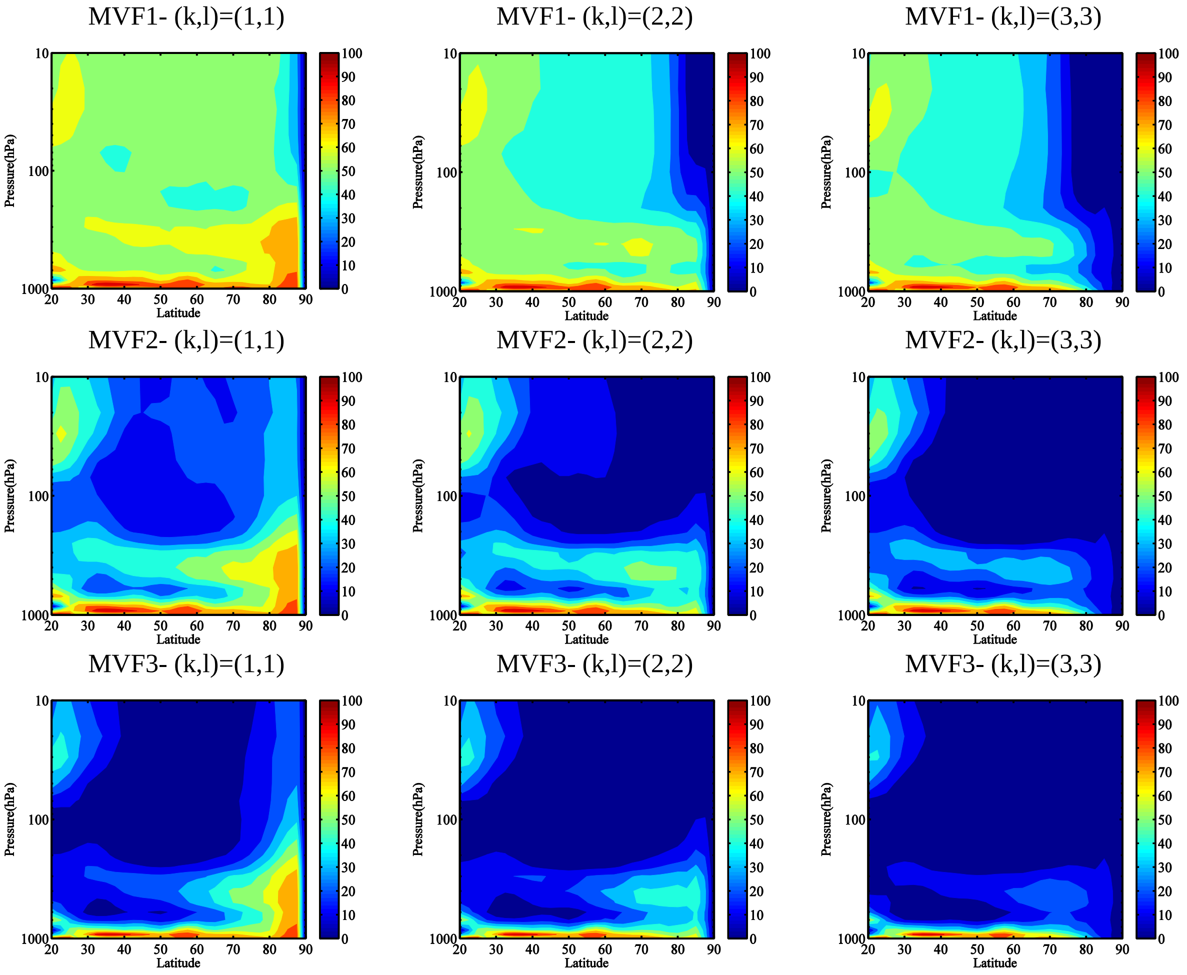

Figure A2. Probability of favorable propagation condition for Rossby waves derived from 50 winters (1961-2010) in the Northern hemisphere based on different MVF values described in Fig. A1.

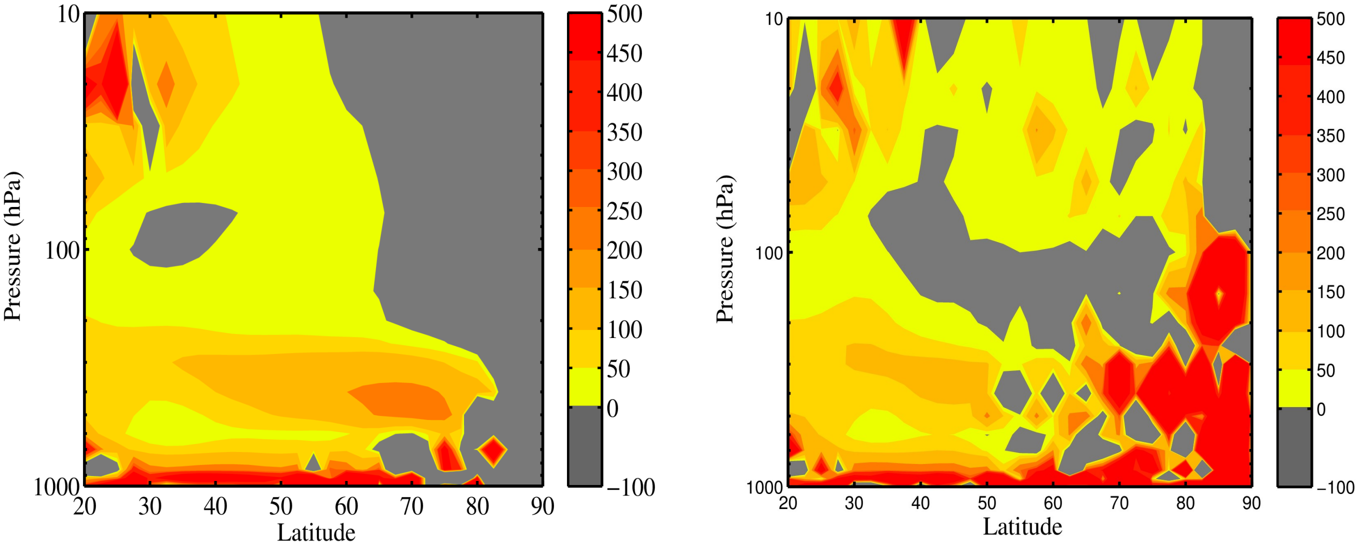

Figure A3. On the left the time averaged zonal mean fields are used to calculate the vertical wavenumber squared (only for (k,l)=(1,1)). On the right the time mean of the rafractive index squared is shown. It is clear that the vertical wavenumber derived from the time averaged zonal mean fields has less noise than the time mean vertical wavenumber squared. We discuss this effect in more detail in the manuscript. Theoretically there are various ways in which one may reduce the level of noise in the time mean of the vertical wavenumber. The advantage of our proposed method is that it maps well and in a physical way on the list of criteria formulated in Table 1. Alternatively one can use other statistical methods like truncated means or trimmed means to reduce the noisiness.