Response to Referee #1

For clarity, the mathematics relies on three assumptions:

(1) The overlap parameters depends only on scale and the two heights a and b.
(2) The mean and variance in cloud cover fraction is the same at both heights a and b.
(3) The mean and variance in cloud cover fraction is also the same in both grid boxes j and j+1.

Major Comments:

The note up to Eq. (12) seemed clear to me. But Eqs. (12-15) and (19-21) could use some extra explanation or the presentation of intermediate mathematical steps. For Eq. (12), I'd note that c_a = c_b = mu and that <c_a'2> = sigma.

(1) It will be noted in the final version of the paper that in Eq (12) $c_a(j) = c_b(j)$. That is, the cloud cover is the same at both heights (a and b) in grid box j. Hence, the mean μ and standard deviation σ in cloud cover are the same at both heights a and b. I.e.

$$\mu = \overline{c_a(j)} = \overline{c_b(j)} \quad \textit{and} \quad \sigma^2 = \overline{c_a^2(j)} - \left(\overline{c_a(j)}\right)^2 = \overline{c_b^2(j)} - \left(\overline{c_b(j)}\right)^2 = \overline{c_a^2(j)} - \mu^2$$

For ease of reading and writing the j is generally dropped in the manuscript. So the above becomes:

$$\mu = \overline{c_a} = \overline{c_b} \quad \textit{and} \quad \sigma^2 = \overline{c_a^2} - (\overline{c_a})^2 = \overline{c_b^2} - (\overline{c_b})^2 = \overline{c_a^2} - \mu^2$$

[NB. The assumption that $c_a(j) = c_b(j)$ will be dropped later on, but the mean and standard deviation are assumed fixed].

By its definition (dropping j) $\overline{c_{rand}} = \overline{c_a} + \overline{c_b} - \overline{c_a c_b}$. This, with (1) above leads to Eq. 12 as:

$$\overline{c_{rand}} = \mu + \mu - \overline{c_a c_b} = 2\mu - \overline{c_a^2} = 2\mu - \sigma^2 - \mu^2$$

For Eq. (13), I'd provide an extra step in the derivation and note that sigma and mu retain their definitions from Eq. (12) (if that's true).

(2) Eq. (13) is more complicated to derive. By its definition, $\overline{C_{RAND}} = \overline{C_a} + \overline{C_b} - \overline{C_a C_b}$. Again, assuming that the cloud cover is the same at both heights, then:

$$\overline{C_{RAND}} = \overline{C_a} + \overline{C_a} - \overline{C_a C_a} = 2\overline{C_a} - \overline{C_a^2}$$

where $C_a = \frac{1}{2}\left(c_a(j) + c_a(j+1)\right)$. Together this leads to:

$$\overline{C_{RAND}} = \overline{c_a(j)} + \overline{c_a(j+1)} - \overline{\frac{1}{2}\left(c_a(j) + c_a(j+1)\right)^2}$$

Multiplying out gives:

$$\overline{C_{RAND}} = \overline{c_a(j)} + \overline{c_a(j+1)} - \frac{1}{4}\overline{\left(c_a(j)\right)^2} - \frac{1}{4}\overline{\left(c_a(j+1)\right)^2} - \frac{1}{2}\overline{c_a(j)c_a(j+1)}$$

Assuming that the mean and variance in cloud cover is the same for both grid boxes j and j+1 (e.g. $\overline{c_a(j)} = \overline{c_a(j+1)} = \overline{c_a}$ *) then sigma and mu retain their definitions from Eq. (12) and j is redundant in the first four terms on the RHS and can be dropped to give:*

$$\overline{C_{RAND}} = \overline{c_a} + \overline{c_a} - \frac{1}{4}\overline{c_a^2} - \frac{1}{4}\overline{c_a^2} - \frac{1}{2}\overline{c_a(j)c_a(j+1)}$$

From (2) this reduces to:

$$\overline{C_{RAND}} = 2\mu - \frac{1}{2}(\sigma^2 + \mu^2) - \frac{1}{2}\overline{c_a(j)c_a(j+1)}$$

By its definition, the co-variance of $c_a(j)$ *and* $c_a(j+1)$ *is given by:*

$$Cov\left(c_a(j), c_a(j+1)\right) = \overline{c_a(j)c_a(j+1)} - \left(\overline{c_a(j)}\right)\left(\overline{c_a(j+1)}\right) = \overline{c_a(j)c_a(j+1)} - \mu^2$$

Hence,

$$\overline{C_{RAND}} = 2\mu - \frac{1}{2}(\sigma^2 + \mu^2) - \frac{1}{2}Cov\left(c_a(j), c_a(j)\right) - \frac{1}{2}\mu^2$$

The (horizontal) cross-correlation coefficient R is, by definition, given by:

$$R = \frac{Cov\left(c_a(j), c_a(j+1)\right)}{\sqrt{Var\left(c_a(j)\right)}\sqrt{Var\left(c_a(j+1)\right)}} = \frac{Cov\left(c_a(j), c_a(j+1)\right)}{\sigma^2}$$

And, so:

$$\overline{C_{RAND}} = 2\mu - \frac{1}{2}(\sigma^2 + \mu^2) - \frac{R}{2}\sigma^2 - \frac{1}{2}\mu^2$$

This gives Eq.(13)

$$\overline{C_{RAND}} = 2\mu - \frac{1}{2}(1 + R) - \mu^2$$

For Eq. (14), I'd clarify how this equation depends on c_max in Eq. (11) and what the value of c_max is.

(3) Initially we are considering the case were $c_a(j) = c_b(j)$ *which implies that:*

$$c_{max} = \max\left(c_a(j), c_b(j)\right) = c_a(j) \textit{ and, so:}$$

$$\overline{c_{max}} = \overline{c_a(j)} = \overline{c_a} = \mu$$

Similarly, as $C_a(j) = C_b(j)$ *then* $C_{MAX} = \max\left(C_a(j), C_b(j)\right) = C_a(j)$ *and again:*

$$\overline{C_{MAX}} = \overline{C_a(j)} = \overline{c_a} = \mu$$

In Eq. (15), what are 'a' and 'b'? Are they related to the two altitude levels 'a' and 'b' (see Eq. 1)? If not, can you change the variable names?

(4) No a and b in this case are the parameters of the Beta distribution. I will change the variable names to avoid this confusion.

In addition, can you write mu and sigma in terms of 'a' and 'b' for the convenience of readers?

(5) This is given in (1) above

line 21, p. 9807: What is *<c_a>* and how is it different than c_a and C_a?

(6) *This is given in (1) above in that <c_a> = mu is the long-term average of c_a*

I don't understand the derivation of Eqs. (19) and (20).

(7) *Eq. (19): For two normally distributed random variables (with the same mean μ and standard deviation σ) that are correlated (with correlation coefficient ρ) then the mean of their maximum (i.e. $\overline{c_{max}}$) is given by:*

$$\overline{c_{max}} = \mu + \sigma^2 \left(\frac{1-\rho}{\pi}\right)^{1/2}$$

This comes from the references mentioned in our 'technical note' or, in a more general form, from:

S. Nadarajah and S. Kotz, "Exact distribution of the max/min of two Gaussian random variables," IEEE Trans. Very Large Scale Integr. Syst.,vol. 16, no. 2, pp. 210–212, Feb. 2008. http://ieeexplore.ieee.org/xpl/articleDetails.jsp?arnumber=4403040

As we are using correlated uniform and triangular distributed random variables (for which I couldn't find a reference for the mean of their maximum) I have used the above equation, which is always correct when ρ=1 (as for the case in (4) above), but for Eqs. (18) and (19) I have replaced the factor σ^2/π with the number that makes $\overline{c_{max}}$ also correct when the cloud cover fractions are independent (and, so ρ=0). This mean is relatively easy to find for pairs of independent variables. For Eq. (19) this is 7/60.

(8) Eq. (20). In a similar way to the definition of R, the vertical correlation coefficient ρ is defined as:

$$\rho = \frac{Cov\left(c_a(j), c_b(j)\right)}{\sqrt{Var\left(c_a(j)\right)}\sqrt{Var\left(c_b(j)\right)}} = \frac{Cov\left(c_a(j), c_b(j)\right)}{\sigma^2} = \frac{\overline{c_a c_b} - \mu^2}{\sigma^2}$$

Hence: $\overline{c_{rand}} = (\overline{c_a} + \overline{c_b}) - \overline{c_a c_b} = 2\mu - \mu^2 - \sigma^2\rho$

[This gives Eq. (12) where ρ = 1.]

For a uniform random variable on the interval (0,1) $\mu = ½$ and $\sigma^2=1/12$ (e.g. Wikipedia). This gives the first part of Eq. (20):

$$\overline{c_{rand}} = 1 - \frac{1}{4} - \frac{1}{12}\rho = \frac{3}{4} - \frac{1}{12}\rho$$

Also:

$$\overline{C_{RAND}} = \overline{C_a} + \overline{C_b} - \overline{C_a C_b} = 2\mu - \overline{C_a C_b} = 2\mu - \tfrac{1}{4}\,\overline{\big(c_a(j) + c_a(j+1)\big)\big(c_b(j) + c_b(j+1)\big)}$$

Multiplying out:

$$\overline{C_{RAND}} = 2\mu - \tfrac{1}{4}\,\overline{c_a(j)c_b(j)} - \tfrac{1}{4}\,\overline{c_a(j)c_b(j+1)} - \tfrac{1}{4}\,\overline{c_a(j+1)c_b(j)} - \tfrac{1}{4}\,\overline{c_a(j+1)c_b(j+1)}$$

As we are only considering the case where R=0 (i.e. no horizontal correlation) this simplifies to:

$$\overline{C_{RAND}} = 2\mu - \frac{1}{4}\overline{c_a(j)c_b(j)} - \frac{1}{4}\overline{c_a(j)}\left(\overline{c_b(j+1)}\right) - \frac{1}{4}\overline{c_a(j+1)}\left(\overline{c_b(j)}\right) - \frac{1}{4}\overline{c_a(j+1)c_b(j+1)}$$

Taking R=0 thus makes j redundant in all the terms above, as the averages are the same for both j and j+1. Hence, collecting terms this becomes:

$$\overline{C_{RAND}} = 2\mu - \frac{1}{2}\mu^2 - \frac{1}{2}\overline{c_a c_b} = 2\mu - \frac{1}{2}\mu^2 \; - \frac{1}{2}(\mu^2 + \sigma^2\rho)$$

$$\overline{C_{RAND}} = 2\mu - \frac{1}{2}\mu^2 \; - \frac{1}{2}\overline{c_a c_b} = 2\mu - \mu^2 \; - \frac{1}{2}\sigma^2\rho$$

For a uniform random variable on (0,1) $\mu = ½$ and $\sigma^2=1/12$ and this gives the second part of Eq. (20):

$$\overline{C_{RAND}} = 1 - \frac{1}{4} - \frac{1}{24}\rho = \frac{3}{4} - \frac{1}{24}\rho$$

Minor Comments:

(9) These will be corrected in the final version, as will be the references missing in the reference list, but given in the text.

line 17, p. 9804: “data is discarded” should be “data are discarded”.
line 19, p. 9804: This would be somewhat redundant, but I would replace “two adjacent grid boxes” with “two horizontally adjacent grid boxes”.
line 10, p. 9805: Replace “Where . . .” with “where”, and do not indent.
line 25, p. 9807: Replace “aproach” with “approach”,