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Technical Note: The horizontal scale-dependence of the cloud overlap parameter alpha

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Abstract

The cloud overlap parameter alpha relates the combined cloud fraction between two altitude levels in a grid box to the cloud fraction as derived under the maximum and random overlap assumptions. In a number of published studies in this and other journals it is found that alpha tends to increase with increasing scale. In this technical note, we investigate this analytically by considering what happens to alpha when two grid boxes are merged to give a grid box with twice the area. Assuming that alpha depends only on scale then, between any two fixed altitudes, there will be a linear relationship between the values of alpha at the two scales. We illustrate this by finding the relationship when cloud cover fractions are assumed to be uniformly distributed, but with varying degrees of horizontal and vertical correlation. Based on this, we conclude that alpha increases with scale if its value is less than the vertical correlation coefficient in cloud fraction between the two altitude levels. This occurs when the cloud are deeper than would be expected at random (i.e. for exponentially distributed cloud depths). However, the degree of scale-dependence is controlled by the horizontal correlation coefficient in the cloud fraction between adjacent grid boxes, being greatest when this correlation is zero. Trivially, there is no scale-dependence when this correlation is one. The observed, generally strong, scale-dependence would thus indicate that the horizontal correlation is small.

1 Introduction

Clouds tend to be represented in GCMs as plane-parallel and horizontally homogeneous, with the combined horizontal cloud fraction between clouds at different altitudes specified according to various overlap schemes (e.g. Smith, 1990; Tiedtke, 1993). These schemes are generally based on a combination of maximum and random overlap. In maximum overlap the clouds are maximally overlapped in height resulting in the minimum of interaction between clouds and downward radiation. Where clouds are randomly overlapped in height the interaction with radiation is greater.

Taking advantage of the fact that clouds close together in altitude are likely maximally overlapped and those significantly different in altitude are likely randomly overlapped Hogan and Illingworth (2000) introduced a cloud overlap scheme that has since been widely taken up within GCMs. In this scheme, the mean combined cloud fraction between two altitude levels is taken to be a weighted average (with weight α) of the mean values given by maximum and random overlap assumption respectively.

The value of α is generally taken to be a function of the height separation (Δz) between the two altitudes and is found to often have an inverse exponential dependence on Δz (e.g. Hogan and Illingworth, 2000). The rate of fall is then determined by a cloud "decorrelation length" L (i.e. $\alpha = e^{-\frac{\Delta z}{L}}$). Since this initial study of Hogan and Illingworth (2000) many others have investigated how α (and L) depend on horizontal scale (e.g. Mace and Benson-Troth, 2002; Oreopoulos and Khairoutdinov, 2003; Pincus et al., 2005; Willén et al., 2005; Barker, 2008a, b; Shonk and Hogan, 2010; Oreopoulos and Norris, 2011; Oreopoulos et al., 2012). Though a number of different definitions for α and methods for deriving L have been used in such studies, they generally find that α (and, hence, L) increases with horizontal scale.

2 The overlap parameter α

From the observed horizontal cloud fractions at altitudes a and b (at a fixed scale), c_a and c_b respectively, the horizontal cloud fractions $c_{\max}$ and c_{rand} can be formed, under the maximum and random overlap schemes, as:

$$c_{\max} = \max(c_a, c_b) \tag{1}$$

$$c_{\text{rand}} = c_a + c_b - c_a c_b \tag{2}$$

From the definition as given by Hogan and Illingworth (2000) for α these are related to the combined horizontal cloud fraction, c_t (jointly covered by the clouds at both altitudes) by:

$$\overline{c_t} = \alpha \overline{c_{\max}} + (1 - \alpha)\overline{c_{\text{rand}}} \quad (3)$$

Where $\overline{c_t}$, $\overline{c_{\max}}$ and $\overline{c_{\text{rand}}}$ are the averages (over time) of c_t, $c_{\max}$ and c_{rand} respectively. Provided $\overline{c_{\max}}$ and $\overline{c_{\text{rand}}}$ are not equal to each other, which is unlikely (as this could only happen if the cloud cover fraction was always zero or one) Eq. (3) can be rearranged to give:

$$\alpha = \frac{\overline{c_t} - \overline{c_{\text{rand}}}}{\overline{c_{\max}} - \overline{c_{\text{rand}}}} \quad (4)$$

As pointed out in Pincus et al. (2005), this is only one way to define α. Another method is to determine a set of values for α using Eq. (3) based on the individual (unaveraged) values of c_t, $c_{\max}$ and c_{rand} and, from these, find an average value for α. However, this approach leads to data being discarded, as (the values for) α are not uniquely defined when either $c_a = 0$ or $c_b = 0$, potentially giving rise to truncated statistics. As the probability that $c_a = 0$ or $c_b = 0$ decreases with increasing grid size (e.g. Astin and Girolamo, 1999) it seems prudent, when considering the scale-dependence, to use Eq. (4) to define α (in which no data is discarded).

3 The horizontal scale-dependence of α

To investigate the scale-dependence of α, we will consider what happens when two adjacent grid boxes, which we label j and $j+1$ respectively, are combined to give a single larger grid box with double the area. In this case, the cloud fractions C_a and C_b at the two altitudes (a and b) in the larger grid box are given by:

$$\left.\begin{aligned} C_a &= \tfrac{c_a(j)+c_a(j+1)}{2} \\ C_b &= \tfrac{c_b(j)+c_b(j+1)}{2} \end{aligned}\right\} \quad (5)$$

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Where $c_x(y)$ is the cloud fraction in grid box y at altitude x. Again, the cloud overlap C_{MAX} and C_{RAND} (at the larger scale) are formed, under the maximum and random overlap assumptions, by:

$$C_{\text{MAX}} = \max(C_a, C_b) \tag{6}$$

$$C_{\text{RAND}} = C_a + C_b - C_a C_b \tag{7}$$

The combined cloud fraction, C_T, at the large scale is given by:

$$C_T = \frac{c_t(j) + c_t(j+1)}{2} \tag{8}$$

Where $c_t(y)$ is the combined cloud fraction in grid box y.

To continue, let α_1 be the value of α at the original scale and α_2 be the value of α when the two grid boxes are merged. As in Eq. (4), the value of α_2 is given by:

$$\alpha_2 = \frac{\overline{C_T} - \overline{C_{\text{RAND}}}}{\overline{C_{\text{MAX}}} - \overline{C_{\text{RAND}}}} \tag{9}$$

Where $\overline{C_T}$, $\overline{C_{\text{MAX}}}$ and $\overline{C_{\text{RAND}}}$ are the time averages of C_T, C_{MAX} and C_{RAND} respectively.

Assuming that α depends only on scale (and a and b) then (using Eq. 3) Eq. (8) becomes:

$$\overline{C_T} = \frac{\alpha_1 \overline{c_{\max}(j)} + (1-\alpha_1)\overline{c_{\text{rand}}(j)} + \alpha_1 \overline{c_{\max}(j+1)} + (1-\alpha_1)\overline{c_{\text{rand}}(j+1)}}{2} \tag{10}$$

Where the averages in Eq. (10) are those for grid boxes j and $j+1$ respectively.

If a and b are fixed altitudes, Eqs. (9) and (10) together imply that $\alpha_2 = m\alpha_1 + c$, where m and c are constants. If the averages in Eq. (10) are the same for both grid boxes j and $j+1$ then, dropping the j and $j+1$ dependences, Eq. (9) gives:

$$\alpha_2 = \frac{\overline{c_{\max}} - \overline{c_{\text{rand}}}}{\overline{C_{\text{MAX}}} - \overline{C_{\text{RAND}}}}\alpha_1 + \frac{\overline{c_{\text{rand}}} - \overline{C_{\text{RAND}}}}{\overline{C_{\text{MAX}}} - \overline{C_{\text{RAND}}}} \tag{11}$$

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This does not necessarily imply that a linear relationship between α_1 and α_2 will be observed, since data from different altitudes (likely having differing values of m and c) are combined in published studies. However, we can use Eq. (11) to investigate the conditions in which $\alpha_2 > \alpha_1$ (i.e. where α would increase with scale).

As an example, where the cloud cover varies between grid boxes, but is constant in altitude (i.e. $c_a(j) = c_b(j)$) then $\overline{c_{\text{max}}} = \overline{c_{\text{MAX}}}$ and so $\alpha_2 = m\alpha_1 + (1-m)$. Hence, in this case, the value of m is uniquely defined by the value of α_2 when α_1 equals zero (e.g., if $\alpha_2 = 0.2$ when $\alpha_1 = 0$ then $m = 0.8$, and $\alpha_2 = 0.8\alpha_1 + 0.2$).

It is instructive to consider this case further by studying the value of m analytically. As the cloud cover is constant with altitude, the mean, given by μ, and the variance, given by σ^2, in cloud fraction are the same at both altitudes (a and b). From its definition $\overline{c_{\text{rand}}}$ is given (from Eq. 2) by:

$$\overline{c_{\text{rand}}} = 2\mu - \sigma^2 - \mu^2 \tag{12}$$

Similarly, the average $\overline{C_{\text{RAND}}}$ is given (From Eq. 7) by:

$$\overline{C_{\text{RAND}}} = 2\mu - \frac{1}{2}(1+R)\sigma^2 - \mu^2 \tag{13}$$

Where R is the horizontal correlation coefficient in cloud fraction between the adjacent (smaller) grid boxes. Using Eq. (11), the value of m becomes:

$$m = \frac{\mu - \sigma^2 - \mu^2}{\mu - \frac{1}{2}(1+R)\sigma^2 - \mu^2} \tag{14}$$

As an example, if the cloud fraction can be modelled as a Beta (a, b) distribution (e.g. Falls, 1974; Tompkins, 2002) then:

$$m = \frac{2(a+b)}{2(a+b) + (1-R)} \tag{15}$$

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Giving:

$$\alpha_2 = \frac{2(a+b)}{2(a+b)+(1-R)}\alpha_1 + \frac{(1-R)}{2(a+b)+(1-R)} \tag{16}$$

In the simplest case where the cloud fraction in each grid box is uniformly distributed (i.e. Beta(1,1)) this becomes:

$$\alpha_2 = \frac{4}{5-R}\alpha_1 + \frac{1-R}{5-R} \tag{17}$$

(Thus, where $R = 0$ then $\alpha_2 = 0.8\alpha_1 + 0.2$).

In this contrived case (where the cloud cover is constant with height) α will always increase with scale (i.e. $\alpha_2 > \alpha_1$) provided the horizontal correlation coefficient, R, in cloud fraction between adjacent grid boxes is positive and less than 1. Trivially, when $R = 1$ there is no scale-dependence to alpha (as $m = 1$). However, as R decreases to zero the degree of the scale-dependence increases and maximises where $R = 0$. This is displayed in Fig. 1, which shows the relationship between between α_1 and α_2 for a range of values for R in the case where the cloud fraction in the adjacent grid boxes are assumed to be uniformly distributed (e.g. LeTreut and Li, 1991). For this case, the scale-dependence is strongest when $R = 0$ giving $\alpha_2 = 0.8\alpha_1 + 0.2$.

So far, we have looked at the scale-dependence where the cloud fraction varies from grid box to grid box, but does not vary with altitude. That is, when the vertical correlation between the cloud fraction at the two altitudes is $\rho = 1$. Let us now consider what happens when $\overline{c_a} = \overline{c_b}$, but $c_a(j) \neq c_b(j)$ (i.e. $\rho \neq 1$).

For illustration, and to simplify the mathematics we will take the extreme case where $R = 0$ and the cloud fractions are correlated uniform distributions, with correlation coefficient ρ. As they are uniformly distributed, $\overline{c_a} = \frac{1}{2} = \overline{c_b}$. Similarly, where $\rho = 0$ then $\overline{c_{\max}} = \frac{2}{3}$ and where $\rho = 1$ then $\overline{c_{\max}} = \frac{1}{2}$. Following the aproach of Clarke (1961), $\overline{c_{\max}}$ can be approximated, using these values, by:

$$\overline{c_{\max}} \cong \frac{1}{2} + \frac{1}{6}(1-\rho)^{1/2} \tag{18}$$

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Similarly, the value $\overline{C_{\mathrm{MAX}}}$, which is the maximum of two triangularlly distributed random variables (being the sum of two correlated Uniform distributed random variables) is given by:

$$\overline{C_{\mathrm{MAX}}} \cong \frac{1}{2} + \frac{7}{60}(1-\rho)^{1/2} \tag{19}$$

The means $\overline{c_{\mathrm{rand}}}$ and $\overline{C_{\mathrm{RAND}}}$ are themselves given by:

$$\left.\begin{aligned} \overline{c_{\mathrm{rand}}} &= \tfrac{3}{4} - \tfrac{1}{12}\rho \\ \overline{C_{\mathrm{RAND}}} &= \tfrac{3}{4} - \tfrac{1}{24}\rho \end{aligned}\right\} \tag{20}$$

Putting the above values into Eq. (11) gives:

$$\alpha_2 \cong \alpha_1 \left(\frac{30 - 10\rho - 20(1-\rho)^{1/2}}{30 - 5\rho - 14(1-\rho)^{1/2}}\right) + \left(\frac{5\rho}{30 - 5\rho - 14(1-\rho)^{1/2}}\right) \tag{21}$$

Though this is an approximate result, the simulated values given in Fig. 2 show that Eq. (21) can be taken as exact for all values of ρ. Hence, when $\rho = 0$ (i.e. where the cloud cover at both altitudes are uncorrelated) $\alpha_2 = \frac{5}{8}\alpha_1$ and so α will always decrease with scale (i.e. $\alpha_2 < \alpha_1$), except where $\alpha_1 = 0$.

It seems likely, given the linear relationship between the values of alpha at the two scales, that for every value of ρ there will be a unique value for α that does not change with scale (being the point-of-intersection with the $\alpha_1 = \alpha_2$ line). This is illustrated in Fig. 2, where the relationship between between α_1 and α_2 is displayed for a range of values for ρ (all with $R = 0$). From Fig. 2 this value seems to be where $\alpha_1 = \alpha_2 \approx \rho$. Also where $\alpha_1 > \rho$ then α will always decrease with scale and where $\alpha_1 < \rho$ then α will always increase with scale.

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4 Conclusions

Based on the definition of α and the scale invariance of the combined cloud fraction, it is clear that if α depends only on scale then the value of alpha, α_2, at one scale is linearly related the value of alpha, α_1, at the other scale (i.e. $\alpha_2 = m\alpha_1 + c$), provided the two altitudes are fixed.

The values of m and c depend on a number of parameters including the mean and variance in cloud fraction at each altitude. However, the most important parameters are the horizontal correlation coefficient, R, between the cloud fractions in adjacent grid boxes (at a given altitude) and the vertical correlation coefficient, ρ, between the cloud fractions at the two altitudes.

Dependent on the relative values of α and ρ it is possible for α to increase, decrease or stay the same with increasing scale. However, the strength of the dependence is controlled by R. Published results obscure the linear relationship between α_2 and α_1 by combining data from pairs of altitudes of fixed height separation with differing cloud cover means and variances. However, our results indicate that an "on average" increase of α with scale implies that "on average" α must generally be smaller than ρ.

In Astin and Di Girolamo (2006) we showed that on average $\alpha \approx \rho$ when cloud depths follow an exponential distribution. Hence, we conclude that published increase of alpha with scale is a consequence of clouds being generally deeper than would be expected at random (i.e. in a Random Markov Field).

As the scale-dependence disappears when $R = 1$ and is strongest when $R = 0$, we conclude that published strong dependence on scale would imply that R must be small. This supports the argument that there is a "dominant horizontal scale for cloud size", since R would tend to unity for very small or very large clouds.

Supplementary material related to this article is available online at http://www.atmos-chem-phys-discuss.net/14/9801/2014/acpd-14-9801-2014-supplement.zip.

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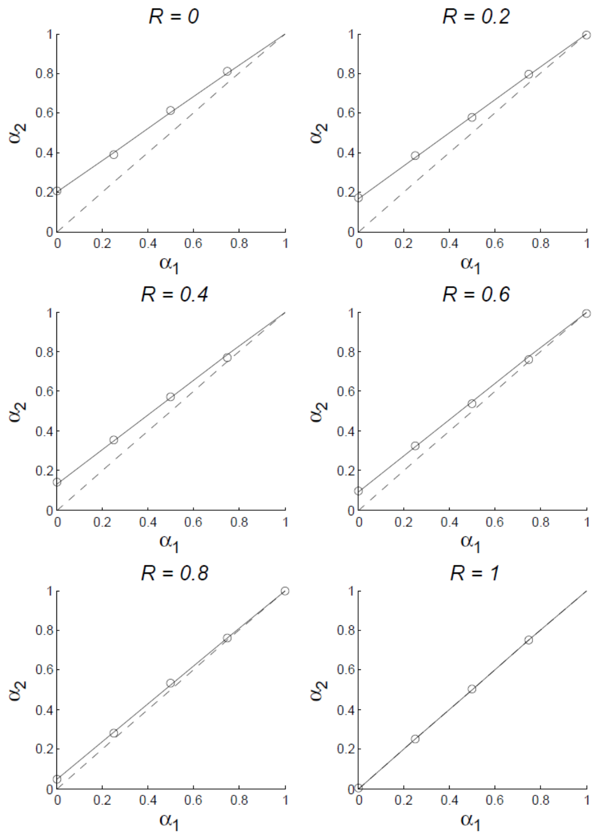

Fig. 1. The dependence of α_2 on α_1 for cloud fractions (in adjacent grid boxes) that are uniformly distributed, where the vertical correlation coefficient in cloud cover $\rho = 1$ and the horizontal correlation coefficient in cloud cover is R (solid line). The dashed line is where there would be no scale dependence to α (i.e. $\alpha_2 = \alpha_1$). The circles are values given by simulation.

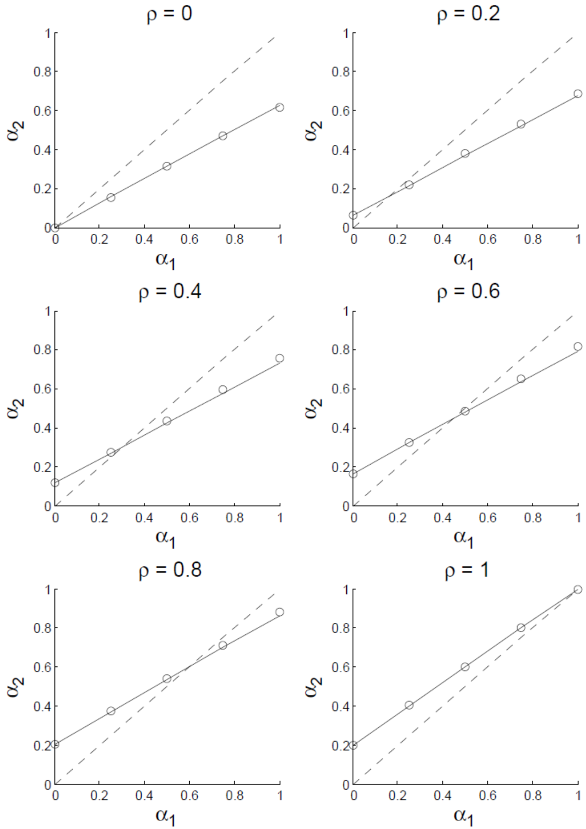

Fig. 2. The dependence of α_2 on α_1 for cloud fractions that are uniformly distributed (solid line), where the horizontal correlation coefficient in cloud cover is $R = 0$, and the vertical correlation coefficient in cloud cover is ρ. The dashed line is where there would be no scale dependence to α (i.e. $\alpha_2 = \alpha_1$). The circles are values from simulation.

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