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Ergodicity test of the eddy correlation method

J. Chen, Y. Hu, Y. Yu, and S. Lü

Key Laboratory of Land Surface Process and Climate Change in Cold and Arid Regions; Cold and Arid Regions Environment and Engineering Institute, Chinese Academy of Sciences, Lanzhou 730000, China

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Correspondence to: J. Chen (chenjinbei@lzb.ac.cn)

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Abstract

The turbulent flux observation in the near-surface layer is a scientific issue which researchers in the fields of atmospheric science, ecology, geography science, etc. are commonly interested in. For eddy correlation measurement in the atmospheric surface layer, the ergodicity of turbulence is a basic assumption of the Monin–Obukhov (M–O) similarity theory, which is confined to steady turbulent flow and homogenous surface; this conflicts with turbulent flow under the conditions of complex terrain and unsteady, long observational period, which the study of modern turbulent flux tends to focus on. In this paper, two sets of data from the Nagqu Station of Plateau Climate and Environment (NaPlaCE) and the cooperative atmosphere–surface exchange study 1999 (CASE99) were used to analyze and verify the ergodicity of turbulence measured by the eddy covariance system. Through verification by observational data, the vortex of atmospheric turbulence, which is smaller than the scale of the atmospheric boundary layer (i.e., its spatial scale is less than 1000 m and temporal scale is shorter than 10 min) can effectively meet the conditions of the average ergodic theorem, and belong to a wide sense stationary random processes. Meanwhile, the vortex, of which the spatial scale is larger than the scale of the boundary layer, cannot meet the conditions of the average ergodic theorem, and thus it involves non-ergodic stationary random processes. Therefore, if the finite time average is used to substitute for the ensemble average to calculate the average random variable of the atmospheric turbulence, then the stationary random process of the vortex, of which spatial scale was less than 1000 m and thus below the scale of the boundary layer, was possibly captured. However, the non-ergodic random process of the vortex, of which the spatial scale was larger than that of the boundary layer, could not be completely captured. Consequently, when the finite time average was used to substitute for the ensemble average, a large rate of error would occur with use of the eddy correction method due to losing the low frequency component information of the larger vortex. When the multi-station observation was compared with the single-station observation, the wide sense of stationary random

process originating from the multi-station observation expanded from a vortex which was about 1000 m smaller than a boundary layer scale to the turbulent vortex, which was larger than the boundary layer scale of 2000 m. Therefore, the calculation of the turbulence average or variance and turbulent flux could effectively meet the ergodic assumption, and the results would be approximate to the actual values. Regardless of vertical velocity and temperature, if the ergodic stationary random processes could be met, then the variance of the vortexes in the different temporal scales could follow M–O similarity theory; in the case of the non-ergodic random process, its vortex variance deviated from the M–O similarity relations. The exploration of ergodicity in the atmospheric turbulence measurements is doubtlessly helpful to understanding the issues in atmospheric turbulent flux observation, and provides a theoretical basis for overcoming related difficulties.

1 Introduction

The land surface process, of which the core is mass–energy exchange, between ecosystem and atmosphere under complicated conditions, has been a scientific issue which urgently requires study in the fields of atmospheric science, ecology, geography science, etc. (Running et al., 1999; Geider et al., 2001). A core goal of FLUXNET and relevant scientific research is to determine the turbulent flux of mass (moisture and CO^2) and energy (sensible heat) between ecosystem and atmosphere, and thus the eddy correlation method, which is used to measure atmospheric turbulent flux, is widely applied (Dennis et al., 2001). Being generally based on the assumed constant flux layer and Monin–Obukhov (M–O) similarity theory, the whole layer atmospheric turbulent flux is determined by the eddy correlation method in the atmospheric surface layer. According to the spectral gap (around 60 min) between the turbulence scale and synoptic scale of the wind velocity spectrum in the atmospheric surface layer, the observational data (McMillen, 1988; Moore, 1986) are first amended to eliminate the interference of synoptic scale motion during the turbulence observation. After the observational errors

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of the instruments had been eliminated, the average, $\overline{u}$, was determined within 15–60 min, the turbulence component, $u' = u - \overline{u}$, was obtained, and finally the turbulent flux of the mass and energy between ecosystem and atmosphere was calculated and determined by means of variance and covariance. With respect to the M–O similarity theory, the constant flux layer requires that the flow field is steady and homogeneous, i.e., the average vertical velocity does not exist. Therefore, many experiments of atmospheric boundary layer focus on seeking ideal homogeneous surfaces as much as possible. In addition, the rotation of the coordinate is highlighted in the error correction of the eddy correlation method (Finnigan, 1983; Wilczak et al., 2001), and such correction is originally used to eliminate the vertical velocity due to the inclined installation of the instrument. However, the turbulent flux is often measured under complex terrain conditions in FLUXNET, and the rotation of the coordination in the error correction eliminates the effects of the average vertical velocity of terrain on the turbulent flux. After analysis, Finnigan (2004) found that the rotation of coordination eliminated the low frequency effect caused by natural terrain. When surface energy imbalance, NEE (Net Ecosystem Exchange) estimation error, and other problems occurred, and it was necessary to consider the low frequency effect (Foken et al., 2006; Segal et al., 1988; Mahrt et al., 1993; Sun et al., 1997; Finnigan et al., 1995; Sakai et al., 2001; Jia-yi et al., 2006), and many methods were proposed to estimate the low frequency effect on the turbulent transport flux (Lee, 1998; Zhang et al., 2010; Baldocchi, 2000; Aubinet et al., 2003; Staebler et al., 2004; Jinbei et al., 2007, 2013). In the rotation of coordinates for correction in the eddy correction method, eliminating the average vertical velocity and estimating the low frequency effect of the vortex of the transport flux were essentially contradictory. According to Kaimal and Wyngaard (1990), the atmospheric turbulence theory and observational method were feasible and led to success under ideal conditions (including a short period, steady state and homogeneous surface, and through observation in the 1950s–1970s) but these conditions are rare in reality. In the land surface process and ecosystem, the observations must be implemented under conditions such as with complex terrain, heterogeneous surface, long period and unsteady state. More modern observational tools and theories will be applied with new perspectives in future research.

The basic principle of the turbulence measurement average is the ensemble average of space, time and state. However, it is impossible to set numerous observational instruments in space and have enough time to obtain all states of the turbulent vortex to realize the ensemble average in actual turbulence measurement experiments. Therefore, based on the ergodic assumption that it is temporally steady and spatially homogeneous, the time average of one spatial point, which is long enough for observation, was used to substitute for the ensemble average (Stull, 1988; Wyngaard, 2010; Aubinet, 2012). The ergodic assumption was first raised by Boltzmann (1871) in his study of ensemble theory of statistical dynamics, performed in 1871 (Uffink, 2004). He argued that an isolated system began from any initial state would undergo all possible microstates after a certain amount of time. At the beginning of the 20th century, the P. Ehrenfest duo proposed the ergodic hypothesis and changed the term "experience" in the aforesaid ergodic hypothesis to "infinitely approximate". The basic point of the ergodic hypothesis or accurate ergodic hypothesis was recognizing that the macroscopic property of the system in the equilibrium state was the average of the microcosmic quantity in a certain amount of time. Nevertheless, the ergodic or accurate ergodic hypothesis was never proven theoretically. The proof of the ergodic hypothesis in physics aroused the interest of mathematicians, and Neumann et al. (1932) first theoretically proved the ergodic theorem (Birkhoff, 1931) in topological space. Krengel (1985) then systematically summarized related achievements. However, the ergodic theorem expressed in the time series by the theory of stationary random process is further visualized in physics. The stationary random process is a random process in which the statistical properties do not vary with time. When the limit of the autocorrelation function of the stationary random process converges to its average square, this random process is ergodic and is identical to the ergodic theorem of the stationary random process. The ergodic theorem also provides the necessary and sufficient conditions for the ergodicity of the stationary random process. The ergodic theorem is adopted in this

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paper, and there will be a detailed statement in the following section. Mattingly (2003) reviewed the research progress of ergodicity of random Navier–Stokes equations which had been made in recent years, and Galanti (2004) solved the random Navier–Stokes equation by numerical value simulation to prove that the turbulence which was temporally steady and spatially homogeneous was ergodic (Lennaert et al., 2006). However, he also indicated that such partially turbulent flows acting as mixed layer, wake flow, jet flow, flow around and boundary layer flow may be non-ergodic turbulence. Gabriel et al. (2004) qualitatively analyzed the problems in ergodicity regarding atmospheric turbulence, and believed that it was common for neutral and unstable stratified atmosphere near the surface layer to reach ergodicity, while it was difficult for the stable layer to reach ergodicity. However, the author did not perform quantitative testing or theoretical demonstration of direct observational experiments related to the ergodicity of the atmospheric turbulence. Therefore, it is clear that there is a need to reevaluate turbulence measurement technology, to test the ergodicity of atmospheric turbulence quantitatively by means of observational experiments.

In the spatial scale, the atmospheric turbulence from the dissipation range, inertial sub-range to energy range, and further the large eddy of turbulent flow is extremely broad (Stull, 1988). Such spatial and temporal size of vortexes include the isotropous 3-D vortex structure of high frequency turbulence and orderly coherent structure of low frequency turbulence (Li et al., 2002). The vortexes in different scales are also different in terms of their spatial structure and physical properties, and even their transport characteristics are not all the same. It is thus reasonable that the vortexes with different transport characteristics are separated, processed and studied by using different methods (Zuo et al., 2012).

Based on the aforesaid analysis, in this study the data from the Nagqu Station of Plateau Climate and Environment were used to measure turbulence by the eddy correlation method under the homogeneous surface and the Fourier transform band-pass filtering method was used to make filtering of different scales. Then the ergodicity of different scale vortexes of atmospheric turbulence was directly tested quantitatively on the basis of the observational data. In addition, the cooperative surface layer turbulence data of the Kansas, US prairie (CASE99) were used to verify the ergodicity of the turbulence measured by multi-station observations. The characteristics of the M–O variance similarity relations of the vortexes in different scales were compared and analyzed to test the feasibility of the M–O similarity of the ergodic and non-ergodic turbulence. The problems of the eddy correlation method in the atmospheric turbulence observation in the surface layer were further explored on the basis of the study on the ergodicity and M-O variance similarity relations of the vortexes in different scales in this paragraph in order to provide an experimental basis for utilizing the M–O similarity theory and developing the transport theory of turbulence in atmospheric boundary layers with complex underlying surfaces.

2 Theories and methods

2.1 Ergodic theorem of stationary random process

The stationary random process is a random process which will not vary with time, i.e., for observed quantity A, its spatial x_i and temporal t_i functions meet the following conditions:

$$A(x_1, x_2, \ldots, x_n; t_1, t_2, \ldots, t_n) = A(x_1, x_2, \ldots, x_n; t_1 + \tau, t_2 + \tau, \ldots, t_n + \tau), \quad (1)$$

where τ is a time period, defined as the relaxation time.

The average μ_A of random variable A and autocorrelation function $R_A(\tau)$ are respectively defined as follows:

$$\mu_A = \lim_{T\to+\infty} \frac{1}{T} \int_0^T A(t)\mathrm{d}t = E[A(t)], \tag{2}$$

$$R_A(\tau) = \lim_{T\to+\infty} \frac{1}{T} \int_0^T A(t)A(t+\tau)\mathrm{d}t = E[A(t)A(t+\tau)]. \tag{3}$$

Autocorrelation function $R_A(\tau)$ is a temporal second-order moment. In the case of $\tau = 0$, the autocorrelation function $R_A(\tau)$ is the variance of a random variable. The necessary and sufficient condition of the stationary random process average to have ergodicity is the average ergodic function Ero(A) (Wang et al., 2009), as shown below:

$$\mathrm{Ero}(A) = \lim_{T\to\infty} \frac{1}{2T} \int_0^{2T} \left(1 - \frac{\tau}{2T}\right) \left[R_A(\tau) - |\mu_A|^2\right] \mathrm{d}\tau = 0. \tag{4}$$

The average ergodic function Ero(A) is the time integral of variance between autocorrelation function $R_A(\tau)$ of variable A and its average, μ_A. If the average ergodic function Ero(A) converges to zero, then the stationary random process will be ergodic. In other words, if the autocorrelation function $R_A(\tau)$ of variable A converges to the square of its average μ_A, this stationary random process is average ergodic. Equation (4) is the average ergodic theorem. For discrete variables, Eq. (4) can be rewritten as the following:

$$\mathrm{Ero}(A) = \lim_{n\to\infty} \sum_{i=0}^{n} \left(1 - \frac{\tau_i}{n}\right) \left[R_A(\tau_i) - |\mu_A|^2\right] = 0. \tag{5}$$

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Equation (5) is the average ergodic theorem of the discrete variable. Hence, Eqs. (4) and (5) can be used as the basis to determine the average ergodicity.

The necessary and sufficient condition the stationary random process must meet for the autocorrelation ergodic theorem is the autocorrelation ergodic function Er(A):

$$\mathrm{Er}(A) = \lim_{T\to\infty} \frac{1}{2T} \int_0^{2T} \left(1 - \frac{\tau'}{2T}\right) \left[B(\tau') - |R_A(\tau)|^2\right] \mathrm{d}\tau' = 0, \tag{6a}$$

$$B(\tau') = E\left\{A(t+\tau+\tau')A(t+\tau')[A(t+\tau)A(t)]\right\}. \tag{6b}$$

where $B\ (\tau')$ is the temporal fourth-order moment of variable A. Autocorrelation ergodic function Er(A) is the time integral of variance between the temporal fourth-order moment $B\ (\tau')$ of variable A and autocorrelation function $R_A\ (\tau)$. If the autocorrelation ergodic function Er(A) converges to zero, then the stationary random process will be of autocorrelation ergodicity, and thus the autocorrelation ergodicity means that the fourth-order moment of the variable of the stationary random process will converge to the square of its autocorrelation function $R_A\ (\tau)$. Equation (6a) is the autocorrelation ergodic theorem. The autocorrelation ergodic function of the corresponding discrete variable can be determined as follows:

$$\mathrm{Er}(A) = \lim_{n\to\infty} \sum_{i=0}^{n} \left(1 - \frac{\tau_i'}{n}\right) \left[B\left(\tau_i'\right) - \left|R_A\left(\tau_j\right)\right|^2\right] = 0, \tag{7a}$$

$$B\left(\tau_i'\right) = E\left\{\sum_{j=0}^{n} A\left(t+\tau_j+\tau_i'\right) A\left(t+\tau_i'\right) \left[A\left(t+\tau_j\right) A(t)\right]\right\}. \tag{7b}$$

Equation (7a) is the ergodic theorem of the autocorrelation function of the discrete variable. Hence, Eqs. (6a) and (7a) can also be used as the basis to test the autocorrelation ergodicity.

The stationary random process conforms to Eqs. (4) and (5) signifying that it agrees with the average ergodic theorem, or that the random process is of average ergodicity; if the stationary random process conforms to Eqs. (6a) and (7a), then it meets the autocorrelation ergodic theorem, or the random process is of autocorrelation ergodicity. If the stationary random process is only of average ergodicity, then it is a strictly stationary random process or narrowly stationary random process. If the stationary random process is of both average ergodicity and autocorrelation ergodicity, then it is a wide sense stationary random process. It is thus clear that the ergodic random process is stationary, but the stationary process may not be ergodic.

With respect to the random process theory, when its average and autocorrelation functions are calculated, a large amount of repeated observations of the random process is required to determine sample function $A_k(t)$. If it is a stationary random process and meets the ergodic conditions, then the time average of a sample of the whole time shaft can be used to substitute for the overall or ensemble average. The conditions of Eqs. (4), (5), (6a) and (7a) can be used as the basis to judge whether or not the random variable meets the average and autocorrelation ergodicity. The ergodic random process must be stationary, and the stationary random process is defined as Eq. (1), and thus the random process is stationary in relaxation time τ. If conditions such as Eqs. (4) and (5) of the average ergodicity are met, then a time average in finite relaxation time τ can be used to substitute for the infinite time average to calculate average Eq. (2) of the random variable; similarly, the finite time average can be used for substitution to calculate the covariance or variance of random variable (Eq. (3)) if conditions such as Eqs. (6a) and (7a) of the autocorrelation ergodicity are met. In a similar manner, the basic principle of the turbulence measurement average is the ensemble average of space, time and state, and it is necessary to conduct mass observation for a long period of time in the whole space. This observation requires a very large investment and is hardly feasible. If the turbulence signal meets the ergodic conditions, the time average in relaxation time τ by multi-station observation, and even single-station observation, can be used to substitute for the ensemble average. In fact, the precondition to estimate the turbulent features (including turbulent flux) by the eddy correlation method is that the turbulence meets the ergodic conditions. Therefore, conditions such as Eqs. (4), (5), (6a) and (7a) will also be the basis for testing the authenticity of the observed results by the eddy correlation method.

2.2 Band-pass filtering

The turbulence in the atmospheric boundary layer is large in scale. A major goal of our study is to understand what type of vortex in the scale can meet the ergodic conditions. Another goal is to use the time average of the signal measured by a single station for the accurate measurement of the turbulent features. To study the ergodicity of the vortexes in different scales, Fourier transform was used as band-pass filtering to separate the vortexes in different scales. That is to say, we set the frequency spectrum to be removed when filtering to zero in the Fourier transform, then determined the signal after filtering by means of Fourier inverse transformation. The specific formula is shown below:

$$F_A(n) = \frac{1}{N}\sum_{k=0}^{N-1} A(k)\cos\left(\frac{2\pi nk}{N}\right) - \frac{i}{N}\sum_{k=0}^{N-1} A(k)\sin\left(\frac{2\pi nk}{N}\right), \tag{8}$$

$$A(k) = \sum_{n=a}^{n=b} F_A(n)\cos\left(\frac{2\pi nk}{N}\right) + i^2\sum_{n=a}^{n=b} F_A(n)\sin\left(\frac{2\pi nk}{N}\right). \tag{9}$$

In Eqs. (8) and (9), $A(k)$ indicates N data points from $k = 0$ to $k = N-1$, and n is the cycle index of the observational time range. Through high-pass filtering it is possible to cut off the low frequency turbulence and obtain a high frequency turbulence signal. Although the aliasing of a half high frequency turbulence after the Fourier transformation cannot be avoided, the correction for high frequency response will compensate for the loss. In order to acquire a purely high frequency signal, the band-pass filtering results from $n = j$ to $n = N-j$ of the high frequency signal were obtained in the filtering

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process. This is referred to as j time filtering in this paper. Finally, the ergodicity of the vortexes in the different scales was analyzed using Eqs. (4)–(6).

2.3 M–O similarity of turbulence variance

The M–O similarity relations of the turbulence variance can be regarded as an effective measure to verify whether or not the turbulent flow field is steady and homogeneous (Foken et al., 2004). Under ideal conditions, the local M–O similarity relations of variance of wind velocity, temperature and other factors can be expressed as follows:

$$\sigma_i/u_* = \varphi_i(z/L), \quad (i = \boldsymbol{u}, \boldsymbol{v}, \boldsymbol{w}), \tag{10}$$

$$\sigma_s/|s_*| = \varphi_s(z/L), \quad (s = \theta, q, \rho_c), \tag{11}$$

where σ is the turbulence variance; corner mark i is the wind velocity u, v or w; s stands for scalar, such as potential temperature θ, humidity q and CO_2 concentration ρ_c; u_* is the friction velocity and defined as $u_* = \left(\overline{\boldsymbol{u}'\boldsymbol{w}'}^2 + \overline{\boldsymbol{v}'\boldsymbol{w}'}^2\right)^{1/4}$; s_* is the turbulent feature of the related scalar and is defined as $s_* = -\overline{\boldsymbol{w}'s'}/u_*$; and M–O length L is defined as shown below:

$$L = u_*^2\theta/[\kappa g(\theta_* + 0.61\theta q_*/\rho_d)]. \tag{12}$$

A large number of research results show that, in the case of unstable stratification, $\phi_i(z/L)$ and $\phi_s(z/L)$ can be expressed in the following forms (Panofsky et al., 1977; Padro, 1993; Katul et al., 1999), under ideal conditions:

$$\phi_i\left(z/L\right) = c_1\left(1 - c_2 z/L\right)^{1/3}, \tag{13}$$

$$\phi_s\left(z/L\right) = \alpha_s\left(1 - \beta_s z/L\right)^{-1/3}, \tag{14}$$

where c_1, c_2, α and β are the undetermined coefficients. In the case of stable stratification, $\phi_s(z/L)$ is approximate to the constant and $\phi_i(z/L)$ is still the $1/3$ function of

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z/L. The turbulence characteristics of the vortexes in the different temporal and spatial scales in the atmosphere are compared and analyzed with Eqs. (13) and (14), to test the feasibility of the M–O similarity under conditions of the ergodic and non-ergodic turbulence.

3 Observation site and data processing

Two sets of data were used in the study. The first included the atmospheric surface layer data measured by a 10 Hz 3-D ultrasonic wind and temperature tester (CSAT3) and infrared gas analyzer (Li7500) at the Nagqu Station of Plateau Climate and Environment, Chinese Academy of Sciences, from 23 July 2011 to 13 September 2011. The second data set was collected from the 20 Hz atmospheric surface layer at seven observation points (CASE99) in the Kansas prairie (Poulos et al., 2002; Chang et al., 2002). The two sets of data, collected for completely different purposes, were compared to test the universality of the research results. The geographic coordinates of Nagqu Station are 31.37° N, 91.90° E, and its altitude is 4509 m a.s.l. The observation station covers an area of 8000 m^2, is built on flat and wide area, the ground surface is mainly composed of sandy soil mixed with some fine stones, and an alpine meadow with vegetation 10–20 cm in height grows in the area (see Fig. 1a). The CASE99 data used included the data measured by a 10 m high 3-D wind and temperature tester (ATI) and on the central tower (37.65° N, –96.74° W) of 55 m height; and the turbulence data were measured by a 10 m high 3-D ultrasonic wind velocity system (ATI) and infrared gas analyzer (Li7500) on six small towers surrounding the main tower. The small towers, sn1, sn2 and sn3 were located 100 m away from the main tower, the sn4 tower was 280 m away, and towers sn5 and sn6 were located 300 m away. The specific positions were as shown in Fig. 1b. Similar to Nagqu Station, the CASE99 observation field was flat and there were grasses of 20–50 cm in height present during the test period. The displacement height of the underlying surface of the Nagqu meadow was determined to 0.03 m by calcu-

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lation, while the displacement height of the CASE99 underlying surface was 0.06 m (Martano, 2002).

The inaccurate data in the measurements caused by circuit, pulse and other factors were deleted before data analysis. In addition, ultrasonic temperature pulse was corrected to absolute temperature pulse (Schotanus et al., 1983; Kaimal et al., 1991). Then the coordinate was rotated using the plane fitting method to improve the installation level (Wilczak, 2001). In the view that moisture and CO_2 were components of the air, their pulsation was also a constituent part of the air density pulsation, and the turbulent pulsation of air temperature is actualized by air molecule pulsation. Therefore, there was no related correction of the humidity or CO_2 pulsation caused by air density fluctuation. In addition, according to our preliminary analysis, such correction may also cause the results to unreasonably deviate from the prediction shown in Eq. (14). The Webb correction (Webb et al., 1980) is the component of the surface energy balance in physical nature, but not the component of the turbulent vortex. We thus did not perform Webb correction on our research objectives of the ergodicity of the vortexes in the different scales.

4 Result analysis

4.1 Verification of average ergodic theorem of vortexes in different temporal scales

We took the data measured at the Nagqu Station at the height of 3.08 m during three time frames, namely 03:00–04:00, 07:00–08:00 and 13:00–14:00 China Standard Time on 25 August, in clear weather, as the case to test the average ergodicity of the vortexes in different temporal scales. These three time frames can represent three situations, namely the nocturnal stable boundary layer, early neutral boundary layer and midday convective boundary layer.

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The trend correction (McMillen, 1988; Moore, 1986) of the data measured in the eddy correlation method has been widely accepted. In nature, this is a type of high-pass filtering which is used to exclude the influence of the low frequency effect of temperature and other diurnal variations on turbulent flux. In order to acquire the effective information of the vortexes in the different temporal scales, first Eqs. (8) and (9) were used to perform band-pass filtering of the Nagqu 3.08 m turbulence data, which was equivalent to the correction of the high-pass filtering. In addition, the results of the time band-pass filtering from $n = j$ to $n = N - j$ corresponding to Eqs. (8) and (9) indicated the information of the vortex in the corresponding temporal scale. The band-pass filtering information of the different time frames was thereby utilized to study the turbulence characteristics and the ergodicity of the vortexes in the different temporal scales of the six time frames, including 2 min, 3 min, 5 min, 10 min, 30 min and 60 min.

Considering that the features of the different scale vortexes of atmospheric turbulence varied with the atmospheric stability parameter z/L, a local M–O stratification stability parameter that was limited in the certain scale range of vortexes was defined as $(z-d)/L_c$, so as to analyze the relationship between the stratification stability and average ergodicity of the wind velocity, temperature and other factors of the vortexes in the different scales. It is noted that the local stability is different from the M–O stratification stability $(z-d)/L$.

We took the local stability of the vortexes in the different temporal scales of the three time frames from nighttime to daytime as an example, as shown in Table 1.

The results show that the local stability parameter $(z-d)/L_c$ of the vortex below 2 min in temporal scale during the time frame of 03:00–04:00 (nighttime) was 0.59, thus it was stable stratification. For the vortex for which the temporal scale gradually increased from 3 min, 5 min and 10 min to 60 min, the $(z-d)/L_c$ also gradually decreased to 0.31 and 0.28. In addition, beginning from the vortex of 10 min in the temporal scale, even the local stability decreased, namely from −0.01 to −0.07. It seemed that the local stability gradually varied from stability to instability as the temporal scale of vortex increased. During the time frame of 07:00–08:00 (morning), the $(z-d)/L_c$ of the

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vortexes from 2 min to 60 min in the temporal scale eventually decreased from 0.52, 0.38, 0.16 and 0.15 to a minimum of −1.29, which meant that the vortex in the temporal scales of 30 min and 60 min had high local instability. However, during the time frame of 14:00–15:00 (midday), the $(z-d)/L_c$ of the vortex from 2 min to 60 min in the temporal scale was unstable. As the scale of the vortex increased, the local instability of the vortexes on the scale of 2 min to 3 min also increased, and the instability value reached the maximum of 0.44 when the scale of the vortex was 5 min; the scale of the vortex continuously increased, but the local instability of the vortex decreased.

The M–O local stability of a vortex is not entirely the same as the M–O stratification stability of the boundary layer in terms of physical significance, and the M–O stratification stability of the boundary layer indicates that the overall effect of the atmospheric stratification in the boundary layer on the stability of all vortexes in integral effect. The local stability of the vortex is only a local effect of the atmospheric stratification on the stability of the vortex in a certain scale. As the scale of the vortex increases, the local stability of the vortex will vary accordingly. The aforesaid results indicate that the local stability of small-scale vortexes was stable in the nocturnal stable boundary layer, but the nocturnal stable boundary layer was possibly unstable for the large-scale vortexes, so as to result in a sink effect on the vortexes in the small-scale vortexes but a buoyancy effect on the large-scale vortexes. However, in the diurnal unstable boundary layer, the local stability of the vortex of 3 min in scale reached the maximum, the instability of the smaller vortexes decreased, but the instability gradually decreased as the scale of the vortex increased. Therefore, the vortex of 3 min in the scale bore maximum buoyancy, but the buoyancy of the vortex decreased as the scale of the vortex increased. In addition, the small-scale vortexes was more stable than the vortexes in the large scale in the nocturnal stable boundary layer; while the large-scale vortexes were more stable than the vortexes in the small scales in the diurnal unstable and convective boundary layers. The above observations signify that it is common for the small-scale vortexes to exist in the nocturnal stable boundary layer, and it is also common for the large-scale vortexes to exist in the diurnal convective boundary layer. Therefore, it is clear that the

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small-scale vortexes are dominant in the nocturnal stable boundary layer, while the large-scale vortexes are dominant in the diurnal convective boundary layer.

Finally, we calculated the autocorrelation function of the vortexes in the different temporal scales using Eq. (5), as well as the variation of the average ergodic function Ero(A) with relaxation time τ if relaxation time $\tau_{i=n}$ was cut off, and verified the ergodic theorem of average value. The average ergodic function Ero(A) of the vertical velocity, temperature and specific humidity of the vortexes in the different scales of the three time frames of 03:00–04:00, 07:00–08:00 and 13:00–14:00 China Standard Time were measured at the Nagqu Station at the height of 3.08 m, and varied with relaxation time τ, as shown in Figs. 2–4a, b and c, respectively. To facilitate comparison, Fig. 5 shows the variation of the average ergodic function Ero(A) of vertical velocity (a), temperature (b) and specific humidity (c) before filtering during the time frame of 14:00–15:00 (midday) with relaxation time τ. Since the ergodic function varied within a large range, the ergodic functions were normalized according to the features of their variables. That is to say, the functions in the following figures are dimensionless ergodic functions.

The characteristics of the average ergodicity of turbulence, as well as comprehensive analysis on related causes, are as follows:

1. Verifying average ergodic theorem of vortexes in different scales: according to the average ergodic theorem of vortexes, Eq. (4), the average ergodic function Ero(A) will converge to 0 when time approaches infinity. This is a theoretical result of the stationary random process. However, the calculated average ergodic function was obtained under the condition that relaxation time $\tau_{i=n}$ was cut off. If the average ergodic function Ero(A) is approximately 0 in relaxation time $\tau_{i=n}$, it will be considered that A approximately meets the average ergodic theorem; if the average ergodic function deviates further from zero, the average ergodicity will be far lower, so as to approximately determine whether or not the average ergodic theorem of the vortexes in different scales is established. Figures 2–4 clearly show that, regardless of vertical velocity, temperature or humidity, the Ero(A) of vortexes below 10 min in the temporal scale will fluctuate around zero within a small range;

thus we may conclude that the average ergodic function Ero(A) of the vortex below 10 min in the temporal scale converges to zero and can effectively meet the conditions of the average ergodic theorem. For the vortexes of 30 min and 60 min, if the vortex is larger in scale, then the average ergodic function Ero(A) will deviate further from zero. In particular, the average ergodic function Ero(A) of the vortexes of 30 min and 60 min of the temperature or humidity does not converge, and even diverges. The above results show that the average ergodic function of the vortexes of 30 min and 60 min cannot converge to zero or meet the conditions of the average ergodic theorem.

2. Comparison of the convergence of the average ergodic function of vertical velocity, temperature and humidity: as seen from Figs. 2–4, if the dimensionless average ergodic function of the vertical velocity is compared with the function value of the temperature or humidity, it is 3–4 magnitudes less than those in the nocturnal stable boundary layer; 1–2 magnitudes less than those in the early neutral boundary layer; and around 2 magnitudes less than those in the midday convective boundary layer. For example, during the time frame of 03:00–04:00 (nighttime), the dimensionless average ergodic function of the vertical velocity is 10^{-5} in magnitude, while the respective magnitudes of the function value of the temperature and humidity are 10^{-1} and 10^{-2}; during the time frame of 07:00–08:00 (morning), the magnitude of the dimensionless average ergodic function of the vertical velocity is 10^{-4}, while the respective magnitudes of the function value of the temperature and humidity are 10^{-2} and 10^{-3}; during the time frame of 13:00–14:00 (midday), the magnitude of the dimensionless average ergodic function of the vertical velocity is 10^{-4}, while the magnitudes of the function values of the temperature and humidity are both 10^{-2}. These results show that the dimensionless average ergodic function of the vertical velocity converges to zero more frequently than the function value of the temperature and humidity, and that the vertical velocity meets the conditions of the average ergodic theorem more easily than the temperature and humidity.

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3. Temporal scale and spatial scale of turbulent vortex: for wind velocity of 1–2 ms^{-1}, the spatial scale of the vortex of 2 min in the temporal scale is around 120–240 m, and the spatial scale of the vortex of 10 min in the temporal scale is around 600–1200 m. The spatial scale of the vortex of 2 min in the temporal scale is equivalent to the height of the surface layer, and the special scale of the vortex of 10 min in the temporal scale is equivalent to the height of the atmospheric boundary layer. The spatial space of the vortex within 30–60 min in the temporal scale is around 1800–3600 m, and this spatial scale exceeds the height of the atmospheric boundary layer. According to stationary random process definition (1) and the average ergodic theorem, the stationary random process must be stable in relaxation time τ. The vortex below 10 min in the temporal scale in the height of the atmospheric boundary layer is a stationary random process, and can effectively meet the conditions of the average ergodic theorem. However, the vortexes of 30 min and 60 min in the temporal scale exceed the height of the atmospheric boundary layer and do not meet the conditions of the average ergodic theorem, and thus these vortexes belongs to the nonstationary random process.

4. Ergodicity of turbulence of all vortexes in the possible scales of the atmospheric boundary layer: Fig. 5 shows the unfiltered average ergodic function of the vortexes in possible scales in the atmospheric boundary layer. When Fig. 5 is compared with Figs. 2c, 3c and 4c, for the turbulence of all vortexes in possible scales in the boundary layer, during the time frame of 14:00–15:00 (midday), the average ergodic function Ero(A) of the vertical velocity, temperature and humidity of the convective boundary layer before filtering is greater than the average ergodic function of the turbulence of the vortexes in the different scales after filtering. As shown in Figs. 2c, 3c and 4c, the magnitude of the vertical velocity is 10^{-4} and the magnitudes of the temperature and specific humidity are both 10^{-2}; according to Fig. 5, the magnitude of the vertical velocity Ero(A) is 10^{-3} and the magnitudes of the temperature and specific humidity are both 10^{0}, therefore 1–2 magnitudes are almost increased. In addition, all trend upward (vertical velocity and temper-

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ature) or downward (specific humidity), deviating from zero. It is thus clear that, even if the time of day is 14:00–15:00, the average ergodic function of all vortexes in the possible scales in the convective boundary layer cannot converge to zero before filtering, and thus local circulation in the convective boundary layer cannot meet the conditions of the average ergodic theorem. We argue that, under general conditions, the vortex below 10 min in the temporal scale or within 600–1200 m in the spatial scale within the height of the atmospheric boundary layer is the ergodic stationary random process, and the turbulence of the vortexes in all possible scales including the boundary layer may belong to the non-ergodic stationary random process.

5. Relations between ergodicity and local stability of vortexes in different scales: the corresponding local stability parameters $(z-d)/L_c$ of vortexes at different times in different scales (see Table 1) show that the local stability parameters $(z-d)/L_c$ of the vortexes in the different scales are different, due to the fact that the temperature stratification in the atmospheric boundary layer has different effects on the stabilities of the vortexes in the different scales. Entirely different results can occur, and the stratification which can cause the vortexes in the large scale to rise may cause the vortexes in the small scale to descend at the same time. However, the analysis results in Figs. 2–4 show that the ergodicity is mainly related to the vortex scale, and its relation with the atmospheric temperature stratification is not significant.

4.2 Verification of autocorrelation ergodic theorem for vortexes in different scales

In the following section, Eqs. (7a) and (7b) are used to verify the autocorrelation ergodic theorem. It was identified in Sect. 3.1 that the turbulent vortexes below 10 min in the temporal scale meet the average ergodic conditions in the various time frames, i.e., the turbulent vortexes below 10 min in the temporal scale are at least in strictly stationary random processes or narrow stationary random processes in the nocturnal stable boundary layer, early neutral boundary layer and midday convective boundary layer. Then these vortexes are used to further analyze whether or not the turbulent vortexes in the different scales which meet the average ergodic conditions also meet the autocorrelation ergodic conditions, so as to verify whether atmospheric turbulence is in the narrow stationary random process or wide sense stationary random process. The ergodic theorem of the autocorrelation function of the turbulence variable under the condition of truncated relaxation time $\tau_{i=n}$ were calculated according to Eq. (7a) to determine the variation of the ergodic theorem of autocorrelation function Er(A) with relaxation time τ. As with the average ergodic function Ero(A), if the ergodic theorem of the autocorrelation function Er(A) of the vortexes of 2 min, 3 min, 5 min, 10 min, 30 min and 60 min in the temporal scale within the relaxation time $\tau_{i=n}$ is approximate to 0, then A shall be deemed to be approximately ergodic; the more the ergodic theorem of the autocorrelation function deviates from 0, the worse the autocorrelation ergodicity becomes. Therefore, this method should be used for approximating whether vortexes in the different scales meet the conditions of the autocorrelation ergodic theorem or ergodicity.

For example, Fig. 6 shows the variation of the ergodic theorem of autocorrelation function Er(A) of the turbulent vortexes of 2 min, 3 min, 5 min, 10 min, 30 min and 60 min in the temporal scale of vertical velocity during the time frames of 03:00–04:00, 07:00–08:00 and 13:00–14:00 with relaxation time τ. Some basic conclusions are drawn from Fig. 6:

1. After comparing Fig. 6a–c with Fig. 2a–c, the dimensionless average ergodic function Ero(A) of the vertical velocity with the dimensionless ergodic theorem of autocorrelation function Er(A) of the vertical velocity, two basic characteristics are very clear. First, the magnitudes of the dimensionless ergodic theorem of autocorrelation function Er(A), regardless of whether in the nocturnal stable boundary layer, early neutral boundary layer or midday convective boundary layer, are all greatly reduced. In Fig. 2a–c, the magnitudes of Ero(A) are respectively 12^{-5}, 10^{-4} and

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10^{-4}, and the magnitudes of Er(A) are respectively 12^{-7}, 10^{-5} and 10^{-5}, as shown in Fig. 6a–c. The magnitudes of Er(A) are reduced by 1–2 compared with those of Ero(A). Second, all ergodic theorem of autocorrelation functions Er(A) of the vortexes of 30 min and 60 min in the temporal scale, regardless of whether they are in the stable boundary layer, natural boundary layer or convective boundary layer, are all reduced and approximate to Ero (A) of the vortex below 10 min in the temporal scale.

2. The above two basic characteristics imply that the ergodic theorem of the autocorrelation function Er(A) of the stable boundary layer, neutral boundary layer or convective boundary layer converges to 0 faster than the average ergodic function Ero(A); the ergodic theorem of the autocorrelation function of the vortexes of 30 min and 60 min in the temporal scale also converge to 0 and meet the conditions of the autocorrelation ergodic theorem, except for the fact that the ergodic theorem of autocorrelation function Er(A) of the vortex below 10 min in the temporal scale can converge to 0 and meet the conditions of the autocorrelation ergodic theorem.

3. According to the ergodic theorem of the autocorrelation function theorem, both vortexes of 30 min and 60 min and the vortex below 10 min in the temporal scale, regardless of whether they are in the stable boundary layer, neutral boundary layer or convective boundary layer, can meet the conditions of the ergodic theorem of autocorrelation function Eq. (7a), i.e., they can meet the conditions of the ergodic theorem. Therefore, in general the turbulence in the atmospheric boundary layer is in the autocorrelation ergodic stationary random process.

4. The above observation results show that the vortexes below 10 min in the temporal scale in the nocturnal stable boundary layer, early neutral boundary layer and midday convective boundary layer can not only meet the conditions of the average ergodic theorem, but they can also meet the conditions of the autocorrelation ergodic theorem. Therefore, the vortexes below 10 min in the temporal scale are

wide sense stationary random processes. Although the vortexes of 30 min and 60 min in the temporal scale in the stable boundary layer, neutral boundary layer and convective boundary layer can meet the conditions of the autocorrelation ergodic theorem, they cannot meet the conditions of the average ergodic theorem. Therefore, the vortexes of 30 min and 60 min in the temporal scale are only autocorrelation ergodic random processes, rather than narrow stationary random processes or wide sense stationary random processes.

4.3 Verification of ergodic theorem of vortexes in different scales measured by multiple stations

The basic principle of the turbulence measurement average is the ensemble average of space, time and state. Sections 3.1 and 3.2 verify the average ergodic theorem and ergodic theorem of the autocorrelation function of the atmospheric turbulence during the stationary random process using observational data, so that the finite time average of a single station is used to substitute for the ensemble average. This section examines the ergodicity of the vortexes in different scales according to the observational data collected at the CASE99 tower and six observation sites (seven stations). When the data were selected, it was considered that if the vortex was not evenly distributed at the seven stations, then the observational results at the seven stations may have originated from many vortexes in a large scale. For this reason, we first compared the high frequency variance spectrum above 0.1 Hz. Based on the observational error, if the difference of all high frequency variances does not exceed the average by ±10 %, then it is assumed that the turbulence is evenly distributed at the seven observation stations. Finally, 17 data sets were collected from among the turbulence observational data from 5 to 30 October, and these data sets refer to the results of strong turbulence at noon on a sunny day. As an example, the same method as described in Sects. 3.1 and 3.2 is used to respectively calculate the variation of the average ergodic function and ergodic theorem of the autocorrelation function of the vertical velocity at 10:00–11:00 on 7 October with relaxation time τ. Next, the observational data collected from

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the seven stations are built into a data set, and the time series of the data set are filtered at 2 min, 3 min, 5 min, 10 min, 30 min and 60 min; the variation of the average ergodic function Ero(W) and ergodic theorem of the autocorrelation function Er(W) of the vertical velocity with relaxation time τ is analyzed to test the ergodicity of vortexes in the different scales in the multi-station observational data. Figure 7a shows the variation of the average ergodic function Ero(W) of the vertical velocity with relaxation time τ, and Fig. 7b shows the variation of the ergodic theorem of the autocorrelation function Er(W) with relaxation time τ. The results are as follows:

1. Ergodic characteristics of the vortexes in the different scales measured at the multi-stations: Fig. 7a shows that the average ergodic function of the vortexes below 30 min in the temporal scale converges to 0 very well, except for the fact that the average ergodic function of vortex of 60 min in the temporal scale clearly deviates upward from 0. Figure 7b shows that all ergodic theorems of the autocorrelation functions of the vortexes in the different scales, including the vortex of 60 min in the temporal scale, gradually converge to 0. Therefore, the vortexes below 30 min in the temporal scale measured at the multi-stations meet the conditions of both the average and autocorrelation ergodic theorems, while the vortex of 60 min in the temporal scale only meets the conditions of autocorrelation ergodic theorem, but cannot meet the conditions of the average ergodic theorem. These observations demonstrate that the vortexes below 30 min in the temporal scale are wide sense stationary random processes in the data series composed of observational data collected from the seven stations. This signifies that the temporal scale of the vortex during the wide sense stationary random process has extended from below 10 min to 30 min in the data series composed of observational data collected from multiple stations, compared with the observational data collected from a single station. As analyzed above, if the vortex below 10 min in the temporal scale is deemed to be a turbulent vortex in the 1000 m boundary layer and the vortex of 30 min in the temporal scale is deemed to be a local circulated vortex in the greater than 2000 m boundary layer, then multiple station

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observations can completely capture the local circulated vortex of 30 min in the temporal scale in the boundary layer.

2. Average time problem of turbulent feature average: according to the average ergodic theorem, if the condition of average ergodic theorems Eqs. (4) or (5) is met, then a time average of finite relaxation time τ is used to substitute for the average of the infinite time and calculate the average random variable Eq. (2). This signifies that the calculation of the turbulence average is determined by the average ergodic theorem, and is also closely related to the scale of the turbulent vortex. The analysis on the ergodicity of vortexes in the different scales in the above two sections demonstrates that the vortexes below 10 min in temporal scale at τ=30 min in the stable boundary layer, neutral boundary layer and convective boundary layer can not only meet the conditions of the average ergodic theorem, but can also meet the conditions of the autocorrelation ergodic theorem. That is to say, they are both wide sense stationary random processes. Therefore, the finite time average of 30 min within relaxation time τ is used for substituting for the ensemble average to calculate average random variable Eq. (2). However, the vortexes of 30 min and 60 min in the temporal scale in the stable boundary layer and neutral boundary layer are only autocorrelation ergodic random processes, rather than narrow and wide sense random processes. Therefore, when the finite time average of 30 min is used for substituting for the ensemble average to calculate average random variable Eq. (2), it may capture the stationary random processes of the vortex below 10 min in the temporal scale, but not completely capture the nonstationary random process of the vortexes above 30 min in the temporal scale. In the observation performed using the eddy correlation method, the substitution of the ensemble average with finite time average of 30 min inevitably results in a high level of error, due to lack of low frequency component information of the large-scale vortexes. However, although the vortexes of 30 min and 60 min in the temporal scale in the convective boundary layer are not wide sense stationary land processes, they are autocorrelation ergodic random pro-

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cesses. This may imply that the average random variable which is calculated with the finite time average in the convective boundary layer to substitute for the ensemble average is often superior to the results of the stable boundary layer and neutral boundary layer. In addition, the results in the previous sections also show that the dimensionless average ergodic function of the vertical velocity may more easily converge to 0 than the functions corresponding to the temperature and humidity, and the vertical velocity may more easily meet the conditions of average ergodic theorem than the temperature and humidity. Therefore, in the observation performed using the eddy correlation method, the results of the vertical velocity are often superior to those of the temperature and humidity. In this section, the results also point out that multi-station observation is capable of completely capturing the circulated vortex in the local boundary layer. Therefore, ergodic assumption is more likely to be met, and its results are much closer to the true values when calculating the turbulence average, variance or turbulent flow with the multi-station observational data.

4.4 M–O similarity of turbulent vortexes in different scales and its relation with ergodicity

Turbulent variance is the most basic turbulent feature. Turbulence velocity variance, which represents turbulence intensity, and the variance of scalars, such as temperature and humidity, effectively describes the structural characteristics of turbulence. In order to test the relation of the M–O similarity of the turbulent vortexes in the different scales and ergodicity, and take it as an example of the above ergodic testing, the vertical velocity and temperature data of Nagqu from 23 July to 13 September are used to determine the M–O similarity of the vertical velocity and temperature variances for the vortexes in the different scales, and analyze its relation with the ergodicity. The following data were deleted during data processing: data involving changes in weather over a short time period and rainfall; nonstationary random process data when the average temperature changes rapidly within 10 min in the morning or evening; and data involving irregular variation of positive and negative values of local stability $(z-d)/L_c$ of vortexes.

Figures 8 and 9 respectively show the similarity curves of the vortexes in the different scales for the vertical velocity and temperature variances in Nagqu, where (a), (b) and (c) are respectively the similarity curve of vortexes of 10 min, 30 min and 60 min in the temporal scale; Table 2 also shows the below fitting curve of the similarity of the vertical velocity variance and relevant parameters:

$$\phi_i\left(z/L\right) = c_1\left(1-c_2 z/L\right)^{1/3}, z/L < 0, \tag{15}$$

$$\phi_i\left(z/L\right) = c_1\left(1+c_2 z/L\right)^{1/3}, z/L > 0. \tag{16}$$

The correlation coefficient and residual in the fitting curve are respectively expressed with R and S.

Figure 8 and Table 2 show that the parameters of the fitting curve are greatly different, even if the fitting curve of similarity of the vertical velocity variance for the vortexes in the different temporal scales is the same. The correlation coefficients of the fitting curve of similarity of the vertical velocity variance at unstable stratification are large, but the correlation coefficients at stable stratification are small. At unstable stratification, the correlation coefficient of the vortex of 10 min in the temporal scale reaches 0.97, while the residual is only 0.16; at stable stratification, the correlation coefficient reduces to 0.76, but the residual increases to 0.25. With the increase of the temporal scale of the vortex from 10 min (Fig. 8a) to 30 min (Fig. 8b) and 60 min (Fig. 8c), the correlation coefficients of similarity of the vertical velocity variance gradually reduce, but the residual increases. The correlation coefficient in 60 min reaches a minimum; it is only 0.83 at unstable stratification, and only 0.30 at stable stratification.

The temperature variance is shown in Fig. 9. The below function is fitted from the vortex of 10 min in the temporal scale at unstable stratification:

$$\phi_\theta\left(z/L_c\right) = 4.9\left(1-79.7 z/L_c\right)^{-1/3}. \tag{17}$$

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As shown in Fig. 9a, the correlation coefficient of the fitting curve is −0.91 and residual is 0.38. With the increase of the temporal scale of the vortex, the discreteness of similarity of the temperature variance is enlarged quickly, and an appropriate curve is not fitted.

The above results show that the discreteness of similarity of the turbulence variance is enlarged with the increase of either the vertical velocity or temperature with the temporal scale of the vortex. The data points collected during the stationary process basically gather near the fitting curve of the variance similarity, while all data points during the nonstationary process deviate significantly from the fitting curve. However, the similarity of the vertical velocity variance is greater than the similarity of the temperature variance. These observations are the same as the testing conclusions of ergodicity for the vortexes in the different scales described in Sects. 3.1–3.3. The ergodicity of the vortex in the small scale is superior to that of the vortex in the larger scale, and the vortex of 10 min in the temporal scale has the best variance similarity function. These observations also signify that when the vortex at the stationary random process meets the ergodic conditions, then both the vertical velocity variance and temperature variance of the vortexes in the different temporal scales comply with the M–O similarity theory very well; but, as for the vortex during nonstationary random process or with poor ergodicity, the vortex variance deviates from the M–O similarity relation.

5 Conclusion and discussion

The below preliminary conclusions are drawn after the above results were verified by partial observational data:

1. The turbulence in the atmospheric boundary layer is a vortex structure; when the temporal scale of the turbulent vortex in the atmosphere surface layer is about 2 min, the corresponding spatial scale is about 120–240 m; when the temporal scale of the turbulent vortex in the atmospheric boundary layer is about 10 min,

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the corresponding spatial scale is about 600–1200 m. As for the vortexes in the larger temporal and spatial scale, such as the vortexes of 30–60 min in the temporal scale, the corresponding spatial scale is about 1800–3600 m. spatial scale exceeds the height of the atmospheric boundary layer.

2. As for the atmospheric turbulent vortex below the scale of the atmospheric boundary layer, i.e., the vortex below 1000 m in the spatial scale and below 10 min in the temporal scale, its average ergodic function Ero(A) and ergodic theorem of autocorrelation function Er(A) converge to 0, and they can meet the conditions of the average ergodic theorem and autocorrelation ergodic theorem. However, as for the atmospheric turbulent vortex above 2000–30 000 m in the spatial scale and above 30–60 min in the temporal scale, its average ergodic function does not converge to 0, that is, it cannot meet the conditions of the average ergodic theorem. Therefore, the atmospheric turbulent vortex below the scale of the atmospheric boundary layer belongs to the wide sense stationary random process, but the atmospheric turbulent vortex above the scale of the atmospheric boundary belongs to the non-ergodic random process, or even the nonstationary random process.

3. Although the atmospheric temperature stratification has different effects on the vortexes in the different scales of stability, the ergodicity is mainly related to the local stability of the vortexes, and its relation with the stratification stability of the atmospheric boundary layer is not significant.

4. Due to the fact that the atmospheric turbulent vortex below 10 min in the temporal scale and below 1000 m of the atmospheric boundary layer in the spatial scale belongs to the wide sense stationary random process, the vortex above 30 min in the temporal scale magnitude and above 2000 m in the spatial scale cannot meet the ergodic conditions, and belongs to the non-ergodic stationary process. When an average of finite time is used for substituting for the ensemble average of infinite time to calculate the average random variable of the atmospheric turbulence, it may capture the stationary random process information of the vortex

below 10 min in the temporal scale and below 1000 m of the atmospheric boundary layer in the spatial scale, which meets the conditions of the average ergodic theorem, but it does not completely capture the nonstationary random information of the turbulent vortex above 30 min in the temporal scale and above 2000 m in the spatial scale. This will inevitably cause a high level of error due to the lack of low frequency component information of the vortex in the large scale when the average of finite time is used to substitute for the ensemble average in the observation using the eddy correlation method.

5. In the data set composed of observational data collected from the seven stations, the vortexes below 30 min in the temporal scale belong to the wide sense stationary random processes. The temporal scale and spatial scale of the vortex during the wide sense stationary random process have extended from below 10 min to 30 min, and from below 1000 m to 2000 m in the data series composed of observational data collected from many stations, compared with the observational data collected from a single station. This signifies that the ergodic assumption is more likely to be met and the observational results produced with the eddy correlation method are much closer to the true values when calculating the turbulence average, variance or turbulent flow with multi-station observational data.

6. If the stationary random process of the ergodic conditions is more effectively met, then the turbulence variance of the vortexes in the different temporal scales can comply with M–O similarity theory very well; however, the turbulence variance during the non-ergodic random process deviates from the M–O similarity relation.

The below important lessons are made based on the above preliminary conclusions:

1. Galanti (2004) proved that the turbulence which was temporally steady and spatially homogeneous was ergodic, but “partially turbulent flows” such as the mixed layer, wake flow, jet flow, flow around and boundary layer flow may be non-ergodic turbulence. According to Galanti, it is clear that the turbulence in the atmospheric

boundary layer is "partially turbulent flow", and it may be non-ergodic. However, it has been proven through observational data that the ergodicity of turbulence is related to the scale of the turbulent vortex. The average ergodic theorem and autocorrelation ergodic theorem for the turbulent vortex in the small scale in the atmospheric boundary layer is applicative, and the vortex in the large scale was non-ergodic. Since the vortex in the large scale in the atmospheric boundary layer may be strongly influenced by the boundary disturbance, it belongs to "partial turbulence"; however, since the atmospheric turbulence in the small scale might not be influenced by boundary disturbance, then it belongs to the ergodic stationary process, which is temporally steady and spatially homogeneous.

2. As analyzed in Sect. 3.1, the corresponding spatial scale of the vortex of 2 min in the temporal scale is 100 m, which amounts to the height range of the atmosphere surface layer; the corresponding spatial scale of the vortex of 10 min in the temporal scale is 1000 m, which is equal to the height range of the atmospheric boundary layer; since the vortex in the large scale may have exceeded the height range of the boundary layer, it may be influenced by the heterogeneous surface in the large scale, and thus it is temporally unsteady and spatially heterogeneous, and non-ergodic. The same conclusion was drawn from different perspectives, in accordance with Galanti's opinion.

3. Monin–Obukhov similarity theory is used for the measurement of atmospheric turbulent flow, which is developed on the conditions of steady time and homogeneous surface. The homogeneous and steady conditions are in line with the ergodic conditions, i.e., temporally steady and spatially homogeneously, as described by Galanti. Therefore, the eddy correlation method for turbulence measurement is based on the ergodic assumption and similarity theory of the atmosphere surface layer. We realized from these conclusions that the vortex in the large scale may include non-ergodic random process components which exceeded the height of the atmospheric boundary layer. The eddy correlation

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method for the measurement and calculation of turbulent variance and covariance may not capture the information of the large scale vortex outside the boundary layer, thus resulting in large error.

The turbulent flux observation in the near-surface layer is a scientific issue which commonly interests researchers in the fields of atmospheric science, ecology, geography science, etc. For eddy correlation measurement in the atmospheric surface layer, the ergodicity of the turbulence is a basic assumption for M–O similarity theory, which is confined to steady turbulent flow and homogenous surface. This conflicts with turbulent flow under the conditions of complex terrain and unsteady, long observational period, which are focused on by the study of turbulent flux. In the present paper, the study of ergodicity during the measurement of atmospheric turbulent flow undoubtedly contributes to providing evidence for overcoming the challenges which occur during the modern measurement of atmospheric turbulent flow. It is clear that such studies are preliminary, and many problems require further research. In the future, we shall further study the ergodic problems during the measurement of atmospheric turbulent flow under the conditions of complex terrain and unsteady, long observational period, to determine effective solutions.

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Table 1. Local stability parameter $(z-d)/L_c$ of the vortexes in different temporal scales on 25 August.

Time bucket Vortex scale	03:00–04:00	07:00–08:00	14:00–15:00
$\le$ 2 min	0.59	0.52	−0.38
$\le$ 3 min	0.31	0.38	−0.44
$\le$ 5 min	0.28	0.16	−0.40
$\le$ 10 min	−0.01	0.15	−0.34
$\le$ 30 min	−0.04	−0.43	−0.27
$\le$ 60 min	−0.07	−1.29	−0.30

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Table 2. Parameters of similarity and fitting curve of vertical velocity variance.

	10 min		30 min		60 min	
	$z/L < 0$	$z/L > 0$	$z/L < 0$	$z/L > 0$	$z/L < 0$	$z/L > 0$
c_1	1.08	1.17	1.06	1.12	0.98	1.06
c_2	4.11	3.67	3.64	3.27	4.62	2.62
R	0.97	0.76	0.94	0.56	0.83	0.30
S	0.19	0.25	0.17	0.27	0.25	0.31

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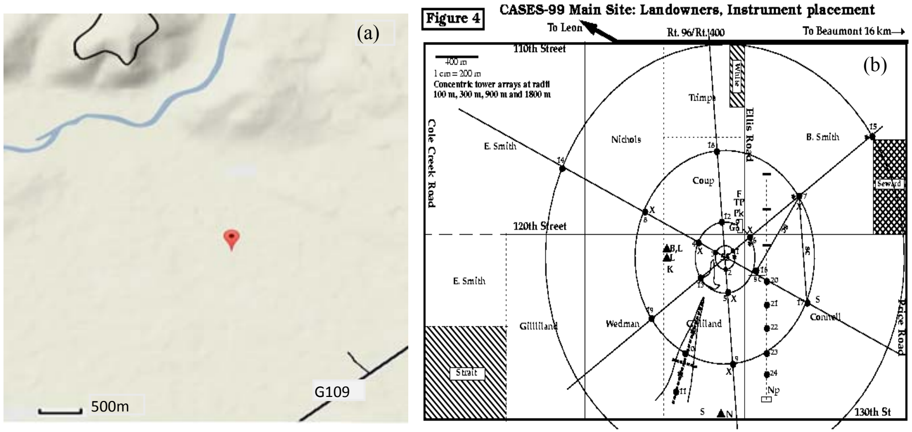

Figure 1. Overview diagram of Nagqu Station of Plateau Climate and Environment **(a)** and CASE99 observation station **(b)**.

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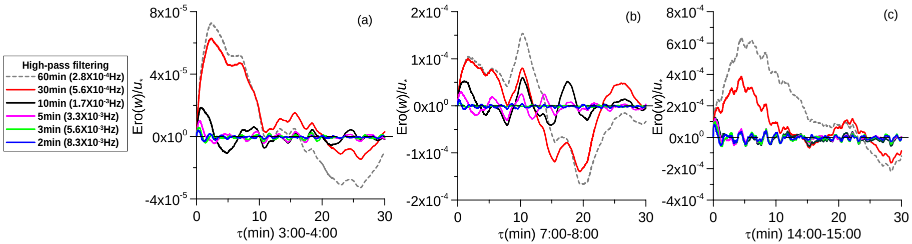

Figure 2. Variation of average ergodic function Ero(W) of vortexes in different scales of vertical velocity measured at Nagqu Station at the height of 3.08 m in the three time frames with relaxation time. Panels **(a)**, **(b)** and **(c)** are the respective results of the three time frames. If their average ergodic function is more approximate to zero, then the average of the vortexes in the corresponding temporal scale will more closely meet the ergodic conditions.

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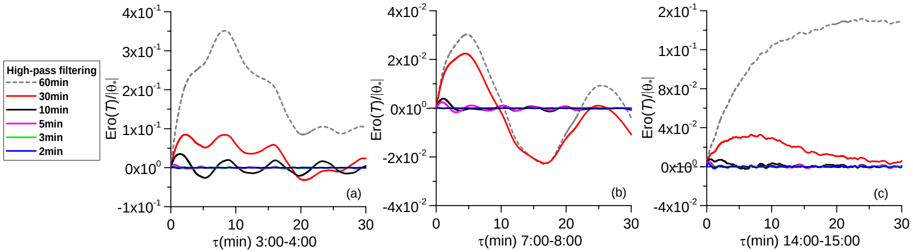

Figure 3. Variation of average ergodic function Ero(θ) of the vortexes in the different scales of temperature with relaxation time (other conditions are similar to Fig. 2, and the same applies to the following figures).

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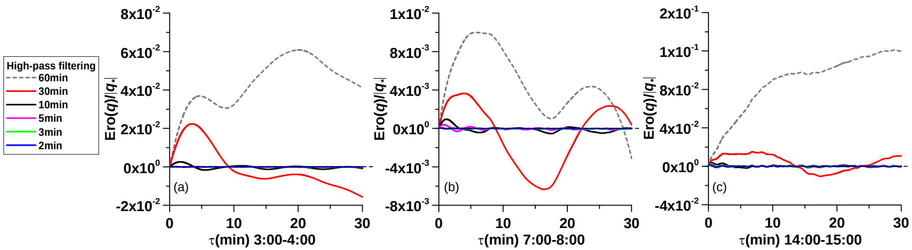

Figure 4. Variation of average ergodic function Ero(q) of the vortexes in the different scales of humidity with relaxation time.

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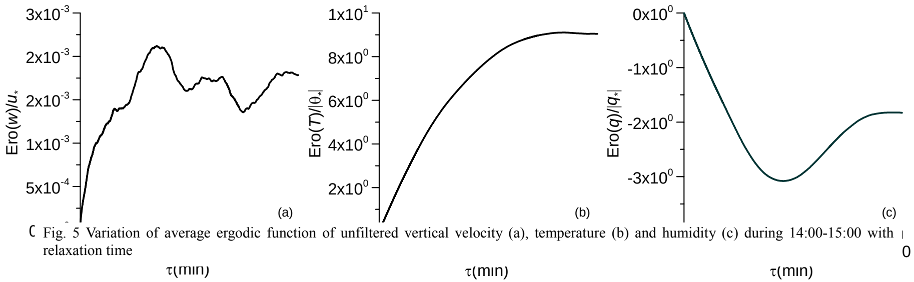

Figure 5. Variation of average ergodic function of unfiltered vertical velocity **(a)**, temperature **(b)** and humidity **(c)** during 14:00–15:00 with relaxation time.

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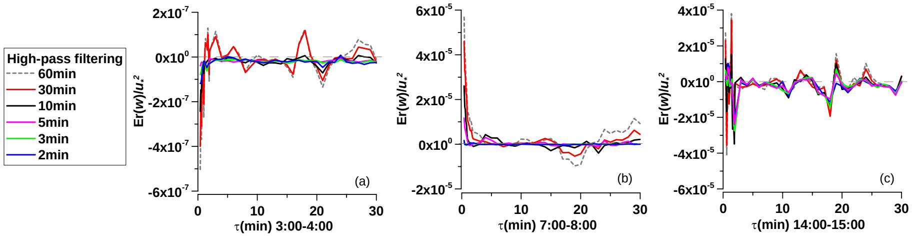

Figure 6. Variation of ergodic theorem of autocorrelation function of the vortexes in the different scales of vertical velocity with relaxation time.

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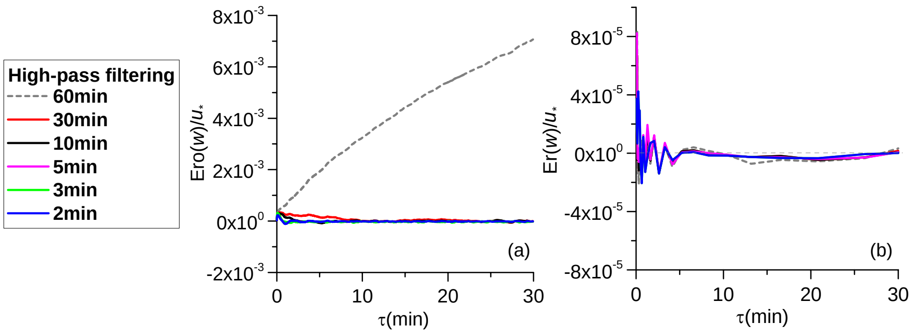

Figure 7. Variation of average ergodic function **(a)** and ergodic theorem of autocorrelation function **(b)** of the vortexes in the different scales of the vertical velocity with relaxation time at the seven observation locations of CASE99.

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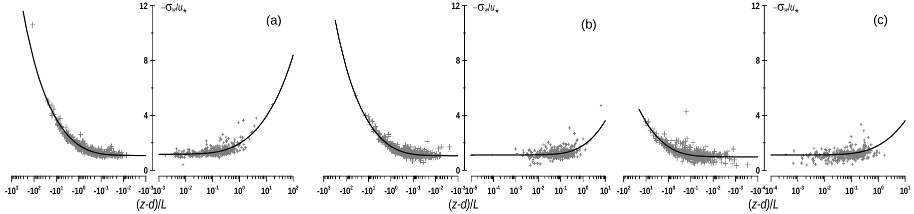

Figure 8. Similarity relations of vertical velocity variances of vortexes in different scales of Nagqu; Panels **(a)**, **(b)** and **(c)** respectively represent the similarity of vortexes of 10 min, 30 min and 60 min in the temporal scale.

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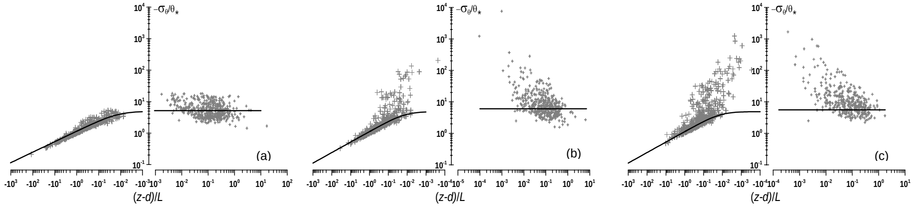

Figure 9. Similarity relations of temperature variance of vortexes in different scales of Nagqu; Panels **(a)**, **(b)** and **(c)** respectively represent the similarity of the vortexes of 10 min, 30 min and 60 min in the temporal scale.

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