Ergodicity Test of the Eddy Correlation Method

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Abstract

The ergodic hypothesis is a basic hypothesis typically invoked in atmospheric ~~turbulent~~ Surface layer (ASL) experiments. The ergodic theorem of ~~the~~ stationary random processes is introduced ~~first into the turbulence in atmospheric surface layer (ASL)~~ to analyze and verify the ergodicity of atmospheric turbulence measured ~~by the~~ using eddy covariance ~~system~~ technique with two sets of field observational data, ~~of the ASL.~~ The results show that eddies of atmospheric turbulence, of which the scale is smaller than the scale of the atmospheric boundary layer (ABL), i.e., the spatial scale is less than 1,000 m and temporal scale is shorter than 10 min, can effectively satisfy the ergodic theorems. ~~Therefore,~~ Under these restrictions, a ~~the~~ finite time average can be used as a ~~to~~ substitute for the ensemble average of atmospheric turbulence. Whereas, eddies that are larger than ABL's scale, cannot satisfy the mean ergodic theorem. Consequently, when a ~~the~~ finite time average is used to substitute for the ensemble average, the eddy correction method incurs ~~would occur a~~ large errors ~~rate~~ due to ~~the~~ loss ~~losing~~ low frequency information associated with ~~of the~~ larger eddies. A ~~The~~ multi-station observation is compared with a ~~the~~ single-station, and then the scope that satisfies the ergodic theorem~~s~~ is expanded from the smaller scale about 1000 m of ABL's scale to about 2000 m, even it exceeds ABL's scale. Therefore, the calculation of averages, variances and fluxes of ~~the~~ turbulence can effectively satisfy the ergodic assumption, and the results are more approximate to the actual values. Regardless of vertical velocity or temperature, the variance of eddies at ~~in~~ different scales follow ~~can more efficiently follow~~ Monin-Obukhov Similarity Theory (MOST) better if the ergodic theorem can be satisfied; ~~or else it deviates from MOST~~. The ergodicity exploration of ~~the~~ atmospheric turbulence is ~~doubtlessly~~ helpful in ~~to~~ understanding the issues in atmospheric turbulent observations and provides a theoretical basis for overcoming related difficulties.

Please rewrite, I do not understand this sentence.

Keywords: Ergodic hypothesis; eddy-correlation method; Monin-Obukhov similarity theory (MOST); atmospheric surface layer (ASL); high-pass filtering

1 Introduction

The basic principle of the average of ~~the~~ a turbulence measurement is ~~the~~ based on to make ensembles averaged over ~~of~~ space, time and state. However, it is impossible ~~that~~ an actual turbulence measurement with enough ~~sufficient enough~~ ~~numerous~~ observational instruments in space for sufficient ~~enough~~ time to obtain all states of turbulent eddies to achieve the goal of an ensemble average. Therefore, based on the ergodic hypothesis, the time average at one spatial point, which is long enough for observation, is used as a ~~to~~ substitute for the ensemble average for temporally steady and spatially homogeneous surfaces (Stull 1988; Wyngaard 2010; Aubinet 2012). The ergodic hypothesis is a basic assumption in ~~atmospheric~~ turbulence experiments in ~~of~~ the atmospheric boundary layer (ABL) and atmospheric surface layer (ASL). ~~The~~ stationarity, homogeneity, and ergodicity are routinely used to link ~~the~~ ensemble statistics (mean and higher-order moments) of field ~~observational~~ experiments in the ABL. Many authors habitually refer to the ergodicity assumption with ~~as some~~ descriptions such as "when satisfying ergodic hypothesis, ……" or "something indicates that ergodic hypothesis is satisfied". ~~Though~~ The success of Monin-Obukhov Similarity Theory (MOST) for unstable and near-neutral conditions is just ~~an~~ evidence of the validity of the ergodic hypothesis ~~validity~~ in the ASL, while ergodicity ~~however it~~ is a ~~only a~~ necessary condition for the success of MOST, it ~~for ergodicity in the ASL experiments~~, does not prove~~s~~ ergodicity (Katul et al., 2004). The success of MOST ~~success is~~ under the conditions of stationary and homogeneity ~~homogeneous surface. It~~ implies that they ~~the stationarity and homogeneity~~ are the important conditions of ASL ergodicity. Therefore, many ABL~~'s~~ experiments focus on seeking ideal homogeneous surface ~~as much as possible~~. ~~And some~~ Test procedures are ~~of availablity are~~ widely applied to establish stationarity (Foken and Wichura 1996; Vickers and Mahrt 1997). Katul et al. (2004) qualitatively analyzed the ergodicity problem~~s~~ in ~~regarding~~ atmospheric turbulence, and believed that it is common for the neutral and unstable ~~stratification in~~ ASL to reach ergodicity, while it is difficult to reach ergodicity in the ~~for the~~ stable ASL ~~layer~~. Eichinger et al. (2001) indicate that LIDAR (Light Detection and Ranging) technique opens up new possibilities for atmospheric measurements and analysis by providing spatial and temporal atmospheric information with simultaneous high-resolution. The stationarity and ergodicity can be tested for such ensembles of experiments. Recent advances in LIDAR measurements offers a promising first step for direct evaluation of such hypotheses for ASL flows (Higgins et al., 2013). Higgins et al. (2013) applied LIDAR of water vapor concentration to investigate the ergodic hypothesis of atmospheric turbulence for the first time. It is clear all the same that there is a need to reevaluate turbulence measurement technology, to test the ergodicity of atmospheric turbulence quantitatively by means of observation experiments.

The ergodic hypothesis was first proposed by Boltzmann (Boltzmann 1871; Uffink 2004) in his study of the ensemble theory of statistical dynamics. He argued that a trajectory traverses *all* points on the energy hypersurface after a certain amount of time. At the beginning of the 20th century, Ehrenfest couple [Ref?] proposed the quasi-ergodic hypothesis and changed the term "traverses *all* points" in the aforesaid ergodic hypothesis to "passes arbitrarily close to every point". The basic points of ergodic hypothesis or quasi-ergodic hypothesis recognize that the macroscopic property of a system in the equilibrium state is the average of microcosmic quantity in a certain amount of time. Nevertheless, the ergodic hypothesis or quasi-ergodic hypothesis were never proven theoretically. The proof of the ergodic hypothesis in physics aroused the interest of mathematicians. The famous mathematician, Neumann et al. (1932) first theoretically proved the ergodic theorem in topological space (Birkhoff 1931, Krengel 1985). Afterward, a banausic ergodic theorem of stationary random processes was proved to provide the necessary and sufficient conditions for the ergodicity of stationary random processes. Mattingly (2003) reviewed research progress on ergodicity ~~of the ergodicity of random~~ for stochastically forced Navier-Stokes equations, and Galanti and Tsinober and ~~(Galanti et al.~~ (2004), Lennaert et al. (2006) solved the ~~random~~ Navier-Stokes equations by numerical simulation to prove that ~~the~~ turbulence which is temporally steady and spatially homogeneous is ergodic. However, Galanti and Tsinober (2004) also indicated that such partially turbulent flows acting as mixed layer, wake flow, jet flow, flow around the boundary layer may be non-ergodic turbulence.

Obviously, the advances of research on ~~the~~ ergodicity in the mathematics and physics have ~~precedence far over~~ led the way for the atmospheric sciences. We try first~~ly~~ to introduce the ergodic theorem of stationary random processes to atmospheric turbulence in the ASL in this paper. ~~And that the~~ The ergodicity of different scale eddies of atmospheric turbulence is directly analyzed and verified quantitatively on the basis of ~~the~~ field observations obtained using the technique.

~~observational data measured by the~~ eddy covariance ~~system.~~

2 Theories and methods

2.1 Ergodic theorems of stationary random processes

~~The~~ stationary random processes are ~~the~~ processes which will not vary with time, i.e., for observed quantity A, its function of ~~spatial~~ space x_i and ~~temporal~~ time t_i satisfies the following condition:

$$A(x_1, x_2, \ldots, x_n; t_1, t_2, \ldots, t_n) = A(x_1, x_2, \ldots, x_n; t_1+\tau, t_2+\tau, \ldots, t_n+\tau), \tag{1}$$

where τ is a time period, defined as the relaxation time.

The mean μ_A of a random variable A and autocorrelation function $R_A(\tau)$ are respectively defined as following:

$$\mu_A = \lim_{T\to+\infty} \frac{1}{T}\int_0^T A(t)dt, \tag{2}$$

$$R_A(\tau) = \lim_{T\to+\infty} \frac{1}{T}\int_0^T A(t)A(t+\tau)dt. \tag{3}$$

The autocorrelation function $R_A(\tau)$ is a temporal second-order moment. In the case of τ=0, the autocorrelation function $R_A(\tau)$ is the variance of random variable. ~~The~~ A necessary and sufficient condition~~s~~ ~~that~~ for a stationary random processes to satisfy the mean ergodicity are the mean ergodic function Ero(A) to zero (Papoulis et al. 1991), as shown below:

$$\mathrm{Ero}(A) = \lim_{T\to\infty} \frac{1}{T}\int_0^{2T}\left(1-\frac{\tau}{2T}\right)\left[R_A(\tau)-\mu_A^2\right]d\tau = 0. \tag{4}$$

The mean ergodic function Ero(A) is a time integral of variation between the autocorrelation function $R_A(\tau)$ of variable A and its mean square, μ_A^2. If the mean ergodic function Ero(A) converges to zero, then the stationary random processes will be ergodic. In other words, if the autocorrelation function $R_A(\tau)$ of variable A converges to its mean square, μ_A^2, the stationary random processes are mean ergodic. The Eq. (4) is namely the mean ergodic theorem to be called as well as ergodic theorem of the *weakly* stationary processes in mathematics. For discrete variables, Eq. (4) can be rewritten as the following:

$$\mathrm{Ero}(A) = \lim_{n\to\infty} \sum_{i=0}^{n}\left(1-\frac{\tau_i}{n}\right)\left[R_A(\tau_i)-\mu_A^2\right] = 0. \tag{5}$$

The Eq. (5) is the mean ergodic theorem of discrete variable. Hence, Eqs. (4) and (5)