Our comment

The ergodic assumption was first raised by Boltzmann (1871) in his study of ensemble theory of statistical dynamics. The stationarity and ergodicity turns into two central concepts (required conditions) used to link field measurements and the NS equations or field measurements to “boundary conditions” at the land-atmosphere interface. The ergodic hypothesis is a basic hypothesis in atmospheric turbulent experiment. Stationarity, homogeneity, and ergodicity are routinely used to link the ensemble statistics (mean and higher-order moments) of turbulence field measurements collected in the ASL and CSL to land surface processes. Many literatures habitually referred to the ergodic assumption, as some descriptions such as “when satisfying ergodicity hypothesis, ……” or “something indicates that ergodicity hypothesis is satisfied”. Though the evidence of the validity of the ergodic hypothesis in the ASL is just the success of Monin-Obukhov similarity theory (MOST) for unstable and near-neutral conditions in atmospheric surface layer, the success of similarity theory, as a necessary condition for ergodicity in the ASL, does not prove ergodicity (Katul et al., 2004). However, the direct testing of the ergodic hypothesis in the ASL has frustrated all experimental efforts and frames the compass of this work (Higgins et al., 2013). So the theoretical demonstration or quantitative testing of direct observational experiment, which is relating to the ergodicity of the atmospheric turbulence, was hardly found.

The lidar technique opens up new possibilities for atmospheric measurements and analysis by providing simultaneous high-resolution spatial and temporal atmospheric information (Eichinger et al., 2001). The stationarity and ergodicity can be tested for such ensembles of experiments. Recent advances in LIDAR (Light Detection and Ranging) measurements offer a promising first step for direct evaluation of such hypotheses for ASL flows (Higgins et al., 2013).

However, after Boltzmann’s ergodic assumption, the notable advances have occurred with the theoretical demonstration and testing of direct experiment in the mathematics and physics, as shown in our article. Especially, the ergodic theorem of the stationary random processes is proved in the mathematics and the ergodicity of turbulence in the physics is tested. And that the necessary and sufficient condition of the ergodicity for stationary random processes is offered. Obviously, the advances of research on the ergodicity in the mathematics and physics are far more quickly than the atmospheric science. This paper tries firstly to introduce the ergodic theorem of the stationary random processes to atmospheric turbulence in surface layer. However, the base of this work is not *assumed in MOST*, but also going from the ergodic theorem for stationary random processes. The results shown MOST is satisfy the ergodicity condition.

It is obvious that the study is conditioned by stationary random processes. Firstly, was the turbulent flow in ASL a stationary random process? In the spatial scale, the atmospheric turbulence from the dissipation range, inertial sub-range to energy range, and further large eddy of turbulent flow is extremely broad. Following the diurnal variation of atmosphere, an eddy of which the temporal scale is larger than 1 hour can hardly meet stationary random process or steady flow, but a smaller-scale eddy (for example 2 min scale) is frequently stationary. The eddies in different scales are also different in terms of their spatial structure and physical properties, and even their transport characteristics are not all the same. It is thus reasonable that the eddies with different transport characteristics are separated and studied by using filtering method. Because the study is just a try to introduce firstly the ergodic theorem of the stationary random processes to atmospheric turbulence, we did not select data involve of complex surface in order to avoid disturbances of terrain. Even so, a neutrally stratified result during 7:00-8:00 has led to a great confusion. Because the 7:00-8:00 is a transition period, there were large differences among the stratifications of the eddies in different scales. But the small-scale eddies had kept stationarity in a certain degree. In the future, we will study the ergodicity of turbulence involve of complex surface. We think privately that the complex surface can only influence on the large-scale eddies, but the small-scale eddies are still steady due to the isotropous eddies and the scaling law of -5/3 from the dissipation range to inertial sub-range.

Respond to referee #1

Thank you very much for that you carefully read our paper and your heartfelt comments and recommendations. We respond as following:

The paper attempts to verify and analyze the ergodicity hypothesis of atmospheric turbulence with some eddy covariance data. While I believe that such work is needed given that ergodicity is assumed in MOST, I have some serious concerns about the analysis and the writing style. Also, given that MOST and EC have been around for decades, I am just surprised that this is the first ergodicity test of the eddy correlation method.

The ergodic assumption was first raised by Boltzmann (1871) in his study of ensemble theory of statistical dynamics. The stationarity and ergodicity turns into two central concepts (required conditions) used to link field measurements and the NS equations or field measurements to “boundary conditions” at the land-atmosphere interface. The ergodic hypothesis is a basic hypothesis in atmospheric turbulent experiment. Stationarity, homogeneity, and ergodicity are routinely used to link the ensemble statistics (mean and higher-order moments) of turbulence field measurements collected in the ASL and CSL to land surface processes. Many literatures habitually referred to the ergodic assumption, as some descriptions such as “when satisfying ergodicity hypothesis, ……” or “something indicates that ergodicity hypothesis is satisfied”. Though the evidence of the validity of the ergodic hypothesis in the ASL is just the success of Monin-Obukhov similarity theory (MOST) for unstable and near-neutral conditions in atmospheric surface layer, the success of similarity theory, as a necessary condition for ergodicity in the ASL, does not prove ergodicity (Katul et al., 2004). However, the direct testing of the ergodic hypothesis in the ASL has frustrated all experimental efforts and frames the compass of this work (Higgins et al., 2013). So the theoretical demonstration or quantitative testing of direct observational experiment, which is relating to the ergodicity of the atmospheric turbulence, was hardly found.

The lidar technique opens up new possibilities for atmospheric measurements and analysis by providing simultaneous high-resolution spatial and temporal atmospheric information (Eichinger et al., 2001). The stationarity and ergodicity can be tested for such ensembles of experiments. Recent advances in LIDAR (Light Detection and Ranging) measurements offer a promising first step for direct evaluation of such hypotheses for ASL flows (Higgins et al., 2013). Here, we thank you for the information about article ‘Are atmospheric surface layer f lows ergodic?’

However, after Boltzmann’s ergodic assumption, the notable advances have occurred with the theoretical demonstration and testing of direct experiment in the mathematics and physics, as shown in our article. Especially, the ergodic theorem of the stationary random processes is proved in the mathematics and the ergodicity of turbulence in the physics is tested. And that the necessary and sufficient condition of the ergodicity for stationary random processes is offered. Obviously, the advances of research on the ergodicity in the mathematics and physics are far more quickly than the atmospheric science. This paper tries firstly to introduce the ergodic theorem of the stationary random processes to atmospheric turbulence in surface layer. However, the base of this work is not *assumed in MOST*, but also going from the ergodic theorem for stationary random processes. The results shown MOST is satisfy the ergodicity condition.

MAJOR COMMENTS

The abstract and introduction should simply be rewritten. The authors should seek the help of a native English speaker in doing so.

We had paid much money to a native English speaker in order to present a perfect manuscript. Moreover, an ACPD editor helped us revise the manuscript. On your comments, we have found some errors. We will carefully revise the manuscript to avoid some improper and wrong descriptions.

1. *A few specific comments here: -Include a clear description of the eddy covariance method - Avoid going from a super general to a super specific statement (e.g. p. 18210, l. 6-11) - Each paragraph should convey one main idea.*
   - Provide a clear description of the ergodic hypothesis. In its current form, it could hardly be more confusing.

This paper is a trying firstly to verify and analyze atmospheric turbulence in surface layer by using the ergodic theorem of the stationary random process. A review of develop history about the ergodicity in the mathematics and physics is necessary. Of course, we will replenish some descriptions of the ergodicity in atmospheric sciences and improve on the writing style.

2. *Sections 4.1 and 4.2 are based on 3 h of turbulence data at a single site. How representative is that? 3.The authors rely on three time frames for their analysis, 3:00-4:00, 7:00-8:00 and 13:00-14:00. They claim that the 7:00-8:00 is neutrally stratified, but this is contradictory to the values of z/L they report. Also, this is clearly a transition period (the authors need to report the sunrise time), so I wonder if it is appropriate to test the ergodicity hypothesis when the requirement for stationarity is violated.*

You are right, The ergodicity hypothesis of the turbulent observation was tested by using every 1h data, which is a heavy work. All results can not be demonstrated in the paper. So we selected carefully the representative results of three time frames. The sun just came out for the 7:00-8:00. There primarily involves of a scale problem of eddy. ‘*the 7:00-8:00 is neutrally stratified*’ as shown in the unfiltered data. But the table 1 presents the local stability of different scale eddy of the filtered data. That is different from the integral stability as shown in the unfiltered data. We should give the description of integral stability in the paper. We will supplement a more detailed description.

3. *I am not familiar/comfortable with the concept of "local stability of a vortex". To me, the fact that z/L varies with your "vortex time scales" is simply either because the flow is not stationary or because you are not fully capturing the flux. Can you*

show some Ogive plots for the buoyancy and momentum fluxes for each of your three time frames?

The selected data are stationary, not unsteady in the paper. In order to differentiate between the stability of eddy in the certain scale range and the integral stability of turbulence, "local stability of an eddy" is defined, just as described in p. 18221, 13-19.

You are right. We have recognized the filtered data are not fully capturing the flux. This incompletely captured flux is just the contribution to be filtered eddies. Obviously, this paper is only a tries to verify and analyze atmospheric turbulence in surface layer by using the ergodic theorem of the stationary random process. The conclusion is that the eddy of atmospheric turbulence, which is smaller than the scale of the atmospheric boundary layer (i.e., its spatial scale is less than 1000m and temporal scale is shorter than 10 min) can effectively meet the conditions of the ergodic theorem. The eddy that smaller than the scale of the atmospheric boundary layer just meets MOST. The eddy just is greater than the scale of the atmospheric boundary layer, which just does not meet MOST, it will be filtered.

We are studying further how estimate the integral flux including full eddies with the coupling effect of vertical velocity on the flux (Hu, 2002; Chen et al., 2007). Of course, we will present another paper to discuss Ogive plots for the buoyancy and momentum fluxes because of involving in the more problems.

4. *p. 18223, l. 13: Here you need to define how you normalized your ergodic functions. For the ergodic hypothesis to be validated, these functions needs to be strictly speaking equal to zero, which is not going to happen with experimental data - we agree on this. However, I believe you need to define a threshold to decide whether the flow is ergodic or not.*

We have supplement the definition of normalized ergodic functions.

5. *p. 18225, l. 1-3: To go for temporal to spatial scales you need to invoke Taylor's hypothesis here. It is valid at all?*

The ergodic assumption is more severe criteria than Taylor's hypothesis. If the turbulent flow satisfies ergodic assumption, Taylor's hypothesis must be applicative.

MINOR COMMENTS

1. *I would use the word "eddy" instead of "vortex".*

   We have used "eddy" instead of "vortex".

2. *Several references are not properly cited, e.g. - p. 18209 Dennis et al. 2001 should be Baldocchi et al. 2001 - p. 18212 Gabriel et al. 2004 should be Katul et al.*

2004.

We have revised these references.

3. You should definitely read and cite this work: Higgins et al. 2013. Are atmospheric surface layer flows ergodic? GRL, 40, 3342–3346.

Thank you telling us information about the paper "Higgins et al. 2013. Are atmospheric surface layer flows ergodic? GRL, 40, 3342–3346"! This paper verified an ergodic flow by comparing the single-point time average with ensemble average. It is one of the few articles, which relate to test directly ergodicity of atmospheric turbulence.

4. *Eqn (2): can you define the operator E[]?*

In Equation (2), $E\left[A(t)\right]$ is redundant. Eq. (2) is written as

$$\mu_A = \lim_{T\to+\infty} \frac{1}{T}\int_0^T A(t)dt \ .$$

Eqn (3) also has be revised as

$$R_A(\tau) = \lim_{T\to+\infty} \frac{1}{T}\int_0^T A(t)A(t+\tau)dt \ .$$

5. *Eqn (4): why over a period 2T? Also, do you have another reference than Wang et al. (2009) which is in Chinese. This equation is key to your analysis.*

It is a clerical error and "2T" is revised as "T".

$$\text{Ero}(A) = \lim_{T\to+\infty} \frac{1}{T}\int_0^{2T} \left(1-\frac{\tau}{2T}\right)\left[R_A(\tau)-|\mu_A|^2\right]d\tau = 0$$

Reference is "Papoulis A. and Pillai S. U. 1991. Probability, random variables and stochastic processes (3rd edn). McGraw-Hill. New York. 666 pp"

6. *Section 2.2: Is time averaging not sufficient to act as a band-pass filter?*

In order to cut off the low frequency turbulence without man-made interference, the Fourier transform was used.

7. *Equation (9): Not clear what a and b mean here.*

The *a* and *b* mean low and high constraints of band-pass filter. We rewritten equation as

$$A(k)=\sum_{n=a}^{N-1}F_A(n)\cos\left(\frac{2\pi nk}{N}\right)+i^2\sum_{n=a}^{N-1}F_A(n)\sin\left(\frac{2\pi nk}{N}\right)$$

a is the lower limit wave-number of high-pass filtering.

8. *p. 18218, l. 11: sigma is actually the standard deviation.*

Here, sigma is a standard deviation.

9. *p. 18219, l. 11: The proper acronym is CASES99.*

Thank you! We have used CASES99 instead of every CAES99.

10. The Nagqu station is located at extremely high altitude (4500 m ASL). How do you expect that this will impact your result?

Yes, we concern about the impact of plateau, but the results shown seemingly that the impact is less. The similar results in CASES99 also were obtained.

11. p. 18219, l. 15: is 8000 m2 the size of the measurement footprint? If not, why are you reporting this value?

It is not a size of the measurement footprint. We have deleted it.

12. *p. 18219, l. 19: remove the minus sign or change 'W' to 'E'.*

We have deleted the minus sign.

13. *p. 18220, l. 3-16: The authors keep referring to 'pulses'. Not clear what they mean here.*

It should be "non-stationary". We have rewritten the sentence.

14, p. 18221, l. 13-16: Can you better justify why you are using a local similarity framework here? Do you expect strong advection or z-less stratification at your sites?

We have recognized that the filtered data, which satisfy the condition of ergodic theorem, are not fully capturing whole eddies in surface layer. So we adopt a local similarity framework here.

15. Equation (17) and Table 2: You should compare the coefficients you obtain those from previous studies.

As the above mentions, the eddy scale, which satisfies condition of ergodic theorem, is different from other studies, so the coefficients may be different from other. For the time being, the compare is not necessary. Of course, as studying further the integral flux including full eddies, the compare of coefficients is necessary.

16. I would recommend to split section 5 into "Discussion" and "Conclusions".

We have split section 5 into "5 Discussion" and "6 Conclusions.

17. References: be consistent with title capitalization.

We have revised this mistake.

18. Table 1: It does not seem appropriate to use "bucket" here.

We have deleted "bucket".

19. Figure 1 is useless. Fig. 1a contains little to no useful information. Fig. 1b seems to have been taken directly from another publication (notice the "Figure 4" label on top). There is just no sufficient information to make sense out of both figures.

Fig. 1a shows a flat surface and circumstance around the station. Fig. 1b is taken directly from the reference “Poulos, Gregory S., and Coauthors, 2002: CASES-99: A Comprehensive Investigation of the Stable Nocturnal Boundary Layer. Bull. Amer. Meteor. Soc., 83, 555–581”. We have redrawn Fig. 1b.

Reference

Aubinet, M., Papale, D., Vesala, T.: Eddy Covariance A Practical Guide to Measurement and Data Analysis, Springer Dordrecht Heidelberg London New York. 438, 2012.

Chen, J., Hu Y., and Zhang L.: Principle of cross coupling between vertical heat turbulent transport and vertical velocity and determination of cross coupling coefficient, Adv. Atoms. Sci., 23 (4), 639-648, 2007.

Eichinger, W. E., Parlange, M. B., Katul, G. G.: Lidar measurements of the dimensionless humidity gradient in the unstable atmospheric surface layer, 3, 7-13, Lakshmi, J. Albertson, and J. Schaake, editors. Land Surface Hydrology, Meteorology, and Climate, Water Science and Application, American Geophysical Union, Washington, D. C. 2001.

Higgins, C. W., Katul, G. G., Froidevaux, M., Simeonov, V. and Parlange, M. B.: Atmospheric surface layer flows ergodic? Geophy. Res. Let., 40, 3342-3346, 2013.

Hu Yinqiao, 2002，Application of Linear Thermodynamics to the Atmospheric System. Part I : Linear Phenomenological Relations and Thermodynamic Property of the Atmosphere System. Advances in Atmospheric Sciences, **19**(1), 448-458.

Katul, G., Cava, D., Poggi, D., Albertson, J., and Mahrt, L.: Stationarity, homogeneity, and ergodicity in canopy turbulence, Handbook of micrometeorology a guide for surface flux measurement and analysis, Lee, X., Kluwer Academic Publishers, New York, 161－180, 2004.

Respond to referee #2

Thank your comments!

While the main hypothesis; testing the assumption of stationarity, homogeneity and ergodicity in canopy turbulence is relevant to the field of micrometeorology and to this journal, the article needs to be thoroughly restructured. Apart from the numerous grammatical errors, the ideas expressed in the article are scattered, i.e all over the place. The authors also limit the analysis to 3 hrs and over two sites; in order to extend the validity of their results, the authors must include data from multiple sites. With all this in to consideration, in its current form, the paper is not suitable for publication.

-Influence of land cover needs to be tested. If possible the analysis should include data from multiple sites (forest, crop, urban, mountain etc). The two sites used in this analysis look very homogenous in terms of surface cover; many contemporary eddy flux measurements are devised over heterogeneous landscapes. It would immensely add to the article if the analysis is extended to include the influence of surface characteristics.

We had paid much money to a native English speaker in order to present a perfect manuscript. Moreover, an ACPD editor helped us revise the manuscript. On your comments, we have found some errors. We will carefully revise the manuscript to avoid some improper and wrong descriptions.

The ergodic assumption was first raised by Boltzmann (1871) in his study of ensemble theory of statistical dynamics. The stationarity and ergodicity turns into two central concepts (required conditions) used to link field measurements and the NS equations or field measurements to “boundary conditions” at the land-atmosphere interface. The ergodic hypothesis is a basic hypothesis in atmospheric turbulent experiment. Stationarity, homogeneity, and ergodicity are routinely used to link the ensemble statistics (mean and higher-order moments) of turbulence field measurements collected in the ASL and CSL to land surface processes. Many literatures habitually referred to the ergodic assumption, as some descriptions such as “when satisfying ergodicity hypothesis, ……” or “something indicates that ergodicity hypothesis is satisfied”. Though the evidence of the validity of the ergodic hypothesis in the ASL is just the success of Monin-Obukhov similarity theory (MOST) for unstable and near-neutral conditions in atmospheric surface layer, the success of similarity theory, as a necessary condition for ergodicity in the ASL, does not prove ergodicity (Katul et al., 2004). However, the direct testing of the ergodic hypothesis in the ASL has frustrated all experimental efforts and frames the compass of this work (Higgins et al., 2013). So the theoretical demonstration or quantitative testing of direct observational experiment, which is relating to the ergodicity of the atmospheric turbulence, was hardly found.

However, after Boltzmann’s ergodic assumption, the theoretical demonstration and testing of direct experiment in the mathematics and physics have notable advances, as shown in our article. Especially, the ergodic theorem of the stationary random processes is proved in the mathematics and the ergodicity of turbulence in the physics is tested. And that the necessary and sufficient condition of the ergodicity for stationary random processes is offered. Obviously, the advances of research on the ergodicity in the mathematics and physics are far more quickly than the atmospheric science.

This paper tries firstly to introduce the ergodic theorem of the stationary random processes to atmospheric turbulence in surface layer. And that it is a trying firstly to verify and analyze atmospheric turbulence in surface layer by using the ergodic theorem of the stationary random process. Of course, it does not in the roundly analyze the land surface processes over heterogeneous landscapes. We believe that the results are the success and offer a promising first step for direct evaluation of ergodic hypotheses for ASL flows. Of course, we will extend the validity of their results, which include data from multiple sites as a second step.

Main Comments

-The land cover characteristics of the two sites discussed in this paper needs to explained thoroughly.

Our study needs a flat surface to study a stationary turbulence. The lands of the two sites are uniform and covered by grass.

-Information on data used for the analysis, time periods, stability need to be tabulated. Also all the error correction and data processing need to be explained more thoroughly.

This work is a study to apply the ergodic theorem of the stationary random processes to atmospheric turbulence in surface layer. At first, the raw data were filtered with Fourier transform. So the eddies whose scale is less than 1 hour were retained. Secondly, after deleting artificially spike data, we calculate the six segment variances of 5-min vertical velocity and temperature by using every 1 hour data set that has been filtered with 5-min high-pass filter. When the segment variances were within the range of the relative difference 10%, the selected turbulent flows were considered to be steady.

-Why are the temporal states in local time, stability parameter must be used?

This study discusses the ergodicity of different scale eddies. M-O similarity framework is not appropriate.

-The authors refer to 10 mins, 1000 m as some indices, these numbers have to be presented in a non-dimensional form so it can be compared to other experiments. -The analysis should include fluxes: momentum, heat and humidity to the analysis.

10 mins for 1 hour observational period, 1000 m is a corresponding length under the condition of velocity 1ms^{-1}. As the above mentions, this work is a trying firstly to verify and

analyze atmospheric turbulence in surface layer by using the ergodic theorem of the stationary random process as a first step. The second step of this work will be to extent the analysis included fluxes: momentum, heat and humidity to the analysis.

Reference

Aubinet, M., Papale, D., Vesala, T.: Eddy Covariance A Practical Guide to Measurement and Data Analysis, Springer Dordrecht Heidelberg London New York. 438, 2012.

Eichinger, W. E., Parlange, M. B., Katul, G. G.: Lidar measurements of the dimensionless humidity gradient in the unstable atmospheric surface layer, 3, 7-13, Lakshmi, J. Albertson, and J. Schaake, editors. Land Surface Hydrology, Meteorology, and Climate, Water Science and Application, American Geophysical Union, Washington, D. C. 2001.

Katul, G., Cava, D., Poggi, D., Albertson, J., and Mahrt, L.: Stationarity, homogeneity, and ergodicity in canopy turbulence, Handbook of micrometeorology a guide for surface flux measurement and analysis, Lee, X., Kluwer Academic Publishers, New York, 161 – 180, 2004.