Ergodicity test of the eddy-covariance technique

Jinbei CHEN[1] Yinqiao HU[2] Ye YU Shihua LÜ

Key Laboratory of Land Surface Processes and Climate Change in Cold and Arid Regions; Cold and Arid Regions Environment and Engineering Institute, Chinese Academy of Sciences, Lanzhou 730000,P R China.

Pingliang Land Surface Process & Severe Weather Research Station, Chinese Academy of Science, Pingliang 744015, P R China.

1 E-mail: chenjinbei@lzb.ac.cn

2 Corresponding author: hyq@ns.lzb.ac.cn

Abstract

The ergodic hypothesis is a basic hypothesis typically invoked in atmospheric surface layer (ASL) experiments. The ergodic theorem of stationary random processes is introduced to analyze and verify the ergodicity of atmospheric turbulence measured using the eddy-covariance technique with two sets of field observational data. The results show that the ergodicity of atmospheric turbulence in ABL is not only relative to the atmospheric stratification but also to the eddy scale of atmospheric turbulence. The eddies of atmospheric turbulence, of which the scale is smaller than the scale of the atmospheric boundary layer (ABL), i.e., the spatial scale is less than 1,000 m and temporal scale is shorter than 10 min, effectively satisfy the ergodic theorems. Under these restrictions, a finite time average can be used as a substitute for the ensemble average of atmospheric turbulence. Whereas, eddies that are larger than ABL scale dissatisfy the mean ergodic theorem. Consequently, when a finite time average is used to substitute for the ensemble average, the eddy-covariance technique incurs large errors due to the loss of low frequency information associated with larger eddies. A multi-stations observation is compared with a single-station, and then the scope that satisfies the ergodic theorem is extended from scales smaller than the ABL approximately 1000 m to scales greater than that about 2000 m. Therefore, the calculation results of averages, variances and fluxes of turbulence are more faithfully approximate the actual values due to effectively satisfy the ergodic assumption. Regardless of vertical velocity or temperature, the variance of eddies at different scales follows Monin-Obukhov Similarity Theory (MOST) better if the ergodic theorem can be satisfied, if not it deviates from MOST. The exploration of ergodicity in atmospheric turbulence is doubtlessly helpful in understanding the issues in

atmospheric turbulent observations, and provides a theoretical basis for overcoming related difficulties.

Keywords: Atmospheric turbulence; Ergodic hypothesis; eddy-covariance technique; Monin-Obukhov similarity theory (MOST); atmospheric surface layer (ASL)

1 Introduction

The basic principle of average of the turbulence measurements is based on ensembles averaged over space, time and state. However, it is impossible to make an actual turbulence measurement with enough observational instruments in space for sufficient time to obtain all states of turbulent eddies to achieve the goal of an ensemble average. Therefore, based on the ergodic hypothesis, the time average of one spatial point, taken over a sufficiently long observational time, is used as a substitute for the ensemble average for temporally steady and spatially homogeneous surfaces (Stull 1988; Wyngaard 2010; Aubinet 2012). The ergodic hypothesis is a basic assumption in turbulence experiments in the atmospheric boundary layer (ABL) and atmospheric surface layer (ASL). Stationarity, homogeneity, and ergodicity are routinely used to link ensemble statistics (mean and higher-order moments) of field experiments in the ABL. Many authors habitually refer to the ergodicity assumption with descriptions such as “when satisfying ergodic hypothesis……” or “something indicates that ergodic hypothesis is satisfied”. The success of Monin-Obukhov Similarity Theory (MOST) for unstable and near-neutral conditions is just evidence of the validity of the ergodic hypothesis in the ASL. While ergodicity is only a necessary condition for the success of MOST, but also it does not prove ergodicity (Katul et al. 2004). The success of MOST under the conditions of stationary and homogeneity implies that the stationary and homogeneity are also the important conditions of ASL ergodicity. Therefore, many ABL experiments focus on seeking ideal homogeneous surfaces. Some test procedures are widely applied to establish stationarity (Foken and Wichura 1996; Vickers and Mahrt 1997). Katul and Hsieh (1999) qualitatively analyzed the ergodicity problem in atmospheric turbulence, and believed that it is common for the neutral and unstable ASL to satisfy ergodicity, while it is difficult to reach ergodicity in the stable ASL. Eichinger et al. (2001) indicate that LIDAR technique opens up new possibilities for atmospheric measurements and analyses by providing spatial and temporal atmospheric information with simultaneous high-resolution. The stationarity and ergodicity can be tested for such ensembles of experiments. Recent advance in LIDAR measurements offers a promising first step for direct evaluation of such hypotheses for ASL flows (Higgins et al., 2013). Higgins et al. (2013) applied LIDAR of water vapor concentration to investigate the ergodic hypothesis of atmospheric turbulence for the first time. It is clear all the same that there is a need to reevaluate the technologies of turbulence measurement, to test the ergodicity of atmospheric turbulence quantitatively by means of observation experiments.

The ergodic hypothesis was first proposed by Boltzmann (Boltzmann 1871; Uffink 2004) in his study of the ensemble theory of statistical dynamics. He argued that a trajectory traverses *all* points on the energy hypersurface after a certain amount of time. At the beginning of 20th century, the Ehrenfest couple (Ehrenfest. and Ehrenfest-Afanassjewa 1912; Uffink 2004) proposed a quasi-ergodic hypothesis and changed the term “traverses *all* points” in the aforesaid ergodic hypothesis to “passes arbitrarily close to every point”. The basic points of ergodic hypothesis or quasi-ergodic hypothesis recognize that the macroscopic property of a system in the equilibrium state is an average of microcosmic quantity in sufficient long time. Nevertheless, the ergodic hypothesis or quasi-ergodic hypothesis were never proven theoretically. The proof of the ergodic hypothesis in physics aroused the interest of mathematicians. Famous mathematician, Neumann et al. (1932) first theoretically proved the ergodic theorem in topological space (Birkhoff 1931, Krengel 1985). Afterward, a banausic ergodic theorem of stationary random processes was proved to provide a necessary and sufficient condition for the ergodicity of stationary random processes. Mattingly (2003) reviewed the research progress on ergodicity for stochastically force Navier-Stokes equation, and that Galanti and Tsinober (2004) and Lennaert et al. (2006) solved the Navier-Stokes equation by numerical simulation to prove that turbulence that is temporally steady and spatially homogeneous is ergodic. However, Galanti and Tsinober (2004) also indicated that such partially turbulent flows acting as mixed layer, wake flow, jet flow, flow around the boundary layer may be non-ergodic.

Obviously, the advances of research on ergodicity in the mathematics and physics have led the way for the atmospheric sciences. We try first to introduce the ergodic theorem of stationary random processes to the atmospheric turbulence in this paper. The ergodicity of different scale eddies of atmospheric turbulence is directly analyzed and verified quantitatively on the basis of field observation data obtained using eddy-covariance technique in the ASL.

2 Theories and methods

2.1 Ergodic theorems of stationary random processes

Stationary random processes are processes which will not vary with time, i.e., for observed quantity A, its function of space x_i and time t_i satisfies the following condition:

$$A(x_1, x_2, \ldots, x_n; t_1, t_2, \ldots, t_n) = A(x_1, x_2, \ldots, x_n; t_1+\tau, t_2+\tau, \ldots, t_n+\tau), \tag{1}$$

where τ is a time period, defined as the relaxation time.

The mean μ_A of a random variable A and its autocorrelation function $R_A(\tau)$ are respectively defined as following:

$$\mu_A = \lim_{T \to +\infty} \frac{1}{T} \int_0^T A(t) dt, \tag{2}$$

$$R_A(\tau) = \lim_{T \to +\infty} \frac{1}{T} \int_0^T A(t) A(t+\tau) dt. \tag{3}$$

The autocorrelation function $R_A(\tau)$ is a temporal second-order moment. In the case of τ=0, the autocorrelation function $R_A(\tau)$ is the variance of random variable. A necessary and sufficient condition for the stationary random processes to satisfy the mean ergodicity is the mean ergodic function Ero(A) to zero (Papoulis and Pillai 1991), as shown below:

$$\mathrm{Ero}(A) = \lim_{T \to \infty} \frac{1}{T} \int_0^{2T} \left(1 - \frac{\tau}{2T}\right) \left[R_A(\tau) - \mu_A^2\right] d\tau = 0. \tag{4}$$

The mean ergodic function Ero(A) is a time integral of the difference between the autocorrelation function $R_A(\tau)$ of variable A and its mean square, μ_A^2. If the mean ergodic function Ero(A) converges to zero, then the stationary random processes will be ergodic. In other words, if the autocorrelation function $R_A(\tau)$ of variable A converges to its mean square, μ_A^2, the stationary random processes are mean ergodic. The Eq. (4) is namely mean ergodic theorem to be called as well as ergodic theorem of the *weakly* stationary processes in the mathematics. For discrete variables, Eq. (4) can be rewritten as following:

$$\mathrm{Ero}(A) = \lim_{n \to \infty} \sum_{i=0}^{n} \left(1 - \frac{\tau_i}{n}\right) \left[R_A(\tau_i) - \mu_A^2\right] = 0. \tag{5}$$

Eq. (5) is mean ergodic theorem of the discrete variable. Hence, Eqs. (4) or (5) can be used as a criterion to judge the mean ergodicity.

For the stationary random processes, the necessary and sufficient condition satisfying the autocorrelation ergodicity is the autocorrelation ergodic function Er(A) to zero:

$$\mathrm{Er}(A)=\lim_{T\to\infty}\frac{1}{T}\int_0^{2T}\left(1-\frac{\tau'}{2T}\right)\left[B(\tau')-\left|R_A(\tau)\right|^2\right]d\tau'=0; \tag{6a}$$

$$B(\tau')=E\left\{A(t+\tau+\tau')A(t+\tau')\left[A(t+\tau)A(t)\right]\right\}. \tag{6b}$$

Where τ' is a differential variable for entire relaxation times, and that $B(\tau')$ is temporal fourth-order moment of variable A. The autocorrelation ergodic function Er(A) is a time integral of the difference between the temporal fourth-order moment $B(\tau')$ of variable A and its autocorrelation function square, $\left|R_A(\tau)\right|^2$. If the autocorrelation ergodic function Er(A) converges to zero, then the stationary random processes will be of autocorrelation ergodicity, and thus the autocorrelation ergodicity means that the fourth-order moment of variable of stationary random processes will converge to square of its autocorrelation function $R_A(\tau)$. Eq. (6a) is namely autocorrelation ergodic theorem to be called as well as ergodic theorem of the *strongly* stationary processes in the mathematics. The autocorrelation ergodic function of corresponding discrete variable can be determined as following:

$$\mathrm{Er}(A)=\lim_{n\to\infty}\sum_{i=0}^{n}\left(1-\frac{\tau_i'}{n}\right)\left[B(\tau_i')-\left|R_A(\tau_j)\right|^2\right]=0, \tag{7a}$$

$$B(\tau'_i)=E\left\{\sum_{j=0}^{n}A(t+\tau_j+\tau'_i)A(t+\tau'_i)\left[A(t+\tau_j)A(t)\right]\right\}. \tag{7b}$$

Eq. (7a) is autocorrelation ergodic theorem of the discrete variable. Hence, Eqs. (6a) or (7a) can also be used as a criterion to judge the autocorrelation ergodicity.

The stationary random processes conform to the criterion, Eqs. (4) or (5), then they satisfy the mean ergodic theorem, or are intituled as the mean ergodicity; the stationary random processes conform to the criterion, Eqs. (6a) or (7a), then they satisfy the autocorrelation ergodic theorem, or are intituled as the autocorrelation ergodicity. If the stationary random processes are only of mean ergodicity, they are strict ergodic or narrow ergodic. If the stationary random processes are of both the mean ergodicity and autocorrelation ergodicity, they are namely wide ergodic stationary random processes. It is thus clear that the ergodic random processes are stationary, but the stationary processes may not be ergodic.

In the random process theory, calculating the mean or high-order moment function requires a large amount of repeated observations to acquire a sample function $A_k(t)$. If the stationary random processes satisfy the ergodic condition, then time average of a sample on the whole time shaft can be used to substitute for the ensemble average. Eqs. (4), (5), (6a) and (7a) can be used as the criterion to judge whether or not satisfying the mean and autocorrelation ergodicity. The ergodic random processes must be the stationary random processes to be defined as Eq. (1), and thus are stationary in relaxation time τ. If the condition such as Eqs (4) or (5) of the mean ergodicity is satisfied, then a time average in finite relaxation time τ can be used to substitute for infinite time average to calculate the mean Eq. (2) of random variable; similarly, the finite time average can be used for substitution to calculate the covariance or variance of random variable, Eq. (3), if the condition such as Eqs. (6a) or (7a) of autocorrelation ergodicity is satisfied. In a similar manner, the basic principle of average of the atmospheric turbulence is the ensemble average of space, time and state, and it is necessary to carry through mass observations for a long period of time in the whole space. This is not only a costly observation, even is hardly feasible. If the turbulence satisfies the ergodic condition, then a time average in relaxation time τ by multi-stations observation, even single-station observation, can substitute for the ensemble average. In fact, precondition to estimate turbulent characteristic quantities and fluxes in the ABL by the eddy-covariance technique is that the turbulence satisfies the ergodic condition. Therefore, conditions such as Eqs. (4), (5), (6a) and (7a) will also be the criterion for testing the ergodicity and authenticity of results observed by the eddy-covariance technique.

2.2 Band-pass filtering

The scope of spatial and temporal scale of the atmospheric turbulence, which is from the dissipation range, inertial sub-range to the energy range, and further the turbulent large eddy, is extremely broad (Stull 1988). In such wide spatial and temporal scope, the turbulent eddies include the isotropic 3-D eddy structure of high frequency turbulence and orderly coherent structure of low frequency turbulence (Li et al. 2002). These eddies of different scale are also each other different in terms of their spatial structure and physical properties, and even their transport characteristics are not the same. It is thus reasonable that eddies with different characteristics are separated, processed and studied using different methods (Zuo et al. 2012). A major goal of our study is to understand what type of eddy in the scale can satisfy the ergodic condition. Another goal is that the time averaging of signals measured by a single station determines accurately turbulent characteristic quantities. In order to study the ergodicity of different scale eddies, Fourier transform is used as a band-pass filtering to distinguish different scale eddy. That is to say, we compel to set the Fourier transform coefficient of the part of frequencies, which does not need, as zero, and then acquire the signals after filtering by means of Fourier inverse transformation. The specific formulae are shown below:

$$F_A(n) = \frac{1}{N}\sum_{k=0}^{N-1} A(k)\cos\left(\frac{2\pi nk}{N}\right) - \frac{i}{N}\sum_{k=0}^{N-1} A(k)\sin\left(\frac{2\pi nk}{N}\right), \tag{8}$$

$$A(k) = \sum_{n=a}^{N-1} F_A(n)\cos\left(\frac{2\pi nk}{N}\right) + i^2\sum_{n=a}^{N-1} F_A(n)\sin\left(\frac{2\pi nk}{N}\right). \tag{9}$$

In Eqs. (8) and (9), $F_A(n)$ and $A(k)$ are respectively the Fourier transformation and Fourier inverse transformation including N data points from k=0 to k=N-1, and n is the cycle index of the observation time range. The high-pass filtering can cut off the low frequency signals of turbulence to obtain the high frequency signals. An aliasing of half high frequency turbulence after the Fourier transformation is unavoidable. At this time, the correction for high frequency response will compensate for that loss. In order to acquire purely signals of different scale eddies in filtering processes, we take results of the band-pass filtering from n=j to n=N-j as required signals. This is referred to as j time filtering in this paper. Finally, the ergodicity of different scale eddies is analyzed using Eqs. (4)-(7).

2.3 MOS of turbulent variance

The characteristics of the relations of Monin-Obukhov Similarity (MOS) for the variance of different scale eddies are analyzed and compared to test feasibility of the MOS relation for ergodic and non-ergodic turbulence. In order to provide an experimental basis for utilizing MOST and developing the turbulence theory of ABL under the condition of the complex underlying surfaces, the problems of eddy-covariance technique of the turbulence observation in ASL are further explored on the basis of studying on the ergodicity and MOS relations of the variance of different scale eddies.

The MOS relations of turbulent variance can be regarded as an effective instrumentality to verify whether or not that the turbulent flow field is steady and homogeneous (Foken et al. 2004). Under ideal conditions, the local MOS relations of the variance of wind velocity, temperature and other factors can be expressed as following:

$$\sigma_i / u_* = \phi_i(z/L), \qquad (i = u, v, w), \tag{10}$$

$$\sigma_s / |s_*| = \phi_s(z/L), \qquad (s = \theta, q). \tag{11}$$

where σ is turbulent variance; corner mark i is wind velocity u, v or w; s stands for scalar, such as potential temperature θ and humidity q; u_* is friction velocity and defined as $u_* = \left(\overline{u'w'}^2 + \overline{v'w'}^2\right)^{1/4}$; s_* is turbulent characteristic quantity related to scalar defined as $s_* = -\overline{w's'}/u_*$; and that M-O length L is defined as (Hill 1989):

$$L = u_*^2 \theta / \left[\kappa g\left(\theta_* + 0.61 \theta q_* / \rho_d\right)\right], \tag{12}$$

where ρ_d is dry air density .

A large number of research results show that, in the case of unstable stratification, $\phi_i(z/L)$ and $\phi_s(z/L)$ can be expressed in the following forms (Panofsky et al. 1977; Padro 1993; Katul et al. 1999):

$$\phi_i(z/L) = c_1(1 - c_2\, z/L)^{1/3}; \tag{13}$$

$$\phi_s(z/L) = \alpha_s(1 - \beta_s\, z/L)^{-1/3}. \tag{14}$$

where c_1, c_2, α and β are coefficient to be determined by the field observation. In the case of stable stratification, $\phi_s(z/L)$ approximates a constant and $\phi_i(z/L)$ is still the 1/3 function of z/L. The turbulent characteristics of eddies in different temporal and spatial scale are analyzed and compared with the mean and autocorrelation ergodic theorems, to test feasibility of MOS relations under the condition of the ergodic and non-ergodic turbulence.

3 **The sources and processing of data**

In this study two turbulence data sets are used for completely different purposes. The first turbulence data set is the data measured by the eddy-covariance technique under the homogeneous surface in Nagqu Station of Plateau Climate and Environment (NSPCE), Chinese Academy of Sciences (CAS). The data set in NSPCE/CAS includes the data that are measured by 3-D sonic anemometer and thermometer (CSAT3) with 10 Hz as well as infrared gas analyzer (Li7500) in ASL from 23 July to 13 September 2011. In addition, the second turbulence data set of CASES-99 (Poulos et al. 2002; Chang and Huynh. 2002) is used to verify the ergodicity of turbulence observed by multi-stations. CASES-99 has seven observation sites to be equivalent to seven observation stations. The data in the central tower of CASES-99 include that measured by sonic anemometer and thermometer (CSAT3) with 20 Hz and the infrared gas analyzer (Li7500) at 10m on tower with 55 m height in ASL. The other six sub-sites of CASES-99 surrounding the central tower, sn1, sn2 and sn3 are located 100 m are away from the central tower, the sub-site sn4 is 280 m away, and sub-sites sn5 and sn6 are located 300 m away. The data of sub-sites include that measured by 3-D sonic anemometer (ATI) and Li7500 at 10 m height on the towers. The analyzed results with two data sets are compared each other to test universality of the research results.

The geographic coordinate of NSPCE/CAS is 31.37°N, 91.90°E, and its altitude is 4509 m a.s.l. The observation station is built on flat and wide area except for a hill of about 200 m at 2 km distance in the north, and floor area is 8000m^2. The ground surface is mainly composed of sandy soil mixed with sparse fine stones, and a plateau meadow with vegetation of 10-20 cm. The roughness length and displacement height of underlying surface of NSPCE meadow are respectively 0.009 m and 0.03 m. CASES-99 is located in prairie of Kansas US. The geographic coordinate of CASES-99 central tower is 37.65°N, 96.74°W. The observation field is flat and growth grasses about 20-50 cm during the observation period, while the roughness length and displacement height of CASES-99 underlying surface are 0.012 m and 0.06 m, respectively (Martano 2000).

These data are used to study the ergodicity of turbulent eddies in ABL. Firstly the inaccurate data caused by spike are deleted before data analyses. Subsequently, the data are divided into continuous sections of 5-hour, and the signals of 1-hour are obtained applying filtering of Eqs. (8) and (9) for each 5-hour data. In order to delete further the abnormal inaccurate data, the data are divided once again into 12 continuous fragments of 5-min in 1-hour. The variances of velocity and temperature are calculated and compared each other for the fragments. The data that deviation is less than $\pm 15\%$ including an instrumental error about $\pm 5\%$ are selected to use. Moreover, temperature of the ultrasonic pulse signals is converted to the absolute temperature (Schotanus et al. 1983; Kaimal and Gaynor 1991). Then all data without spike for 25 days are done the coordinate rotation using the plane fitting method to improve the levelness of instrument installation (Wilczak 2001). The trend correction (McMillen 1988; Moore 1986) is used to exclude the influence of low-frequency trend effect caused by the diurnal variations and weather processes. The Webb correction (Webb et al. 1980) is a component of surface energy balance in physical nature, but not the component of turbulent eddy. However, this study is to analyze the ergodicity of turbulent eddies. According to our preliminary analysis about the ergodicity of turbulent eddies, such correction may cause the unreasonable deviation from the prediction with Eq. (14). We thus do not perform the Webb correction in our research on the ergodicity.

4. Result analyses

Applying the two data sets from NSPCE/CAS and CASES-99, the ergodicity of different temporal scale eddies is tested. Here as an example, we select representative data measured at level of 3.08m in NSPCE/CAS during three time frames, namely 3:00-4:00, 7:00-8:00 and 13:00-14:00 China Standard Time (CST) on 25 August in clear weather to test and demonstrate the ergodicity of different temporal scale eddies. These three time frames represent three situations, i.e. the nocturnal stable boundary layer, early neutral boundary layer and midday convective boundary layer.

Eqs. (8) and (9) are used to perform band-pass filtering from $n=j$ to $n=N-j$ to acquire the signals of eddies corresponding temporal scale including 2 min, 3 min, 5 min, 10 min, 30 min and 60 min. The turbulence characteristics and ergodicity of eddies in the different temporal scale including 2 min, 3 min, 5 min, 10 min, 30 min and 60 min are studied using above processed data for three time frames.

4.1 M-O eddy local stability and M-O stratification stability

The M-O stratification stability parameter z/L describes a whole characteristic of the mechanical and buoyancy effect on the ASL turbulence. However this study will decompose the turbulence into different scale eddies. Considering that the property of different scale eddies of the atmospheric turbulence varies with the atmospheric stability parameter z/L, a M-O eddy local stability that is limited in the certain scale range of eddies is defined as z/L_c, so as to analyze relations between the stratification stability and ergodicity of the different scale eddies for the wind velocity, temperature and other factors. It is worth noting that the M-O eddy local stability, z/L_c, is different from the M-O stratification stability, z/L.

As a typical example, the eddy local stabilities, z/L_c, of the different temporal scales for the three time frames from the nighttime to the daytime are shown in Table 1. The results show that the eddy local stability z/L_c below 2 min in temporal scale at time 3:00-4:00 AM(CST) during the nighttime time frame is 0.59, thus it is stable stratification. But as the eddy temporal scale gradually increases from 3 min, 5 min and 10 min to 60 min, the eddy local stability, z/L_c, gradually decreases to 0.31 and 0.28. Even starting from 10 min in the temporal scale, the eddy local stability decreases from -0.01 to -0.07. It seems that the eddy local stability gradually varies from stable to unstable as the eddy temporal scale increases. At 7:00-8:00 AM (CST) during the morning time frame, the eddy local stability z/L_c from 2 min to 60 min in the temporal scale eventually decreases from 0.52, 0.38, 0.16 and 0.15 to -0.43 in 30 min and a minimum of -1.29 in 60 min. It means that eddies in the temporal scales of 30 min and 60 min have high local instability. However, at 14:00-15:00 PM (CST) during the midday time frame, eddies in the temporal scales from 2 min to 60 min are all unstable. Now $-z/L_c$ is defined as eddy local instability. As the eddy scale increases, the eddy local instability in the scales from 2 min to 3 min also increases. And that its value reaches a maximum of 0.44 as the eddy scale is at 5 min. But as the eddy scale increases continuously, the eddy local instability is reduced.

The M-O eddy local stability is not entirely the same as the M-O stratification stability of ABL in the physical significance. The M-O stratification stability of ABL indicates the overall effect of atmospheric stratification in the ABL on the stability including all eddies in integral boundary layer. The M-O stratification stability z/L is stable 0.02 at 3:00-4:00 AM (CST) for no filtering data to include whole turbulent signals, but unstable -0.004 and -0.54 at 7:00-8:00 and 13:00-14:00 PM (CST), respectively. However the eddy local stability is only a local effect of atmospheric stratification on the stability of eddies in a certain scale. As the eddy scale increases, the eddy local stability z/L_c will vary accordingly. The aforesaid results indicate that the local stability of small-scale eddies is stable in the nocturnal stable boundary layer, but it is possibly unstable for the large-scale eddies. As a result, a sink effect on the small-scale eddies in the nocturnal stable boundary layer, but a positive buoyancy effect on the large-scale eddies. However, in diurnal unstable boundary layer, the eddy local instability of 3 min scale reaches a maximum, and then the instability gradually decreases as the eddy scale increases. Therefore, eddies of 3 min scale hold maximum buoyancy, but the eddy buoyancy decreases as the eddy scale increases continuously. Nevertheless, the small-scale eddies are more stable than the large scale eddies in the nocturnal stable boundary layer; the large-scale eddies are more stable than the small scale eddies in the diurnal convective boundary layer with unstable stratification. The above facts signify that it is common that there exist mainly the small-scale eddies in the nocturnal boundary layer with stable stratification. And it is also common that there exist mainly the large-scale eddies in the diurnal convective boundary layer with unstable stratification. Therefore, it can well understand that the small-scale eddies are dominant in the nocturnal stable boundary layer, while the large-scale eddies are dominant in the diurnal convective boundary layer.

4.2 Verification of mean ergodic theorem of eddies in different temporal scale

In order to verify the mean ergodic theorem, we calculate the mean and autocorrelation functions using Eq. (2) and Eq. (3), then calculate the variation of mean ergodic function Ero(A) using Eq. (5) of eddies in the different temporal scale with relaxation time τ to be cut off with $\tau_{i=n}$. The mean ergodic functions, Ero(A), of vertical velocity, temperature and specific humidity of the different scale eddies are calculated using data at level of 3.08m at 3:00-4:00, 7:00-8:00 and 13:00-14:00 (CST) for three time frames in NSPCE/CAS, as shown in Figs. 1-3 respectively. Since the ergodic function varies within a large range, the ergodic functions are normalized according to the characteristic quantity of relevant variables ($A_* = \boldsymbol{u}_*, |\theta_*|, |q_*|$). That is to say, functions in all following figures are the dimensionless ergodic functions, Ero(A)/A_*.

Comprehensive analyses of the characteristics of mean ergodicity of atmospheric turbulence as well as the relevant causes:

4.2.1 Verifying mean ergodic theorem of different scale eddies

According to the mean ergodic theorem, Eq. (4), the mean ergodic function Ero(A)/A_* will converge to 0 if the time approaches infinite. This is only a theoretical result of the stationary random processes. A practical mean ergodic function is calculated under the condition of that relaxation time $\tau_{i=n}$ is cut off. If the mean ergodic function

$Ero(A)/A_*$ converges approximately to 0 in relaxation time $\tau_{i=n,}$ it will be considered that random variable A approximately satisfies the mean ergodic theorem. The mean ergodic function deviates more from zero, the mean ergodicity will be of poor quality. Consequently, we can judge approximately the mean ergodic theorem of different scale eddies whether or not holds. Figs. 1-3 clearly show that, regardless of the vertical velocity, temperature or humidity, the $Ero(A)/A_*$ of eddies below 10 min in the temporal scale will swing around zero within a small range; thus we can conclude that the mean ergodic function $Ero(A)/A_*$ of eddies below 10 min in the temporal scale converges to zero to satisfy effectively the condition of mean ergodic theorem. For eddies of 30 min and 60 min, which are larger scale, the mean ergodic function $Ero(A)/A_*$ will deviate further from zero. In particular, the mean ergodic function $Ero(A)/A_*$ of eddies of 30 min and 60 min for the temperature and humidity does not converge, and even diverges. Above results show that the mean ergodic function of eddies of 30 min and 60 min cannot converge to zero or cannot satisfy the condition of mean ergodic theorem.

4.2.2 Comparison of the convergence of mean ergodic functions of vertical velocity, temperature and humidity

As seen from the Figs. 1-3, dimensionless mean ergodic function of the vertical velocity is compared with respective function of the temperature and humidity, it is 3-4 magnitudes less than those in the nocturnal stable boundary layer; 1-2 magnitudes less than those in the early neutral boundary layer; and about 2 magnitudes less than those in the midday convective boundary layer. For example, at 3:00-4:00 PM (CST) during nighttime time frame, the dimensionless mean ergodic function of vertical velocity is 10^{-5} in magnitude, while respective magnitudes of function value of the temperature and humidity are 10^{-1} and 10^{-2}; at 7:00-8:00 AM (CAT) during morning time frame, magnitude of mean ergodic function of the vertical velocity is 10^{-4}, while the respective magnitudes of function value of the temperature and humidity are 10^{-2} and 10^{-3}; at 13:00-14:00 PM(CST) during midday time frame, magnitude of mean ergodic function of the vertical velocity is 10^{-4}, while the magnitudes of function value of the temperature and humidity are both 10^{-2}. These results show that the dimensionless mean ergodic function of vertical velocity converges to zero much more easily than respective function value of the temperature and humidity, and that the vertical velocity satisfies the condition of mean ergodic theorem to overmatch more than the temperature and humidity.

4.2.3 Temporal scale and spatial scale of turbulent eddies

For wind velocity of 1-2 ms^{-1}, eddy spatial scale in the temporal scale 2 min is in the range of 120-240 m, and eddy spatial scale in the temporal scale of 10 min is in the range of 600-1200 m. The eddy spatial scale in the temporal scale of 2 min is equivalent to ASL height, and the eddy spatial scale in the temporal scale of 10 min is equivalent to ABL height. The eddy spatial scale within the temporal scales of 30-60 min is around 1800-3600 m, and this spatial scale clearly exceeds ABL height to belong to scope of the atmospheric local circulation. According to the stationary random processes definition (1) and mean ergodic theorem, the stationary random processes must be smooth in relaxation time τ. The eddoes below temporal scale of 10 min, i.e., below ABL height, can effectively satisfy the condition of mean ergodic theorem, and must be the stationary random processes of mean ergodicity. However, eddies in the temporal scales of 30 min and 60 min exceed ABL height and do not satisfy the condition of mean ergodic theorem.

4.2.4 Turbulence ergodicity of all eddies in possible scales in ABL

To facilitate comparison, Fig. 4 shows the variation of mean ergodic function Ero(A) of the vertical velocity (a), temperature (b) and specific humidity (c) before filtering with relaxation time τ at 14:00-15:00 PM (CST) during midday time frame in convective boundary layer. It is obvious that Fig. 4 is unfiltered mean ergodic function of eddies in all possible scales in ABL. The Fig. 4 compares with Figs. 1c, 2c and 3c, which are the mean ergodic function Ero(A)/A_* of vertical velocity, temperature and humidity after filtering at 14:00-15:00 PM (CST) during the midday time frame. The result shows that the mean ergodic functions before filtering are greater than that after filtering. As shown in Figs. 1c, 2c and 3c, the magnitude for the vertical velocity is 10^{-4} and the magnitudes for the temperature and specific humidity are both 10^{-2}. According to Fig. 4, the magnitude of vertical velocity Ero(A)/A_* is 10^{-3} and the magnitudes of temperature and specific humidity are both 10^{0}, therefore 1-2 magnitudes are almost decreased after filtering. Moreover, all trend upward deviating from zero for vertical velocity and temperature, but downward deviating from zero for specific humidity. It is thus clear that, at 14:00-15:00 PM (CST) during the midday time frame, when is equivalent to the local time 12:00-13:00, the unfiltered mean ergodic function of eddies in all possible scales in convective boundary layer cannot converge to zero before filtering, i.e., cannot satisfy the condition of mean ergodic theorem. This may be that eddies in all possible scales before filtering include the local circulation in convective boundary layer. So we argue that, under general situations, the eddies only below 10 min in the temporal scale or within 600-1200 m in the spatial scale in ABL must be the stationary random processes of mean ergodicity.

4.2.5 Relation between the ergodicity and local stability of different scale eddies

Table 1 list the corresponding relation of eddy local stabilities z/L_c of eddies of different scales with the different time frames. It shows that the eddy local stabilities z/L_c of different scale eddies are different, due to the fact that the temperature stratification in ABL has different effect on the stability for different scale eddies. Even entirely contrary results can occur. At the same time, the stratification that causes the large scale eddy to ascend with buoyancy may cause the small scale eddy to descend. However, the results in Figs. 1-3 show that the ergodicity is mainly related to the eddy scale, and its relation with the atmospheric temperature stratification seems secondary.

4.3 Verification of autocorrelation ergodic theorem for different scale eddies

In this section, Eqs. (7a) and (7b) are used to verify the autocorrelation ergodic theorem. It is accordant with Sect. 4.2 that the turbulent eddies below 10 min in temporal scale satisfy the mean ergodic condition in the various time frames, i.e., the turbulent eddies below 10 min in temporal scale are at least strictly stationary random processes or narrow stationary random processes whether in the nocturnal stable boundary layer, or in the early neutral boundary layer and midday convective boundary layer. Then we analyze further the different scale eddies that satisfy the mean ergodic condition whether or not also satisfy the autocorrelation ergodic condition, so as to verify atmospheric turbulence is whether narrow or wide stationary random processes. The autocorrelation ergodic function of turbulence variable A under the condition of truncated relaxation time $\tau_{i=n}$ is calculated according to Eq. (7a) to determine the variation of autocorrelation ergodic function Er(A) with relaxation time τ. As with the mean ergodic function Ero(A), if the autocorrelation ergodic function Er(A) of eddies of 2 min, 3 min, 5 min, 10 min, 30 min and 60 min in the temporal scale within the relaxation time $\tau_{i=n}$ approximates 0, then A shall be deemed to be approximately ergodic; the more the autocorrelation ergodic function deviates from 0, the worse the autocorrelation ergodicity becomes. Therefore, this method can be used to judge approximatively whether the different scale eddies satisfy the condition of autocorrelation ergodic theorem.

As an example for the vertical velocity, Fig. 5 shows the variation of normalized autocorrelation ergodic function Ero(w)/u_* of the turbulent eddies of 2 min, 3 min, 5 min, 10 min, 30 min and 60 min in the temporal scale with relaxation time τ at 3:00-4:00, 7:00-8:00 and 13:00-14:00 (CST) during the time frames respectively. Some basic conclusions are drawn from Fig. 5 as following:

1. After comparing the Figs. 5a-c with the Figs. 1a-c, i.e., comparing the dimensionless mean ergodic function Ero(w)/u_* of vertical velocity with the dimensionless autocorrelation ergodic function Er(w)/u_*, two basic characteristics are very clear. First, the magnitudes of the dimensionless autocorrelation ergodic function Er($\mathbf{w}$)/u_*, regardless of whether in the nocturnal stable boundary layer, early neutral boundary layer or midday convective boundary layer, are all greatly reduced. In Figs. 1a-c, the magnitudes of Ero(w)/u_* are respectively 10^{-5}, 10^{-4} and 10^{-4}, and the magnitudes of Er(w)/u_* are respectively 10^{-7}, 10^{-5} and 10^{-5} as shown in Figs. 5a-c. The magnitudes of Er(w)/u_* reduce by 1-2 magnitudes compared with those of Ero(w) /u_*. Second, all autocorrelation ergodic functions Er(w)/u_* of the eddies of 30 min and 60 min in temporal scale, regardless of whether they are in the stable boundary layer, natural boundary layer or convective boundary layer, are all reduced and approximate to Ero (w)/u_* of the eddies below 10 min in temporal scale.
2. The above two basic characteristics imply that the autocorrelation ergodic function Er(w)/u_* of the stable boundary layer, neutral boundary layer or convective boundary layer converges to 0 faster than the mean ergodic function Ero ($\mathbf{w}$)/u_*; the autocorrelation ergodic function of eddies of 30 min and 60 min in temporal scale also converges to 0 and satisfies the condition of autocorrelation ergodic theorem, except for the fact that the autocorrelation ergodic function Er($\mathbf{w}$)/u_* of the eddies below 10 min in temporal scale can converge to 0 and satisfy the condition of autocorrelation ergodic theorem.
3. According to the autocorrelation ergodic function Eq. (7a), the eddies of 30 min, 60 min and below 10 min in the temporal scale, regardless of whether they are in the stable boundary layer, neutral boundary layer or convective boundary layer, all

eddies satisfy the condition of autocorrelation ergodic theorem. Therefore, in general ABL turbulence is the stationary random processes of autocorrelation ergodicity.

4. The above results show that the eddies below 10 min in temporal scale in the nocturnal stable boundary layer, early neutral boundary layer and midday convective boundary layer not only satisfy the condition of mean ergodic theorem, but also they satisfy the condition of autocorrelation ergodic theorem. Therefore, eddies below 10 min in the temporal scale are a wide ergodic stationary random processes. Although the eddies of 30 min and 60 min in temporal scale in the stable boundary layer, neutral boundary layer and convective boundary layer satisfy the condition of autocorrelation ergodic theorem, but they dissatisfy the condition of mean ergodic theorem. Therefore, eddies of 30 min and 60 min in the temporal scale are neither narrow ergodic stationary random processes, nor wide ergodic stationary random processes.

4.4 Ergodic theorem verification of different scale eddies for the multi-stations

The basic principle of turbulence average is an ensemble average of the space, time and state. Sections 4.2 and 4.3 verify the mean ergodic theorem and autocorrelation ergodic theorem of atmospheric turbulence using field observational data, so that the finite time average of a single station can be used to substitute for the ensemble average for the ergodic turbulence. This section examines the ergodicity of different scale eddies using the observational data of center tower and six sub-sites of CASES-99, in all seven sites to be equivalent to seven stations. When the data are selected, it is considered that if the eddies are not evenly distributed at the seven sites, then the observation results at the seven sites may have originated from many eddies in the large scale. For this reason, the high frequency variance spectrum in excess of 0.1 Hz is compared firstly. Based on the observational error, if the scatter of all high frequency variances does not exceed the average by ±10%, then it is assumed that the turbulence is evenly distributed at the seven observation sites. And then, 17 datasets are chosen from among the observed turbulence data from 5 to 30 October, and these data sets represent typical strong turbulence at noon on the sunny day. As an example, the same method as described in Sections 4.2 and 4.3 is used to respectively calculate variation of the mean ergodic function and autocorrelation ergodic function with relaxation time τ for the vertical velocity at 10:00-11:00 AM on 7 October. The time series composed of the above data sets is performed band-pass filtering in 2 min, 3 min, 5 min, 10 min, 30 min and 60 min. The variations of mean ergodic function Ero(w)/u_* and autocorrelation ergodic function Er(w)/u_* with relaxation time τ are analyzed for the vertical velocity to test the ergodicity of different scale eddies for observation of the multi-stations. Fig. 6a shows variation of mean ergodic function Ero(w)/u_* with the relaxation time τ for the vertical velocity, and Fig. 6b shows variation of autocorrelation ergodic function Er(w)/u_* with the relaxation time τ.

The results show ergodic characteristics of different scale eddies measured with the multi-stations as following:

Fig. 6a shows that the mean ergodic function of eddies below 30 min in temporal scale converges to 0 very well, except for the fact that the mean ergodic function of eddies of 60 min in temporal scale clearly deviates upward from 0. Fig. 6b shows that autocorrelation ergodic function of all different scale eddies including 60 min in temporal scale, gradually converges to 0. Therefore, eddies below 30 min in temporal scale measured with the multi-stations satisfy the conditions of both the mean and autocorrelation ergodic theorem, while eddies of 60 min in temporal scale only satisfies the condition of autocorrelation ergodic theorem, but dissatisfy the condition of mean ergodic theorem. These facts demonstrate that eddies below 30 min in temporal scale are the wide ergodic stationary random processes for time series of above data sets composed by the seven stations. This signifies that comparing of data composed of the multi-stations with data from a single station, the eddy temporal scale of wide ergodic stationary random processes is extended from below 10 min to 30 min. As analyzed above, if the eddies below 10 min in temporal scale are deemed to be the turbulent eddies in the ABL with height about 1000 m, and the eddies of 30 min in the temporal scale, which is equivalent to that the space scale is greater than 2000 m, are deemed including eddy components of the local circulation in ABL, in that way the multiple station observations can completely capture the local circulated eddies, which space scale is greater than 2000 m.

4.5 Average time problem of turbulent quantity averaging

The atmospheric observations are impossible to repeat experiments exactly, must use the ergodic hypothesis and replace ensemble averages with time averages. It arises a problem how does determine the averaging time.

The analyses on the ergodicity of different scale eddies in above two sections demonstrate that the eddies below 10 min in temporal scale as relaxation time τ=30 min in the stable boundary layer, neutral boundary layer and convective boundary layer not only satisfy the mean ergodic theorem, but also satisfy the autocorrelation ergodic theorem. That is to say, they are namely wide ergodic stationary random processes. Therefore, a finite time average of 30 min within relaxation time τ can be used for substituting for the ensemble average to calculate mean random variable, Eq. (2). However, the eddies of 30 min and 60 min in temporal scale in the stable boundary layer and neutral boundary layer are only autocorrelation ergodic random processes, neither narrow nor wide sense random processes. Therefore, when the finite time average of 30 min can be used for substituting for the ensemble average to calculate mean random variable Eq. (2), it may capture the eddies below 10 min in temporal scale in stationary random processes, but cannot completely capture the eddies in excess of 30 min in temporal scale. The above results signify that the turbulence average is restricted not only by the mean ergodic theorem, but also is closely related to the scale of turbulent eddies. In the atmospheric observations performed using the eddy-covariance technique, the substitution of ensemble average with finite time average of 30 min inevitably results in a high level of error, due to loss of low frequency component information associating with the large-scale eddies. However, although eddies of 30 min and 60 min in temporal scale in convective boundary layer are not wide ergodic stationary random processes, they are autocorrelation ergodic random processes. This may imply that the mean of atmospheric turbulence in the convective boundary layer, which is calculated to substitute the finite time average for the ensemble average, is often superior to the results of the stable boundary layer and neutral boundary layer. Withal, the results in the previous sections also show that the mean ergodic function of vertical velocity may more easily converge to 0 than functions corresponding to the temperature and humidity, i.e., the vertical velocity may more easily satisfy the condition of mean ergodic theorem than the temperature and humidity. Therefore, in the observation performed using the eddy-covariance technique, the result of vertical velocity is often superior to those of the temperature and humidity. In the previous section, the results also point out that multi-stations observation can completely capture eddy of the local circumfluence in the ABL. Therefore, the multi-stations observation is more likely to satisfy the ergodic assumption, and its results are much closer to the true values. In order to determine the averaging time, Oncley (1996) defined an Ogive function of cumulative integral

$$Og_{x,y}\left(f_0\right)=\int_{\infty}^{f_0} Co_{x,y}\left(f\right)df \tag{15}$$

where x and y are any two variables, and their covariance is $\overline{xy}$, $Co_{xy}(f)$ is the cospectrum of xy. If the Ogive function converges to a constant value at a frequency $f=f_0$, this frequency could be converted to an averaging time. Ogive function of $\overline{u'w'}$ is often used to examine the minimal averaging time. As a comparison, here the variation of Ogive functions of $\overline{w'^2}$ and $\overline{u'w'}$ with frequency at the height 3.08 m in NSPCE/CAS for the three time frames is shown in Fig.7. The Fig.7 shows convergence frequency of Ogive function for $\overline{w'^2}$ in the nighttime stable boundary layer, morningtide neutral boundary layer and midday convection boundary layer is respectively about at 0.01 Hz, 0.0001 Hz and 0.001 Hz. It is equivalent to the averaging times about 2 min, 160 min and 16 min. For $\overline{u'w'}$, it converges about at 0.001 Hz only in the midday convection boundary layer to be equivalent to the averaging time about 16 min; it seems no convergence in the nighttime stable and morningtide neutral boundary layer. It is implied determining the averaging time encounters a bit difficult with the Ogive function in the stable and neutral boundary layer. Fig.7 shows also that when the frequency is lower than 0.0001Hz, Ogive functions $\overline{u'w'}$ ascend in the stable boundary layer, but descend in the morningtide neutral boundary layer and midday convection boundary layer. We must especially note that Ogive function is a cumulative integral. So as Ogive function changes direction from ascending to descending, it implies a possibility that there exists a superimposing of the negative and positive momentum fluxes caused by a cross local circulation effect in nighttime and midday. This cross local circulation in ABL may cause the low frequency effect on the Ogive function. So that the local circulation in ABL may be an important cause that Ogive fails to judge the averaging time. In this work, the choice of averaging time with the ergodic theory seems superior to with the Ogive function.

4.6 MOS of turbulent eddies in different scales and its relation with ergodicity

Turbulent variance is a most basic characteristic quantity of the turbulence. Turbulence velocity variance, which represents turbulence intensity, and the variance of scalars, such as temperature and humidity, effectively describes the structural characteristics of turbulence. In order to test MOS relation of the different scale eddies with ergodicity, the vertical velocity and temperature data of NSPCE/CAS from 23 July to 13 September are used to determine the MOS relationship of variances of vertical velocity and temperature for the different scale eddies, and to analyze its relation with the ergodicity.

The MOS relation of vertical velocity variance as following:

$$\phi_i(z/L) = c_1(1 - c_2 z/L)^{1/3}, \quad z/L < 0, \tag{16}$$

$$\phi_i(z/L) = c_1(1 + c_2 z/L)^{1/3}, \quad z/L > 0. \tag{17}$$

Fig. 8 and 9 respectively shows the MOS relation curves of different scale eddies for the vertical velocity and temperature variances in NSPCE/CAS. The figures (a), (b) and (c) of Fig. 8 and 9 are respectively the similarity curve of eddies of 10 min, 30 min and 60 min in the temporal scale. Table 2 shows the relevant parameters of fitting curve of MOS relation for the vertical velocity variance. The correlation coefficient and residual of fitting curve are respectively expressed with *R* and *S*.

Fig. 8 and Table 2 show that the parameters of fitting curve are greatly different, even if the fitting curve modality of MOS relation of the vertical velocity variance is the same for the eddies in different temporal scales. The correlation coefficients of MOS fitting curve of the vertical velocity variance under the unstable stratification are large, but the correlation coefficients under the stable stratification are small. Under unstable stratification, the correlation coefficient of eddies of 10 min in the temporal scale reaches 0.97, while the residual is only 0.16; under the stable stratification, the correlation coefficient reduces to 0.76, and the residual increases to 0.25. With the increase of eddy temporal scale from 10 min (Fig. 8a) to 30 min (Fig. 8b) and 60 min (Fig. 8c), the correlation coefficients of MOS relation of the vertical velocity variance gradually reduce, and the residuals increase. The correlation coefficient in 60 min reaches a minimum; it is 0.83 under the unstable stratification, and only 0.30 under the stable stratification.

The temperature variance is shown in Fig. 9. MOS function to fit from eddies of 10 min in the temporal scale under the unstable stratification is following:

$$\phi_\theta(z/L_c) = 4.9(1 - 79.7\, z/L_c)^{-1/3}. \tag{18}$$

As shown in Fig. 9a, the correlation coefficient of fitting curve is 0.91 and residual is

0.38. With increase of the eddy temporal scale, discreteness of MOS relation of the temperature variance is enlarged quickly to incur that the appropriate curve cannot be fitted.

The above results show that the discreteness of fitting curve of MOS relation for the turbulence variance is enlarged with the increase of eddy temporal scale, whether it is the vertical velocity or temperature. The points of data during the stationary processes basically gather nearby the fitting curve of variance similarity relation, while all data points during the non-stationary processes deviate significantly from the fitting curve. However, the similarity of vertical velocity variance is superior to that of the temperature variance. These results are consistent to the conclusions of ergodicity test for the different scale eddies described in Sections 4.2-4.4. The ergodicity of the small-scale eddies is superior to that of the larger-scale eddies, and eddies of 10 min in the temporal scale have the best variance similarity relations. These results also signify that when eddies in the stationary random processes satisfy the ergodic condition, both the vertical velocity variance and temperature variance of eddies in the different temporal scales comply with MOST very well; but, as for eddies with poor ergodicity during non-stationary random processes, the variances deviate from MOS relations.

5 Discussions

1. Galanti and Tsinober (2004) proved that the turbulence, which is temporally steady and spatially homogeneous, is ergodic, but '*partially turbulent flows*' such as the mixed layer, wake flow, jet flow, flow around and boundary layer flow may be non-ergodic turbulence. However, it has been proven through atmospheric observational data that the turbulence ergodicity is related to the scale of turbulent eddies. Since the large-scale eddies in ABL may be strongly influenced by the boundary disturbance, thus belong to 'partial turbulence'; however, since the small-scale eddies in atmospheric turbulence may be not influenced by boundary disturbance, may be temporally steady and spatially homogeneous turbulence. So that the mean ergodic theorem and autocorrelation ergodic theorem are applicative for turbulence eddies in the small scale in ABL, but the ergodic theorems aren't applicative for the large-scale eddies, i.e., the small-scale eddies in the ABL are ergodic and the large-scale eddies exceeding the ABL scale are non-ergodic.
2. The eddy-covariance technique for turbulence measurement is based on the ergodic

assumption. A lack of ergodicity related to the presence of large-scale eddy transport can lead to a consider error of the flux measurement. This has already been pointed out by Mauder et al. (2007) or Foken et al. (2011). Therefore, we realize from the above results that the large scale eddies that exceed ABL height may include component of non-ergodic random processes. The eddy-covariance technique cannot capture the signals of large-scale eddies exceeded ABL scale to result in the large error in the measurements of atmospheric turbulent variance and covariance. MOST is developed under the condition of the steady time and homogeneous surface. MOST conditions, steady time and homogeneous underlying surface, are in line with the ergodic conditions, therefore the turbulence variances, even the turbulent fluxes of eddies in different temporal scales may comply with MOST very well, if the ergodic conditions of stationary random processes are more effectively satisfied.

3. According to Kaimal and Wyngaard (1990), the atmospheric turbulence theory and observation method were feasible and led to success under ideal conditions including a short period, steady state and homogeneous underlying surface, and through observation in the 1950s-1970s, but these conditions are rare in reality. In the land surface processes and ecosystem, the turbulent flux observations in ASL turn into a scientific issue, in which commonly interest researchers in the fields of atmospheric sciences, ecology, geography sciences, etc. These observations must be implemented under conditions such as with complex terrain, heterogeneous surface, long period and unsteady state. It is necessary that more neoteric observational tools and theories will be applied with new perspectives in future research.
4. It is successful that the ergodic theorem of stationary random processes is introduced from the mathematics into atmospheric sciences. It undoubtedly provides a profited tool for overcoming the challenges encountering in the modern measurements of atmospheric turbulent flow. At least it offers a promising first step to diagnosticate directly the ergodic hypotheses for ASL flows as a criterion. And that the necessary and sufficient condition of ergodic theorem can use to judge the applicative scope of eddy-covariance technique and MOST, and seek potential disable reasons for using them in the ABL.
5. In the future, we shall keep up to study the ergodic problems for the atmospheric

turbulence measurements under the conditions of complex terrain, heterogeneous surface and unsteady, long observational period, and to seek effective schemes. The above results indicate the atmospheric turbulent eddies below the scale of ABL can be captured by the eddy-covariance technique and comply with MOST very well. Perhaps MOST can be as the first order approximation to deal with the turbulence of eddies below ABL scale satisfying the ergodic theorems, then to compensate the effects of eddies dissatisfying the ergodic theorem, which may be caused by the advection, local circulation, low frequency effect, etc. under the complex terrain, heterogeneous surface. For example, we developed a turbulent theory of non-equilibrium thermodynamics (Hu, Y., 2007; Hu, Y., et al., 2009) to find the coupling effects of vertical velocity, which is caused by the advection, local circulation, and low frequency, on the vertical fluxes. The coupling effects of vertical velocity may be as a scheme to compensate the effects of eddies dissatisfying the ergodic theorems (Hu, Y., 2003; Chen, J., et al., 2007, 2013).

6. It is clear that such studies are preliminary, and many problems require further research, and the attestation of more field experiments is necessary.

6 Conclusions

From the above results, we can draw the below preliminary conclusions:

1. The turbulence in ABL is an eddy structure. When the temporal scale of turbulent eddies in ABL is about 2 min, the corresponding spatial scale is about 120-240 m to be equivalent to ASL height; when the temporal scale of turbulent eddies in ABL is about 10 min, the corresponding spatial scale is about 600-1200 m to be equivalent to the ABL height. For the eddies of larger temporal and spatial scale, such as eddies of 30-60 min in the temporal scale, the corresponding spatial scale is about 1800-3600 m to exceed the ABL height.
2. The above results show that the ergodicity of atmospheric turbulence in ABL is not only relative to the atmospheric stratification but also to the eddy scale of atmospheric turbulence. For the atmospheric turbulent eddies below the ABL scale, i.e. the eddies below about 1000 m in the spatial scale and about 10 min in the temporal scale, the mean ergodic function Ero(A) and autocorrelation ergodic function Er(A) converge to 0, i.e., they satisfy the conditions of mean and autocorrelation ergodic theorem. However, for the atmospheric turbulent eddies

in excess of 2000-3000m in the spatial scale and in excess of 30-60 min in the temporal scale, the mean ergodic function doesn't converge to 0, thus dissatisfy the condition of mean ergodic theorem. Therefore, the turbulent eddies that is below the ABL scale belong to the wide ergodic stationary random processes, but the turbulent eddies that are larger than ABL scale belong to the non-ergodic random processes, or even the non-stationary random processes.

3. Due to above facts, when the stationary random process information of eddies below 10 min in the temporal scale and below 1000 m of ABL height in the spatial scale can be captured, the atmospheric turbulence may satisfy the condition of mean ergodic theorem. Therefore, an average of finite time can be used to substitute for the ensemble average to calculate the mean of random variable as measuring atmospheric turbulence with the eddy-covariance technique. But for the turbulence of eddies to be larger than 30 min in temporal scale, i.e., 2000 m in spatial scale magnitude, it dissatisfy the condition of mean ergodic theorem, so that the eddy-covariance technique cannot completely capture the information of non-stationary random processes. This will inevitably cause a high level of error when the average of finite time is used to substitute for the ensemble average in the experiments due to the loss of low frequency component information associating with the large-scale eddies.
4. Although the atmospheric temperature stratification has different effects on the stability of eddies in the different scales, the ergodicity is mainly related to the eddy local stability, and its relation with the stratification stability of ABL is secondary.
5. The data series composed from seven stations compare with the observational data from a single station. The results show that the temporal and spatial scale of eddies to belong to the wide ergodic stationary random processes are extended from 10 min to below 30 min and from 1000 m to below 2000 m respectively. This signifies that the ergodic assumption is more likely to be satisfied well with multi-stations observation, and observational results produced by the eddy-covariance technique are much closer to the true values when calculating the turbulence averages, variances or fluxes.
6. If the ergodic conditions of stationary random processes are more effectively satisfied, then the turbulence variances of eddies in the different temporal scale

can comply with MOST very well; however, the turbulence variances of the non-ergodic random processes deviate from MOS relations.

Acknowledgements. This study is supported by Project Granted Nos. 91025011, 91437103 of the National Natural Science Foundation of China and Project Granted No. 2010CB951701-2 of the National Program on Key Basic Research. This work was strongly supported by Heihe Upstream Watershed Ecology-Hydrology Experimental Research Station, Chinese Academy of Sciences. We would like to express my sincere regards for their support. And that we thank Dr. Gordon Maclean in NCAR for providing the detailed data of CASES-99 used in this study, and thank referees and editor very much for heartfelt comments, discussions and marked errors.

References

Aubinet, M., Vesala, T., and Papale, D.: Eddy covariance, a practical guide to measurement and data analysis, Springer, Dordrecht, Heidelberg, London, New York, 438, 2012.

Birkhoff, G. D.: Proof of the ergodic theorem, Proc. Nat. Acad. Sci. USA. 18, 656-660, 1931.

Boltzmann, L.: Analytischer beweis des zweiten Haubtsatzes der mechanischen Wärmetheorie aus den Sätzen über das Gleichgewicht der lebendigen Kraft, Wiener Berichte , 63, 712-732, in WAI, paper 20, 1871.

Chang, S. S. and Huynh, G. D.: Analysis of sonic anemometer data from the CASES-99 field experiment. Army Research Laboratory, Adelphi, MD. 2002.

Chen, J., Hu Y., and Zhang L.: Principle of cross coupling between vertical heat turbulent transport and vertical velocity and determination of cross coupling coefficient, Adv. Atoms. Sci., 23 (4), 639-648, 2007.

Chen, J., Hu, Y., Lu, S., and Yu, Ye.: Experimental demonstration of the coupling effect of vertical velocity on latent heat flux, Sci. China. Ser. D-Earth Sci., 56, 1-9, 2013.

Eichinger, W. E., Parlange, M. B., Katul, G. G.: Lidar measurements of the dimensionless humidity gradient in the unstable ASL, Lakshmi, V., Albertson, J. and Schaake, J., Koster, R. D., Duan, Q., Land Surface Hydrology, Meteorology, and Climate, American Geophysical Union, Washington, D. C. 7-13, 2001.

Ehrenfest, P. and T. Ehrenfest-Afanassjewa (1912), The Conceptual Foundations of the Statistical Approach in Mechanics , New York: Cornell University Press, 1959.

Foken, T., Wichura, B.: Tools for quality assessment of surface-based flux measurements. Agric For Meteorol., 78, 83-105, 1996.

Foken, T., Göckede, M., Mauder, M., Mahrt, L., Amiro, B. D., and Munger, J. W.: Post-field data quality control, in: Handbook of micrometeorology: a guide for surface flux measurement and analysis, Lee, X., Massman, W. J., and Law, B.: Kluwer, Dordrecht, 181-208, 2004.

Foken, T., Aubinet, M., Finnigan, J. J., Leclerc, M. Y., Mauder, M., Paw, U. K. T.: Results of a panel discussion about the energy balance closure correction for trace gases. Bull Am. Meteorol. Soc., 92, ES13-ES18, 2011.

Galanti, B. and Tsinober, A.: Is turbulence ergodic? Phys. Lett. A, 330, 173–18, 2004.

Higgins, C. W., Katul, G. G., Froidevaux, M., Simeonov, V. and Parlange, M. B.: Atmospheric surface layer flows ergodic? Geophy. Res. Let., 40, 3342-3346, 2013.

Hill, R. J.: Implications of Monin–Obukhov similarity theory for scalar quantities, J. Atmos. Sci. 46, 2236–2244, 1989.

Hu, Y.: Convergence movement influence on the turbulent transportation in atmospheric boundary layer, Adv. Atoms. Sci., **20**, 794-798, 2003.

Hu, Y., Chen, J., Zuo, H.: Theorem of turbulent intensity and macroscopic mechanism of the turbulence development, Sci China Ser D-Earth Sci, **37**, 789-800, 2007.

Hu, Y., and Chen, J.: Nonequilibrium Thermodynamic Theory of Atmospheric Turbulence, In: Atmospheric Turbulence, Meteorological Modeling and Aerodynamics, Edited by Lang P. R. and Lombargo F. S., Nova Science Publishers, New York., 59-110, 2010.

Kaimal, J. C. and Wyngaard, J. C.: The Kansas and Minnesota experiments, Bound. Lay. Meteor., 50, 31-47, 1990.

Kaimal, J. C. and Gaynor, J. E.: Another look at sonic thermometry, Bound. Lay. Meteor., 56, 401–410, 1991.

Katul, G. G., Hsieh, C. I.: A note on the flux-variance similarity relationships for heat and water vapor in the unstable atmospheric surface layer, Bound. Lay. Meteor., 90, 327–338, 1999.

Katul, G., Cava, D., Poggi, D., Albertson, J., and Mahrt, L.: Stationarity, homogeneity, and ergodicity in canopy turbulence, in: Handbook of micrometeorology: a guide for surface flux measurement and analysis, Lee, X., Massman, W., and Law, B., Kluwer Academic Publishers, Dordrecht/BOSTON/LONDON , 161−180, 2004.

Krengel, U.: Ergodic theorems, de Gruyter, Berlin, New York, 363, 1985.

Lennaert van, V., Shigeo, K., and Genta, K.: Periodic motion representing isotropic turbulence, Fluid Dyn. Res., 38, 19–46, 2006.

Li, X., Hu, F., Pu, Y., Al-Jiboori, M. H., Hu, Z., and Hong, Z.: Identification of coherent structures of turbulence at the atmospheric surface layer, Adv. Atoms. Sci., 19(4), 687-698, 2002.

Martano, P.: Estimation of surface roughness length and displacement height from single-level sonic anemometer data, J. Appl. Meteorol., 39(5), 708–715, 2000.

Mattingly, J. C.: On recent progress for the stochastic Navier Stokes equations, Journées équations aux dérivées partielles, Univ. Nantes, Nantes, Exp. No. XI, 1-52, 2003.

Mauder, M., Desjardins, R. L., MacPherson, J.: Scale analysis of airborne flux measurements over heterogeneous terrain in a boreal ecosystem. J. Geophys. Res. 112, D13, 2007.

McMillen, R. T.: An eddy correlation technique with extended applicability to non simple terrain, Bound. Lay. Meteor., 43, 231-245, 1988.

Moore, C. J.: Frequency response corrections for eddy correlation systems, Bound. Lay. Meteor., 37, 17-35, 1986.

Neumann, J. V.: Proof of the quasi-ergodic hypothesis, Mathematics Proc. N. A. S., 18, 70-82, 1932.

Oncley, S. P., Friehe, C. A., Larue, J. C., Businger, J. A., Itsweire, E. C., Chang, S. S.: Surface-Layer Fluxes, Profiles, and Turbulence Measurements over Uniform Terrain under Near-Neutral Conditions, J. Atmos. Sci., 53 (7), 1029-1044, 1996.

Padro, J.: An investigation of flux-variance methods and universal functions applied to three land-use types in unstable conditions, Bound. Lay. Meteor., 66, 413–425, 1993.

Panofsky, H, A,, Lenschow, D, H,, and Wyngaard, J. C.: The characteristics of turbulent velocity components in the surface layer under unstable conditions. Bound. Lay. Meteor., 11, 355–361, 1977.

Papoulis, A. and Pillai, S. U.: Probability, random variables and stochastic processes. McGraw-Hill. New York. 666, 1991.

Poulos, G. S., Blumen, W., Fritts, D, C., Lundquist, J. K., Sun, J., Burns, S. P., Nappo, C., Banta, R., Newsom, R., Cuxart, J., Terradellas, E., and Balsley, Ben.: CASES-99:

a comprehensive investigation of the stable nocturnal boundary layer. Bull. Amer. Meteor. Soc., 83, 555–581, 2002.

Schotanus, P., Nieuwstadt, F. T. M., and de Bruin, H. A. R.: Temperature measurement with a sonic anemometer and its application to heat and moisture fluxes, Bound. Lay. Meteor., 26, 81–93, 1983.

Stull, R. B.: An introduction to boundary layer meteorology. Kluwer Academic Publ. Dordrecht. 670, 1988.

Uffink, J.: Boltzmann's work in statistical physics, Stanford encyclopedia of philosophy, Edward, N. Z., 2004.

Vickers, D., Mahrt, L.: Quality control and flux sampling problems for tower and aircraft data. J. Atmos. Oceanic. Technol., 14, 512-526, 1997.

Webb, E. K., Pearman, G. I., and Leuning, R.: Correction of the flux measurements for density effects due to heat and water vapor transfer, Q. J. R. Meteorol. Soc., 106, 85–100, 1980.

Wilczak, J. M., Oncley, S. P., and Stage, S. A.: Sonic anemometer tilts correction algorithms. Bound. Lay. Meteor., 99(1), 127-150, 2001.

Wyngaard, J. C.: Turbulence in the atmosphere, getting to know turbulence, Cambridge University Press, New York, 393 pp. 2010.

Zuo, H., Xiao X., Yang Q., Dong L., Chen J., Wang S.: On the atmospheric movement and the imbalance of observed and calculated energy in the surface layer, Sci. China. Ser. D-Earth Sci., 55, 1518-1532, 2012.

Table 1 Local Stability Parameter $(z\text{-}d)/L_c$ of the Eddies in Different Temporal Scales on 25 August

Eddy scale \ Time	3:00-4:00	7:00-8:00	14:00-15:00
≤2 min	0.59	0.52	-0.38
≤3 min	0.31	0.38	-0.44
≤5 min	0.28	0.16	-0.40
≤10 min	-0.01	0.15	-0.34
≤30 min	-0.04	-0.43	-0.27
≤60 min	-0.07	-1.29	-0.30

Table 2 Parameters of the Fitting Curve of MOS relation for Vertical Velocity Variance

	10 min		30 min		60 min	
	$z/L<0$	$z/L>0$	$z/L<0$	$z/L>0$	$z/L<0$	$z/L>0$
c_1	1.08	1.17	1.06	1.12	0.98	1.06
c_2	4.11	3.67	3.64	3.27	4.62	2.62
R	0.97	0.76	0.94	0.56	0.83	0.30
S	0.19	0.25	0.17	0.27	0.25	0.31

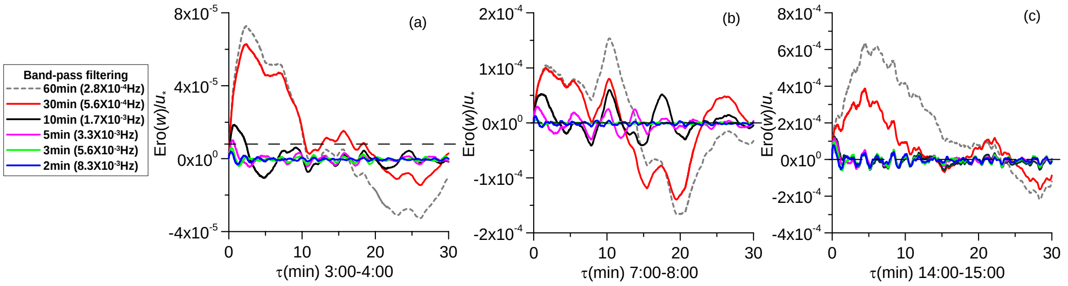

Fig. 1. Variation of mean ergodic function Ero($\boldsymbol{w}$) of vertical velocity measured at the height 3.08 m in NSPCE with relaxation time for the different scale eddies after band-pass filtering. Panels (a), (b) and (c) are the respective results of the three time frames. If their mean ergodic function is more approximate to zero, then eddies in the corresponding temporal scale will more closely satisfy the ergodic conditions.

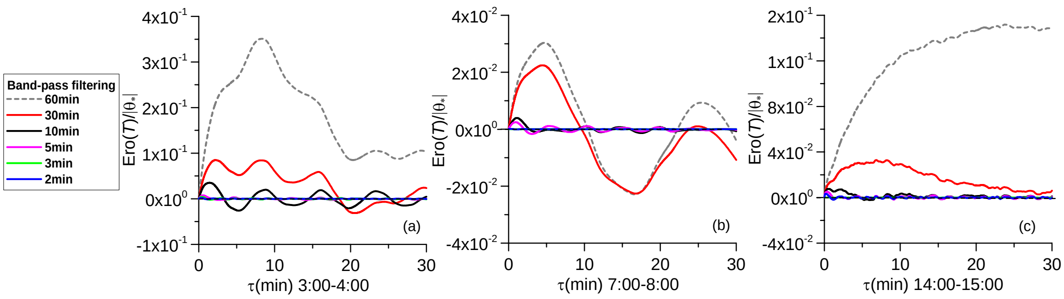

Fig. 2. Variation of mean ergodic function Ero(T) of the different scale eddies of temperature with relaxation time (other conditions are as some as Fig. 2, and the same applies to the following figures).

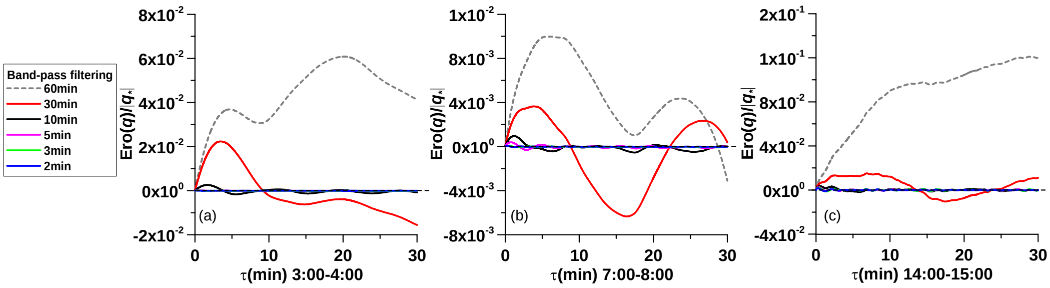

Fig. 3. Variation of mean ergodic function Ero(q) of the different scale eddies of humidity with relaxation time.

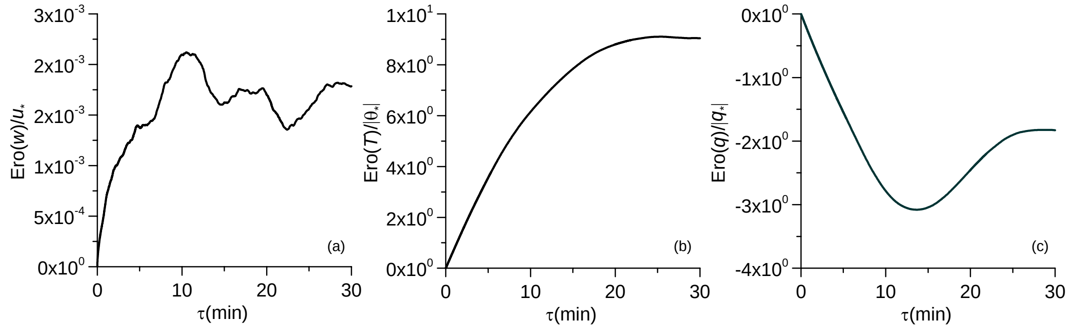

Fig. 4. Variation of mean ergodic function Ero(***w***) of the vertical velocity (a), temperature (b) and specific humidity (c) before filtering at 14:00-15:00 (CST) during midday in NSPCE with relaxation time *τ*.

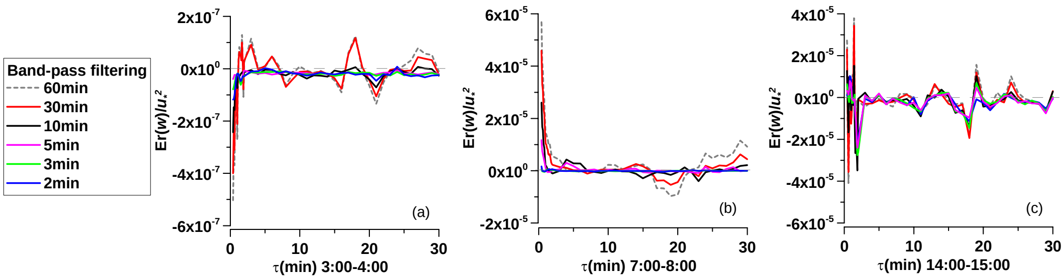

Fig. 5. Variation of the autocorrelation ergodic function of vertical velocity with relaxation time for different scale eddies.

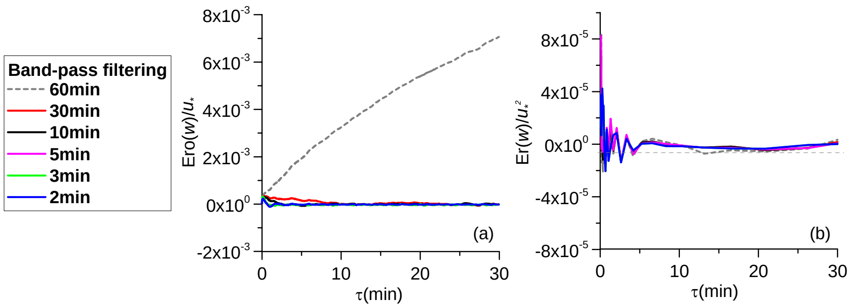

Fig. 6. Variation of mean ergodic function (a) and autocorrelation ergodic function (b) of the vertical velocity with relaxation time for the different scale eddies in the seven stations of CASES-99.

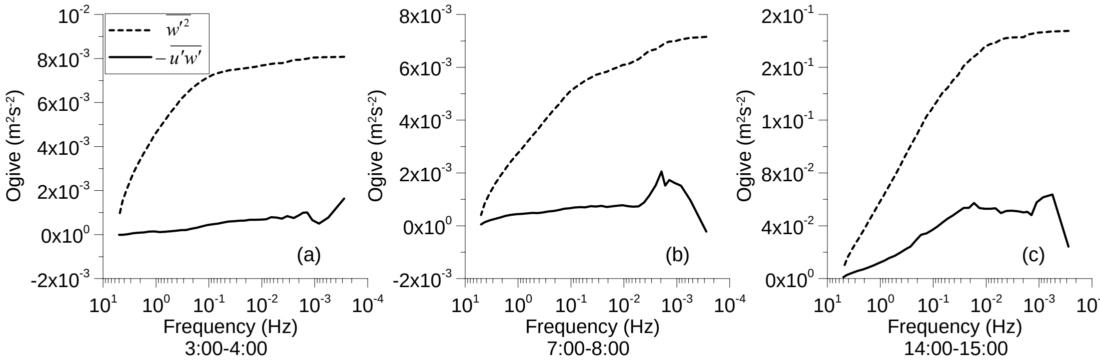

Fig. 7. Variation of Ogive functions of $\overline{w'^2}$ and $-\overline{u'w'}$ with frequency at height 3.08 m for the three time frames in NSPCE.

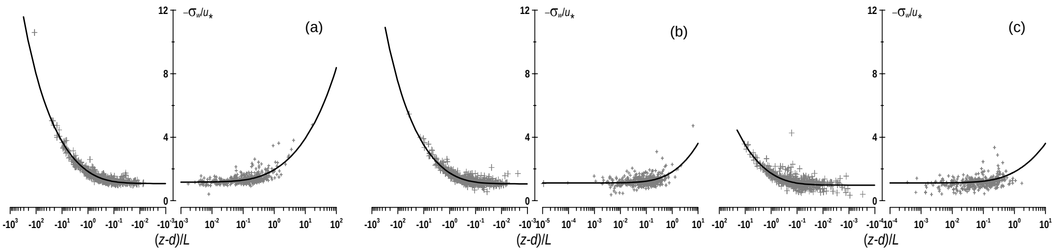

Figure 8. MOS relation of vertical velocity variances of the different scale eddies in NSPCE; Panels (a), (b) and (c) respectively represent the similarity of eddies of 10 min, 30 min and 60 min in the temporal scale.

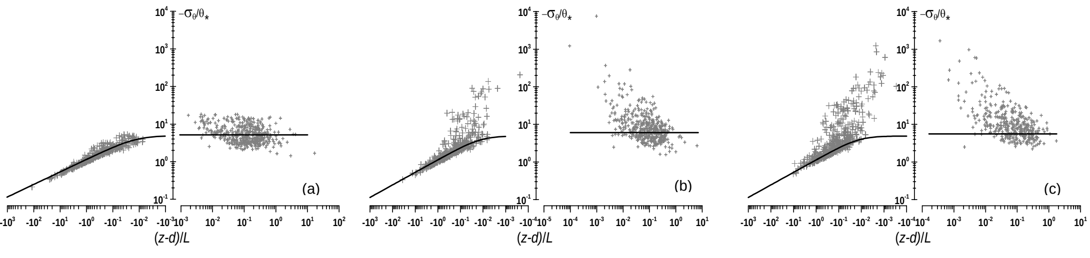

Figure 9. MOS relations of temperature variance of in different scale eddies of NSPCE; Panels (a), (b) and (c) respectively represent the similarity of the eddies of 10 min, 30 min and 60 min in the temporal scale.