Ergodicity Test of the Eddy Correlation Method

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Abstract

The ergodic hypothesis is a basic hypothesis in atmospheric turbulent experiment. The ergodic theorem of the stationary random processes is introduced first into the turbulence in atmospheric surface layer (ASL) to analyze and verify the ergodicity of atmospheric turbulence measured by the eddy covariance system with two sets of field observational data of the ASL. The results show that eddies of atmospheric turbulence, of which the scale are smaller than the scale of atmospheric boundary layer (ABL), i.e. the spatial scale is less than 1,000 m and temporal scale is shorter than 10 min, can effectively satisfy the ergodic theorems. Therefore, the finite time average can be used to substitute for the ensemble average of atmospheric turbulence. Whereas, eddies are larger than ABL's scale, cannot satisfy the mean ergodic theorem. Consequently, when the finite time average is used to substitute for the ensemble average, a large rate of error would occurs with the eddy correction method due to the losing low frequency information of the larger eddies. The multi-station observation is compared with the single-station, and then the scope that satisfies the ergodic theorems is expanded from the smaller scale about 1000 m of ABL's scale to about 2000 m, even it exceeds ABL's scale. Therefore, the calculation of average, variance and fluxes of the turbulence can effectively satisfy the ergodic assumption, and the results are more approximate to the actual values. Regardless of vertical velocity or temperature, the variance of eddies in different scales can more efficiently follow MOST, if the ergodic theorem can be satisfied; or else it deviates from MOST. The exploration of ergodicity of the atmospheric turbulence is doubtlessly helpful to understanding the issues in atmospheric turbulent observation, and provides a theoretical basis for overcoming related difficulties.

Keywords: Ergodic hypothesis; eddy-correlation method; Monin-Obukhov similarity

theory (MOST); atmospheric surface layer (ASL); high-pass filtering

1 Introduction

The basic principle of average of the turbulence measurement is the ensemble average of space, time and state. However, it is impossible that an actual turbulence measurement with numerous observational instruments in space for enough time to obtain all states of turbulent eddies to achieve the goal of ensemble average. Therefore, based on the ergodic hypothesis, the time average of one spatial point, which is long enough for observation, is used to substitute for the ensemble average for temporally steady and spatially homogeneous surface (Stull 1988; Wyngaard 2010; Aubinet 2012). The ergodic hypothesis is a basic assumption in atmospheric turbulent experiment of the atmospheric boundary layer (ABL) and atmospheric surface layer (ASL). The stationarity, homogeneity, and ergodicity are routinely used to link the ensemble statistics (mean and higher-order moments) of field observational experiments in the ABL. Many authors habitually refer to the ergodicity assumption, as some descriptions such as “when satisfying ergodic hypothesis, ……” or “something indicates that ergodic hypothesis is satisfied”. Though the success of Monin-Obukhov similarity theory (MOST) for unstable and near-neutral conditions is just an evidence of ergodic hypothesis validity in the ASL, however it is only a necessary condition for ergodicity in the ASL experiments, does not proves ergodicity (Katul et al., 2004). MOST success is under the conditions of stationary and homogeneous surface. It implies that the stationarity and homogeneity are the important conditions of ASL ergodicity. Therefore, many ABL’s experiments focus on seeking ideal homogeneous surface as much as possible. And some test procedures of availablity are widely applied to establish stationarity (Foken and Wichura 1996; Vickers and Mahrt 1997). Katul et al. (2004) qualitatively analyzed the ergodicity problems in regarding atmospheric turbulence, and believed that it is common for the neutral and unstable stratification in ASL to reach ergodicity, while it is difficult to reach ergodicity for the stable layer. Eichinger et al. (2001) indicate that LIDAR (Light Detection and Ranging) technique opens up new possibilities for atmospheric measurements and analysis by providing spatial and temporal atmospheric information with simultaneous high-resolution. The stationarity and ergodicity can be tested for such ensembles of experiments. Recent advances in LIDAR measurements offers a promising first step for direct evaluation of such hypotheses for ASL flows (Higgins et al., 2013). Higgins et al. (2013) applied LIDAR of water vapor concentration to investigate the ergodic hypothesis of atmospheric turbulence for the first time. It is clear all the same that there is a need to reevaluate turbulence measurement technology, to test the ergodicity of atmospheric turbulence quantitatively by means of observation experiments.

The ergodic hypothesis was first proposed by Boltzmann (Boltzmann 1871; Uffink 2004) in his study of the ensemble theory of statistical dynamics. He argued that a trajectory traverses *all* points on the energy hypersurface after a certain amount of time. At the beginning of 20th century, Ehrenfest' couple proposed the quasi-ergodic hypothesis and changed the term "traverses *all* points" in aforesaid ergodic hypothesis to "passes arbitrarily close to every point". The basic points of ergodic hypothesis or quasi-ergodic hypothesis recognize that the macroscopic property of system in the equilibrium state is the average of microcosmic quantity in a certain amount of time. Nevertheless, the ergodic hypothesis or quasi-ergodic hypothesis were never proven theoretically. The proof of ergodic hypothesis in physics aroused the interest of mathematicians. The famous mathematician, Neumann et al. (1932) first theoretically proved the ergodic theorem in topological space (Birkhoff 1931, Krengel 1985). Afterward, a banausic ergodic theorem of stationary random processes was proved to provide the necessary and sufficient conditions for the ergodicity of stationary random processes. Mattingly (2003) reviewed the research progress of ergodicity of random Navier-Stokes equations, and Galanti (Galanti et al. 2004, Lennaert et al. 2006) solved the random Navier-Stokes equation by numerical simulation to prove that the turbulence which is temporally steady and spatially homogeneous is ergodic. However, Galanti (2004) also indicated that such partially turbulent flows acting as mixed layer, wake flow, jet flow, flow around the boundary layer may be non-ergodic turbulence.

Obviously, the advances of research on the ergodicity in the mathematics and physics have precedence far over the atmospheric science. We try firstly to introduce the ergodic theorem of stationary random processes to atmospheric turbulence in ASL in this paper. And that the ergodicity of different scale eddies of atmospheric turbulence is directly analyzed and verified quantitatively on the basis of the field observational data measured by the eddy covariance system.

2 Theories and methods

2.1 Ergodic theorems of stationary random processes

The stationary random processes are the processes which will not vary with time, i.e., for observed quantity A, its function of spatial x_i and temporal t_i satisfies the following condition:

$$A(x_1, x_2, \ldots, x_n; t_1, t_2, \ldots, t_n) = A(x_1, x_2, \ldots, x_n; t_1+\tau, t_2+\tau, \ldots, t_n+\tau), \tag{1}$$

where τ is a time period, defined as the relaxation time.

The mean μ_A of random variable A and autocorrelation function $R_A(\tau)$ are respectively defined as follows:

$$\mu_A = \lim_{T\to+\infty} \frac{1}{T}\int_0^T A(t)dt, \tag{2}$$

$$R_A(\tau) = \lim_{T\to+\infty} \frac{1}{T}\int_0^T A(t)A(t+\tau)dt. \tag{3}$$

The autocorrelation function $R_A(\tau)$ is a temporal second-order moment. In the case of τ=0, the autocorrelation function $R_A(\tau)$ is the variance of random variable. The necessary and sufficient conditions that stationary random processes satisfy the mean ergodicity are the mean ergodic function Ero(A) to zero (Papoulis et al. 1991), as shown below:

$$\mathrm{Ero}(A) = \lim_{T\to\infty} \frac{1}{T}\int_0^{2T} \left(1-\frac{\tau}{2T}\right)\left[R_A(\tau)-\mu_A^2\right]d\tau = 0. \tag{4}$$

The mean ergodic function Ero(A) is a time integral of variation between the autocorrelation function $R_A(\tau)$ of variable A and its mean square, μ_A^2. If the mean ergodic function Ero(A) converges to zero, then the stationary random processes will be ergodic. In other words, if the autocorrelation function $R_A(\tau)$ of variable A converges to its mean square, μ_A^2, the stationary random processes are mean ergodic. The Eq. (4) is namely the mean ergodic theorem to be called as well as ergodic theorem of the weakly stationary processes in mathematics. For discrete variables, Eq. (4) can be rewritten as the following:

$$\mathrm{Ero}(A) = \lim_{n\to\infty} \sum_{i=0}^{n} \left(1-\frac{\tau_i}{n}\right)\left[R_A(\tau_i)-\mu_A^2\right] = 0. \tag{5}$$

The Eq. (5) is the mean ergodic theorem of discrete variable. Hence, Eqs. (4) and (5) can be used as a criterion to judge the mean ergodicity.

The necessary and sufficient conditions that stationary random processes satisfy the autocorrelation ergodicity are the autocorrelation ergodic function Er(A) to zero:

$$\mathrm{Er}(A)=\lim_{T\to\infty}\frac{1}{T}\int_0^{2T}\left(1-\frac{\tau'}{2T}\right)\left[B(\tau')-\left|R_A(\tau)\right|^2\right]d\tau'=0; \tag{6a}$$

$$B(\tau')=E\left\{A(t+\tau+\tau')A(t+\tau')\left[A(t+\tau)A(t)\right]\right\}. \tag{6b}$$

where $B(\tau')$ is the temporal fourth-order moment of variable A. The autocorrelation ergodic function Er(A) is a time integral of variation between the temporal fourth-order moment $B(\tau')$ of variable A and its autocorrelation function square, $\left|R_A(\tau)\right|^2$. If the autocorrelation ergodic function Er(A) converges to zero, then the stationary random processes will be of autocorrelation ergodicity, and thus the autocorrelation ergodicity means that the fourth-order moment of variable of stationary random processes will converge to square of its autocorrelation function $R_A(\tau)$. The Eq. (6a) is namely the autocorrelation ergodic theorem to be called as well as the ergodic theorem of strongly stationary processes in the mathematics. The autocorrelation ergodic function of corresponding discrete variable can be determined as follows:

$$\mathrm{Er}(A)=\lim_{n\to\infty}\sum_{i=0}^{n}\left(1-\frac{\tau_i'}{n}\right)\left[B(\tau_i')-\left|R_A(\tau_j)\right|^2\right]=0, \tag{7a}$$

$$B(\tau'_i)=E\left\{\sum_{j=0}^{n}A(t+\tau_j+\tau'_i)A(t+\tau'_i)\left[A(t+\tau_j)A(t)\right]\right\}. \tag{7b}$$

The Eq. (7a) is the autocorrelation ergodic theorem of discrete variable. Hence, Eqs. (6a) and (7a) can also be used as a criterion to judge autocorrelation ergodicity.

The stationary random processes conform to the criterion, Eqs. (4) or (5), viz. satisfy the mean ergodic theorem, or are intituled as the mean ergodicity; if the stationary random processes conform to Eqs. (6a) or (7a), then satisfy the autocorrelation ergodic theorem, or are intituled as the autocorrelation ergodicity. If the stationary random processes are only of mean ergodicity, then they are the strict ergodic or narrow ergodic. If the stationary random processes are of both the mean ergodicity and autocorrelation ergodicity, then they are the wide ergodic stationary random processes. It is thus clear that the ergodic random processes are stationary, but the stationary processes may not be ergodic.

With respect to the random process theory, when the mean and high-order moment function is calculated, a large amount of repeated observations of random processes require to acquire the sample function $A_k(t)$. If the stationary random processes satisfy the ergodic conditions, then the time average of a sample on the whole time shaft can be used to substitute for the overall or ensemble average. Eqs. (4), (5), (6a) and (7a) can be used as the criterion to judge whether or not satisfying the mean and autocorrelation ergodicity. The ergodic random processes must be the stationary random processes to be defined as Eq. (1), and thus are stationary in relaxation time τ. If conditions such as Eqs (4) and (5) of the mean ergodicity are satisfied, then a time average in finite relaxation time τ can be used to substitute for the infinite time average to calculate mean Eq. (2) of the random variable; similarly, the finite time average can be used for substitution to calculate the covariance or variance of random variable, Eq. (3), if conditions such as Eqs. (6a) and (7a) of autocorrelation ergodicity are satisfied. In a similar manner, the basic principle of average of atmospheric turbulence measurement is the ensemble average of space, time and state, and it is necessary to carry through mass observation for a long period of time in the whole space. This is not only a costly observation, even is hardly feasible. If the turbulence signals satisfy the ergodic conditions, then the time average in relaxation time τ by multi-station observation, even single-station observation, can substitute for the ensemble average. In fact, the precondition to estimate the turbulent characteristic quantities and fluxes in ABL by the eddy correlation method is that the turbulence satisfies the ergodic conditions. Therefore, conditions such as Eqs. (4), (5), (6a) and (7a) will also be the criterion for testing the authenticity of observed results by the eddy correlation method.

2.2 Band-pass filtering

In the spatial scale, the atmospheric turbulence from the dissipation range, inertial sub-range to energy range, and further large eddy of turbulent flow is extremely broad (Stull 1988). Such spatial and temporal size of eddies include the isotropous 3-D eddy structure of high frequency turbulence and orderly coherent structure of low frequency turbulence (Li et al. 2002). The different scale eddies are also different in terms of their spatial structure and physical properties, and even their transport characteristics are not all the same. It is thus reasonable that eddies with different transport characteristics are separated, processed and studied by using different methods (Zuo et al. 2012). A major goal of our study is to understand what type of eddy in the scale can satisfy the ergodic conditions. Another goal is that the time averaging of signals measured by a single station determines accurately the turbulent characteristic quantities. In order to study the ergodicity of eddies in different scales, Fourier transform is used as band-pass filtering to distinguish the different scale eddies. That is to say, we set by the strong arm not needing part of frequencies as zero in the Fourier transform, and then acquire the signals after filtering by means of Fourier inverse transformation. The specific formulae are shown below:

$$F_A(n)=\frac{1}{N}\sum_{k=0}^{N-1}A(k)\cos\left(\frac{2\pi nk}{N}\right)-\frac{i}{N}\sum_{k=0}^{N-1}A(k)\sin\left(\frac{2\pi nk}{N}\right), \tag{8}$$

$$A(k)=\sum_{n=a}^{N-1}F_A(n)\cos\left(\frac{2\pi nk}{N}\right)+i^2\sum_{n=a}^{N-1}F_A(n)\sin\left(\frac{2\pi nk}{N}\right). \tag{9}$$

In Eqs. (8) and (9), $F_A(n)$ and $A(k)$ are respectively the Fourier transformation and Fourier inverse transformation including N data points from k=0 to k=N-1, and n is the cycle index of the observation time range. The high-pass filtering can cut off the low frequency signals of turbulence to obtain high frequency signals. The aliasing of half high frequency turbulence after the Fourier transformation is unavoidable. At the same time, the correction for high frequency response will compensate for the loss. In order to acquire purely high frequency signals in the filtering processes, we take the results of band-pass filtering from $n=j$ to $n=N-j$ as the high frequency signals. This is referred to as j time filtering in this paper. Finally, the ergodicity of different scale eddies is analyzed using Eqs. (4)-(7).

2.3 M-O similarity of turbulent variance

The characteristics of relations of Monin-Obukhov similarity (MOS) of variance for the different scale eddies are analyzed and compared to test the feasibility of MOS's relation for the ergodic and non-ergodic turbulence. The problems of eddy correlation method in the turbulence observation in ASL are further explored on the basis of the study on the ergodicity and MOS's relations of the variance of different scale eddies in order to provide an experimental basis for utilizing MOST and developing the turbulence theory of ABL with complex underlying surfaces.

The MOS's relations of turbulent variance can be regarded as an effective instrumentality to verify whether or not the turbulent flow field is steady and homogeneous (Foken et al. 2004). Under ideal conditions, the local MOS's relations of variance of wind velocity, temperature and other factors can be expressed as follows:

$$\sigma_i / u_* = \phi_i \left(z/L \right), \qquad \left(i = u,v,w \right), \tag{10}$$

$$\sigma_s / \left| s_* \right| = \phi_s \left(z/L \right), \qquad \left(s = \theta, q \right). \tag{11}$$

where σ is the turbulent variance; corner mark i is the wind velocity u, v or w; s stands for scalar, such as potential temperature θ and humidity q; u_* is the friction velocity and defined as $u_* = \left(\overline{u'w'}^2 + \overline{v'w'}^2 \right)^{1/4}$; s_* is the turbulent characteristic quantity of the related scalar and defined as $s_* = -\overline{w's'}/u_*$; and that M-O length L is defined as:

$$L = u_*^2 \theta / \left[\kappa g \left(\theta_* + 0.61 \theta q_* / \rho_d \right) \right]. \tag{12}$$

A large number of research results show that, in the case of unstable stratification, $\phi_i \left(z/L \right)$ and $\phi_s \left(z/L \right)$ can be expressed in the following forms (Panofsky et al. 1977; Padro 1993; Katul et al. 1999):

$$\phi_i \left(z/L \right) = c_1 \left(1 - c_2\, z/L \right)^{1/3}; \tag{13}$$

$$\phi_s \left(z/L \right) = \alpha_s \left(1 - \beta_s\, z/L \right)^{-1/3}. \tag{14}$$

where c_1, c_2, α and β are coefficients to be determined by the field observation. In the case of stable stratification, $\phi_s \left(z/L \right)$ is approximate to a constant and $\phi_i \left(z/L \right)$ is still the 1/3 function of z/L. The turbulent characteristics of eddies in the different temporal and spatial scales in are analyzed and compared with the mean and autocorrelation ergodic theorems, to test the feasibility of MOS's relations under conditions of the ergodic and non-ergodic turbulence.

3 **The sources and processing of data**

In this study, the first turbulence data that were measured by the eddy correlation method under the homogeneous surface in the Nagqu Station of Plateau Climate and Environment (NSPCE), Chinese Academy of Sciences (CAS) are used. The data set in NSPCE/CAS includes that measured by 3-D sonic anemometer and thermometer (CSAT3) with 10 Hz as well as infrared gas analyzer (Li7500) in ASL from 23 July 2011 to 13 September 2011. In addition, the second turbulence data set of CASES-99 (Poulos et al. 2002; Chang et al. 2002) is used to verify the ergodicity of turbulence observed by multi-station. The data set is ASL's data in seven observation points. The sub-towers, sn1, sn2 and sn3 are located 100 m away from the central tower, the sn4 is 280 m away, and tower sn5 and sn6 are located 300 m away. For CASES-99, the data of sonic anemometer and thermometer (CSAT3) with 20 Hz and the infrared gas analyzer (Li7500) in ASL at 10m on the central tower with 55 m height. And other turbulence data include 3-D sonic anemometer (ATI) and Li7500 at 10 m height on six sub-towers surrounding the central tower. The two sets of data collected for completely different purposes are compared to test the universality of the research results.

The geographic coordinate of NSPCE/CAS is 31.37°N, 91.90°E, and its altitude is 4509 m a.s.l. The observation station is built on flat and wide area except for a hill of about 200 m at 2 km distance in the north, and floors area 8000m^2. The ground surface is mainly composed of sandy soil mixed with sparse fine stones, and an alpine meadow with vegetation of 10-20 cm. The displacement height of underlying surface of NSPCE's meadow is determined to 0.03 m by calculation. CASES-99 is located in prairie of Kansas US. The geographic coordinate of CASES-99's central tower is 37.65°N, 96.74°W. The observation field is flat and growth grasses about 20-50 cm during the observation period, while the displacement height of the CASES-99's underlying surface is 0.06 m (Martano 2002).

This study is conditioned to the stationary random processes. So the inaccurate data in the measurements caused by circuit pulse are deleted before data analysis. Subsequently, the data are divides into continuous sections of 5-hour, and the 1-hour high frequency signals are obtained by applying filtering of Eqs. (8) and (9) on each 5-hour data. In order to conform to the stationary random condition and to select the steady turbulent data, the 12 fragments of 5-min variances of the velocity and temperature in 1-hour are calculated and compared with each other. When their deviations are less than $\pm 15\%$ including an instrumental error about $\pm 5\%$, the data are selected to study the ergodicity of the observed eddies. Moreover, the ultrasonic temperature pulsation is corrected to absolute temperature pulsation (Schotanus et al. 1983; Kaimal et al. 1991). Then the coordinate is rotated using the plane fitting method to improve the installation level (Wilczak 2001). The moisture is components of the air; their pulsation is also a constituent part of the air density pulsation. Therefore, there is no relevant correction on the humidity pulsation caused by air density fluctuation. According to our preliminary analysis, such correlation may also cause the unreason deviation from the prediction shown in Eq. (14). The Webb correction (Webb et al. 1980) is the component of surface energy balance in physical nature, but not the component of turbulent eddy. We thus do not perform Webb correction on our research objectives of the ergodicity.

4. Result analysis

Applying the two sets of data from NSPCE and CASES-99, we have tested the ergodicity of eddies in different temporal scales under the condition of steady turbulence. Here, we carefully select the representative data measured at the level of 3.08m in NSPCE during three time frames, namely 3:00-4:00, 7:00-8:00 and 13:00-14:00 China Standard Time (CST) on 25 August in clear weather to test and demonstrate the ergodicity of eddies in different temporal scales. These three time frames can represent three situations, i.e. the nocturnal stable boundary layer, early neutral boundary layer and midday convective boundary layer.

The trend correction (McMillen 1988; Moore 1986) is used to exclude the influence of low-frequency trend effect. In order to acquire the effective information of eddies in the different temporal scales, Eqs. (8) and (9) are used to perform band-pass filtering of the turbulence data at 3.08 m in NSPCE, which is equivalent to the correction of the high-pass filtering. In addition, the results of the time band-pass filtering from $n=j$ to $n=N-j$ corresponding to Eqs. (8) and (9) acquire the information of eddies in the corresponding temporal scale. The band-pass filtering information of different time frames is thereby utilized to study the turbulence characteristics and ergodicity of eddies in the different temporal scales of six time frames, including 2 min, 3 min, 5 min, 10 min, 30 min and 60 min.

4.1 M-O eddy local stability and M-O stratification stability

The M-O stratification stability z/L describe a whole characteristic between the mechanical and buoyancy effects in ASL's turbulence, but this study will decompose the turbulence into the different scale eddies. Considering that the features of different scale eddies of the atmospheric turbulence varied with the atmospheric stability parameter z/L, a M-O eddy local stability that is limited in the certain scale range of eddies is defined as z/L_c, so as to analyze the relation between the stratification stability and ergodicity of the wind velocity, temperature and other factors of the different scale eddies. It is noted that the M-O eddy local stability, z/L_c, is different from the M-O stratification stability, z/L.

As an example, the eddy local stability z/L_c in the different temporal scales of the three time frames from nighttime to daytime is as shown in Table 1. The results show that the eddy local stability z/L_c below 2 min in temporal scale during the nighttime time frame of 3:00-4:00 is 0.59, thus it is stable stratification. For the eddies of which the temporal scale gradually increases from 3 min, 5 min and 10 min to 60 min, but the eddy local stability, z/L_c, gradually decreases to 0.31 and 0.28. In addition, beginning from eddies of 10 min in the temporal scale, even the eddy local stability decreases from -0.01 to -0.07. It seems that the eddy local stability gradually varies from stable to unstable as the eddy temporal scale increases. During the morning time frame of 7:00-8:00, the eddy local stability z/L_c from 2 min to 60 min in the temporal scale eventually decreases from 0.52, 0.38, 0.16 and 0.15 to a minimum of -1.29. It means that eddies in the temporal scales of 30 min and 60 min have high local instability. However, during the midday time frame of 14:00-15:00, eddies in the temporal scales from 2 min to 60 min are unstable. As the eddy scale increases, the local instability of eddies in the scales from 2 min to 3 min also increases, and the instability value reaches the maximum of 0.44 when the eddy scale is 5 min; the eddy scale continuously increases, but the eddy local instability decreases.

The M-O eddy local stability is not entirely the same as the M-O stratification stability of ABL in the physical significance. The M-O stratification stability of ABL indicates that the overall effect of atmospheric stratification of the ABL on the stability including all eddies in integral boundary layer. The M-O stratification stability z/L of the no filtering data to include the whole turbulent signals is stable 0.02 for 3:00-4:00 (CST), but unstable -0.004 and -0.54 for 7:00-8:00 and 13:00-14:00 (CST), respectively. But the eddy local stability is only a local effect of atmospheric stratification on the stability of eddies in a certain scale. As the eddy scale increases, the eddy local stability z/L_c will vary accordingly. The aforesaid results indicate that the local stability of small-scale eddies is stable in the nocturnal stable boundary layer, but the nocturnal stable boundary layer is possibly unstable for the large-scale eddies, so to result in a sink effect on the small-scale eddies, but a positive buoyancy effect on the large-scale eddies. However, in the diurnal unstable boundary layer, the eddy local stability of 3 min scale reaches the maximum, than which the instability of smaller scale eddy decreases. But the instability gradually also decreases as the eddy scale increases. Therefore, eddies of 3 min scale hold maximum buoyancy, but the eddy buoyancy decreases as the eddy scale increases. However, the small-scale eddies are more stable than eddies in the large scale in the nocturnal stable boundary layer; while the large-scale eddies are more stable than the eddies in the small scales in the diurnal unstable and convective boundary layers. The above facts signify that it is common that there exist mainly the small-scale eddies in the nocturnal boundary layer with stable stratification. And it is also common that there exist mainly the large-scale eddies in the diurnal convective boundary layer with unstable stratification. Therefore, it can well understand that the small-scale eddies are dominant in the nocturnal stable boundary layer, while the large-scale eddies are dominant in the diurnal convective boundary layer.

4.2 Verification of mean ergodic theorem of eddies in different temporal scales

In order to verify the mean ergodic theorem, we calculated the mean and autocorrelation functions using Eq. (2) and Eq. (3), then calculated the variation of mean ergodic function Ero(A) using Eq. (5) of eddies in the different temporal scales with relaxation time τ to be cut off with $\tau_{i=n}$. The mean ergodic functions, Ero(A), of vertical velocity, temperature and specific humidity of the different scale eddies are calculated by using the data at level of 3.08m for the three time frames of 3:00-4:00, 7:00-8:00 and 13:00-14:00 (CST) in NSPCE, as shown in Figs. 1-3 respectively. Since the ergodic function varies within a large range, the ergodic functions are normalized according to the characteristic quantity of relevant variables ($A_* = \boldsymbol{u}_*, |\theta_*|, |q_*|$). That is to say, the functions in all following figures are dimensionless ergodic functions, Ero(A)/A_*.

The comprehensive analyses of the mean ergodicity characteristics of atmospheric turbulence and the relevant causes:

4.2.1 Verifying mean ergodic theorem of different scale eddies

According to the mean ergodic theorem, Eq. (4), the mean ergodic function Ero(A)/A_* will converge to 0 if the time approaches infinite. This is a theoretical result of the stationary random processes. However, the practical mean ergodic function is calculated under the condition of that relaxation time $\tau_{i=n}$ is cut off. If the mean ergodic function Ero(A)/A_* verges approximately to 0 in relaxation time $\tau_{i=n}$, it will be considered that random variable A approximately satisfies the mean ergodic theorem. The mean ergodic function deviates more from zero, the mean ergodicity will be of poor quality. So as we can judge approximately whether or not the mean ergodic theorem of eddies in different scales holds. Figs. 1-3 clearly show that, regardless of the vertical velocity, temperature or humidity, the Ero(A)/A_* of eddies below 10 min in the temporal scale will swing around zero within a small range; thus we can conclude that the mean ergodic function Ero(A)/A_* of eddies below 10 min in the temporal scale converges to zero to satisfy effectively the conditions of mean ergodic theorem. For eddies of 30 min and 60 min, which are larger scale, then the mean ergodic function Ero(A)/A_* will derivate further from zero. In particular, the mean ergodic function Ero(A)/A_* of eddies of 30 min and 60 min of the temperature or humidity does not converge, and even diverges. The above results show that the mean ergodic function of eddies of 30 min and 60 min cannot converge to zero or cannot satisfy the conditions of mean ergodic theorem.

4.2.2 Comparison of the convergence of mean ergodic functions of vertical velocity, temperature and humidity

As seen from Figs. 1-3, the dimensionless mean ergodic function of the vertical velocity is compared with the respective function of the temperature and humidity, it is 3-4 magnitudes less than those in the nocturnal stable boundary layer; 1-2 magnitudes less than those in the early neutral boundary layer; and around 2 magnitudes less than those in the midday convective boundary layer. For example, during nighttime time frame of 3:00-4:00 (CST), the dimensionless mean ergodic function of vertical velocity is 10^{-5} in magnitude, while the respective magnitudes of function value of the temperature and humidity are 10^{-1} and 10^{-2}; during morning time frame of 7:00-8:00 (CAT), the magnitude of mean ergodic function of the vertical velocity is 10^{-4}, while the respective magnitudes of function value of the temperature and humidity are 10^{-2} and 10^{-3}; during midday time frame of 13:00-14:00 (CST), the magnitude of mean ergodic function of the vertical velocity is 10^{-4}, while the magnitudes of function value of the temperature and humidity are both 10^{-2}. These results show that the dimensionless mean ergodic function of vertical velocity converges to zero much more easily than respective function value of the temperature and humidity, and that the vertical velocity satisfies the conditions of mean ergodic theorem more easily than the temperature and humidity.

4.2.3 Temporal scale and spatial scale of turbulent eddy

For wind velocity of 1-2 ms^{-1}, the eddy spatial scale in the temporal scale 2 min is around 120-240 m, and the eddy spatial scale in the temporal scale10 min is around 600-1200 m. The eddy spatial scale in the temporal scale 2 min is equivalent to the ASL's height, and the eddy spatial scale in the temporal scale 10 min is equivalent to ABL's height. The eddy spatial scale within the temporal scale 30-60 min is around 1800-3600 m, and this spatial scale clearly exceeds ABL's height to belong to the scope of atmospheric local circulation. According to the stationary random processes definition (1) and the mean ergodic theorem, the stationary random processes must be smooth in the relaxation time τ. The eddoes below temporal scale 10 min, i.e. below ABL's height are the stationary random processes, and can effectively satisfy the conditions of mean ergodic theorem. However, eddies in the temporal scale 30 min and 60 min exceed the ABL's height do not satisfy the conditions of mean ergodic theorem, thus these eddies belong to the non-stationary random processes.

4.2.4 Ergodicity of the turbulence of all eddies of possible scale in ABL

To facilitate comparison, Fig. 4 shows the variation of mean ergodic function Ero(A) of the vertical velocity (a), temperature (b) and specific humidity (c) before filtering with relaxation time τ during midday 14:00-15:00 (CST) in the convective boundary layer. It is obvious that Fig. 4 is unfiltered mean ergodic function of eddies in all possible scale in ABL. The Fig. 4 compares with Figs. 1c, 2c and 3c, which are the mean ergodic function Ero(A)/A_* of vertical velocity, temperature and humidity after filtering during the midday time frame of 14:00-15:00 (CST). The result shows that the mean ergodic functions before filtering are greater than that after filtering. As shown in Figs. 1c, 2c and 3c, the magnitude for the vertical velocity is 10^{-4} and the magnitudes for the temperature and specific humidity are both 10^{-2}. According to Fig. 4, the magnitude of vertical velocity Ero(A)/A_* is 10^{-3} and the magnitudes of temperature and specific humidity are both 10^{0}, therefore 1-2 magnitudes are almost decreased after filtering. Moreover, all trend upward for vertical velocity and temperature and downward for specific humidity, deviating from zero. It is thus clear that, even if the midday 14:00-15:00 (CST). when is equivalent to local time 12:00-13:00, the mean ergodic function of eddies in all possible scale in the convective boundary layer cannot converge to zero before filtering, i.e. cannot satisfy the conditions of mean ergodic theorem. That may be that eddies in all possible scale before filtering including the local circulation in convective boundary layer. So we argue that, under general situations, the eddies only below 10 min in the temporal scale or within 600-1200 m in the spatial scale in ABL are the ergodic stationary random processes, but the turbulence including the eddies with all possible scale in ABL may belong to the non-ergodic stationary random processes.

4.2.5 Relation between the ergodicity and local stability of different scale eddies

The corresponding eddy local stability z/L_c of eddies at different times in different scales (see Table 1) show that the eddy local stability z/L_c of the different scale eddies are different, due to the fact that the temperature stratification in ABL has different effects on the stability of in the different scale eddies. Even entirely contrary results can occur. At the same time the stratification which can cause the large scale eddy to ascend with buoyancy may cause the small scale eddy to descend. However, the analysis results in Figs. 1-3 show that the ergodicity is mainly related to the eddy scale, and its relation with the atmospheric temperature stratification seems unimportance.

4.3 Verification of autocorrelation ergodic theorem for different scale eddies

In this section, Eqs. (7a) and (7b) are used to verify the autocorrelation ergodic theorem. It is identified in Sect. 4.2 that the turbulent eddies below 10 min in the temporal scale satisfy the mean ergodic conditions in the various time frames, i.e. the turbulent eddies below 10 min in the temporal scale are at least in strictly stationary random processes or narrow stationary random processes in the nocturnal stable boundary layer, early neutral boundary layer and midday convective boundary layer. Then we analyze further the different scale eddies which satisfy the mean ergodic conditions whether or not also satisfy the autocorrelation ergodic conditions, so as to verify whether atmospheric turbulence is in the narrow or wide stationary random processes. The autocorrelation ergodic function of turbulence variable A under the condition of truncated relaxation time $\tau_{i=n}$ are calculated according to Eq. (7a) to determine the variation of autocorrelation ergodic function Er(A) with relaxation time τ. As with the mean ergodic function Ero(A), if the autocorrelation ergodic function Er(A) of the eddies of 2 min, 3 min, 5 min, 10 min, 30 min and 60 min in the temporal scale within the relaxation time $\tau_{i=n}$ is approximate to 0, then A shall be deemed to be approximately ergodic; the more the autocorrelation ergodic function deviates from 0, the worse the autocorrelation ergodicity becomes. Therefore, this method can be used to judge approximatively whether the different scale eddies satisfy the conditions of autocorrelation ergodic theorem.

For example, Fig. 5 shows the variation of normalized autocorrelation ergodic function Ero(w)/u_* of the turbulent eddies of 2 min, 3 min, 5 min, 10 min, 30 min and 60 min in the temporal scale with relaxation time τ for the vertical velocity during the time frames of 3:00-4:00, 7:00-8:00 and 13:00-14:00 (CST). Some basic conclusions are drawn from Fig. 5:

1. After comparing Figs. 5a-c with Figs. 1a-c, i.e. comparing the dimensionless mean ergodic function Ero(w)/u_* of vertical velocity with the dimensionless autocorrelation ergodic function Er(w)/u_*, two basic characteristics are very clear. First, the magnitudes of the dimensionless autocorrelation ergodic function Er($\boldsymbol{w}$)/u_*, regardless of whether in the nocturnal stable boundary layer, early neutral boundary layer or midday convective boundary layer, are all greatly reduced. In Figs. 1a-c, the magnitudes of Ero(w)/u_* are respectively 10^{-5}, 10^{-4} and 10^{-4}, and the magnitudes of Er(w)/u_* are respectively 10^{-7}, 10^{-5} and 10^{-5}, as shown in Figs. 5a-c. The magnitudes of $Er(w)/u_*$ reduce by 1-2 magnitudes compared with those of Ero(w) /u_*. Second, all autocorrelation ergodic functions Er(w)/u_* of the eddies of 30 min and 60 min in temporal scale, regardless of whether they are in the stable boundary layer, natural boundary layer or convective boundary layer, are all reduced and approximate to Ero (w)/u_* of the eddies below 10 min in temporal scale.
2. The above two basic characteristics imply that the autocorrelation ergodic function Er(w)/u_* of the stable boundary layer, neutral boundary layer or convective boundary layer converges to 0 faster than the mean ergodic function Ero ($\boldsymbol{w}$)/u_*; the autocorrelation ergodic function of eddies of 30 min and 60 min in temporal scale also converges to 0 and satisfies the conditions of autocorrelation ergodic theorem, except for the fact that the autocorrelation ergodic function Er($\boldsymbol{w}$)/u_* of the eddies below 10 min in temporal scale can converge to 0 and satisfy the conditions of autocorrelation ergodic theorem.
3. According to the autocorrelation ergodic function Eq. (7a), the eddies of 30 min, 60 min and below 10 min in the temporal scale, regardless of whether they are in the stable boundary layer, neutral boundary layer or convective boundary layer, all eddies can satisfy the conditions of autocorrelation ergodic theorem. Therefore, in general the ABL's turbulence is the stationary random processes of autocorrelation ergodic.

4. The above results show that the eddies below 10 min in temporal scale in the nocturnal stable boundary layer, early neutral boundary layer and midday convective boundary layer can not only satisfy the conditions of mean ergodic theorem, but also they can also satisfy the conditions of autocorrelation ergodic theorem. Therefore, eddies below 10 min in the temporal scale are wide ergodic stationary random processes. Although the eddies of 30 min and 60 min in temporal scale in the stable boundary layer, neutral boundary layer and convective boundary layer can satisfy the conditions of autocorrelation ergodic theorem, they cannot satisfy the conditions of mean ergodic theorem. Therefore, eddies of 30 min and 60 min in the temporal scale are neither narrow ergodic stationary random processes, nor wide ergodic stationary random processes.

4.4 Ergodic theorem verification of different scale eddies for the multiple stations

The basic principle of turbulence average is the ensemble average of space, time and state. Sections 4.2 and 4.3 verify the mean ergodic theorem and autocorrelation ergodic theorem of atmospheric turbulence during the stationary random processes using field observational data, so that the finite time average of a single station can be used to substitute for the ensemble average. This section examines the ergodicity of different scale eddies according to the observational data from the CASES-99 tower and six sub-sites (seven stations). When the data are selected, it is considered that if the eddies are not evenly distributed at the seven stations, then the observation results at the seven stations may have originated from many eddies in the large scale. For this reason, we first compared the high frequency variance spectrum above 0.1 Hz. Based on the observational error, if the difference of all high frequency variances does not exceed the average by ±10%, then it is assumed that the turbulence is evenly distributed at the seven observation stations. Finally, 17 datasets are chosen from among the turbulence observation data from 5 to 30 October, and these data sets refer to the results of strong turbulence at noon on a sunny day. As an example, the same method as described in Sections 4.2 and 4.3 is used to respectively calculate the variation of the mean ergodic function and the autocorrelation ergodic function of vertical velocity in 10:00-11:00 on 7 October with relaxation time τ. Next, the observation data chosen from the seven stations are built into a data set, and the time series of data set are filtered at 2 min, 3 min, 5 min, 10 min, 30 min and 60 min. The variations of mean ergodic function $\mathrm{Ero}(w)/u_*$ and autocorrelation ergodic function

$Er(w)/u_*$ of the vertical velocity with relaxation time τ are analyzed to test the ergodicity of different scale eddies for the observation of multi-station. Fig. 6a shows the variation of mean ergodic function $Ero(w)/u_*$ of the vertical velocity with the relaxation time τ, and Fig. 6b shows the variation of autocorrelation ergodic function $Er(w)/u_*$ with the relaxation time τ.

The results show ergodic characteristics of different scale eddies measured at the multi-stations as following:

Fig. 6a shows that the mean ergodic function of eddies below 30 min in temporal scale converges to 0 very well, except for the fact that the mean ergodic function of eddies of 60 min in temporal scale clearly deviates upward from 0. Fig. 6b shows that all autocorrelation ergodic functions of different scale eddy, including eddies of 60 min in temporal scale, gradually converge to 0. Therefore, eddies below 30 min in temporal scale measured at the multi-stations satisfy the conditions of both the mean and autocorrelation ergodic theorems, while eddies of 60 min in temporal scale only satisfies the conditions of autocorrelation ergodic theorem, but cannot satisfy the conditions of mean ergodic theorem. These facts demonstrate that eddies below 30 min in temporal scale are wide ergodic stationary random processes in the data series composed by the seven stations. This signifies that the comparing of data series composed of multiple stations with data from a single station, the eddy temporal scale for wide ergodic stationary random processes is extended from below 10 min to 30 min. As analyzed above, if the eddies below 10 min in temporal scale are deemed to be the turbulent eddies in the ABL with height about 1000 m and the eddies of 30 min in the temporal scale, which is equivalent to that the space scale is greater than 2000 m, are deemed including the eddy components of local circulation in ABL, then multiple station observations can completely capture the local circulated eddies, which space scale is greater than 2000 m.

4.5 Average time problem of turbulent quantity averaging

The atmospheric observations are impossible to repeat experiment exactly, must use the ergodic hypothesis and replace ensemble averages with time averages. It arises a problem how does determine the averaging time.

The analyses on the ergodicity of different scale eddies in above two sections demonstrate that the eddies below 10 min in temporal scale as τ=30 min in the stable boundary layer, neutral boundary layer and convective boundary layer can not only satisfy the conditions of mean ergodic theorem, but also can also satisfy the conditions of autocorrelation ergodic theorem. That is to say, they are namely wide ergodic stationary random processes. Therefore, the finite time average of 30 min within relaxation time τ can be used for substituting for the ensemble average to calculate mean random variable Eq. (2). However, the eddies of 30 min and 60 min in the temporal scale in the stable boundary layer and neutral boundary layer are only autocorrelation ergodic random processes, neither narrow nor wide sense random processes. Therefore, when the finite time average of 30 min can be used for substituting for the ensemble average to calculate mean random variable Eq. (2), it may capture the eddies below 10 min in temporal scale in stationary random processes, but not completely capture the eddies above 30 min in the temporal scale. The above results signify that the turbulence average is restricted not only by the mean ergodic theorem, but also is closely related to the scale of turbulent eddy. In the observation performed using the eddy correlation method, the substitution of ensemble average with finite time average of 30 min inevitably results in a high level of error, due to lack of low frequency component information of the large-scale eddies. However, although eddies of 30 min and 60 min in the temporal scale in convective boundary layer are not wide ergodic stationary random processes, they are autocorrelation ergodic random processes. This may imply that the mean random variable which is calculated with the finite time average in the convective boundary layer to substitute for the ensemble average is often superior to the results of the stable boundary layer and neutral boundary layer. Withal, the results in the previous sections also show that the mean ergodic function of vertical velocity may more easily converge to 0 than functions corresponding to the temperature and humidity, and the vertical velocity may more easily satisfy the conditions of mean ergodic theorem than the temperature and humidity. Therefore, in the observation performed using the eddy correlation method, the result of vertical velocity is often superior to those of the temperature and humidity. In this section, the results also point out that multi-station observation is cape of completely capturing eddies of local circumfluence in the ABL. Therefore, the ergodic assumption is more likely to be satisfied, and its results are much closer to the true values when calculating the turbulence mean, variance or fluxes with the multi-station observation data.

In order to determine the averaging time, Oncley (1996) defined an Ogive function of cumulative integral

$$Og_{x,y}\left(f_0\right)=\int_{\infty}^{f_0} Co_{x,y}\left(f\right)df \tag{15}$$

where x and y are any two variables whose covariance is $\overline{xy}$, $Co_{xy}(f)$ is the cospectrum of xy. If the Ogive function converges to a constant value at a frequency $f=f_0$, which could be converted to the averaging time of the measurement. The Ogive of $\overline{u'w'}$ is often examined to determine the least averaging time. As a comparison, here the variation of Ogive functions of $\overline{w'^2}$ and $\overline{u'w'}$ with frequency at the height 3.08 m in NSPCE for the three time frames is shown in Fig.7. Fig.7 shows the variation of Ogive convergence frequency for $\overline{w'^2}$ in the nighttime stable conditions, morningtide neutral boundary layer and midday convection boundary layer converges respectively converges about 0.01 Hz, 0.0001 Hz and 0.001 Hz. It is equivalent to the averaging times about 2 min, 160 min and 16 min. However for $\overline{u'w'}$, it converges about 0.001 Hz only in the midday convection boundary layer to be equivalent to the averaging time about 16 min. However it seems no convergence in the nighttime stable and morningtide neutral boundary layer. It is implied determining averaging time seems to have a bit difficult with the Ogive function in the stable and neutral boundary layer. The Fig.7 shows also that when the frequency is lower than 0.0001Hz, Ogive functions $\overline{u'w'}$ ascend in the stable boundary layer, and descend in the morningtide neutral boundary layer and midday convection boundary layer. It may be low frequency effect caused the cross local circulation in the nighttime and midday in ABL. Especially we must note that the Ogive is a function of the cumulative integral. So as Ogive changes direction from ascending to descending, it implies that in the negative momentum flux superimposing positive flux. The foremost reason that there exists the positive up momentum flux at 3m level in the ASL is a local circulation effect highly possible. The local circulation in ABL may be a cause that Ogive fails to judge the averaging time. In this work, the choice of averaging time with the ergodic theory seems superior to with the Ogive function.

4.6 M-O similarity of turbulent eddies in different scales and its relation with ergodicity

Turbulent variance is a most basic characteristic quantity of the turbulence. Turbulence velocity variance, which represents turbulence intensity, and the variance of scalars, such as temperature and humidity, effectively describes the structural characteristics of turbulence. In order to test MOS relations of the different scale eddies with ergodicity, the vertical velocity and temperature data of NSPCE from 23 July to 13 September are used to determine the MOS relationship of variances of vertical velocity and temperature for the different scale eddies, and analyze its relation with the ergodicity.

The MOS relation of vertical velocity variance as following:

$$\phi_i\left(z/L\right)=c_1\left(1-c_2\,z/L\right)^{1/3},\quad z/L<0 \qquad ,\ (16)$$

$$\phi_i\left(z/L\right)=c_1\left(1+c_2\,z/L\right)^{1/3},\quad z/L>0\,. \qquad (17)$$

Fig. 8 and 9 respectively shows the MOS relation curves of different scale eddies for the vertical velocity and temperature variances in NSPCE. The figures (a), (b) and (c) of Fig. 8 and 9 are respectively the similarity curve of eddies of 10 min, 30 min and 60 min in the temporal scale. Table 2 shows the relevant parameters of fitting curve of MOS relation for the vertical velocity variance. The correlation coefficient and residual of fitting curve are respectively expressed with *R* and *S*.

Fig. 8 and Table 2 show that the parameters of fitting curve are greatly different, even if the fitting curve modality of MOS relation of the vertical velocity variance for the eddies in different temporal scales is the same. The correlation coefficients of MOS's fitting curve of the vertical velocity variance under the unstable stratification are large, but the correlation coefficients under the stable stratification are small. Under unstable stratification, the correlation coefficient of eddies of 10 min in the temporal scale reaches 0.97, while the residual is only 0.16; under the stable stratification, the correlation coefficient reduces to 0.76, and the residual increases to 0.25. With the increase of eddy temporal scale from 10 min (Fig. 8a) to 30 min (Fig. 8b) and 60 min (Fig. 8c), the correlation coefficients of MOS relation of the vertical velocity variance gradually reduce, and the residual increases. The correlation coefficient in 60 min is the minimum; it is only 0.83 under the unstable stratification, and only 0.30 under the stable stratification.

The temperature variance is shown in Fig. 9. The MOS's function to fit from eddies of 10 min in the temporal scale under the unstable stratification is following:

$$\phi_\theta\left(z/L_c\right)=4.9\left(1-79.7\,z/L_c\right)^{-1/3}\,. \qquad (18)$$

As shown in Fig. 9a, the correlation coefficient of fitting curve is -0.91 and residual is 0.38. With the increase of eddy temporal scale, discreteness of MOS relation of the temperature variance is enlarged quickly, and an appropriate curve cannot be fitted.

The above results show that the discreteness of fitting curve of MOS relation for the turbulence variance is enlarged with the increase of eddy temporal scale for either the vertical velocity or temperature. The points of data during the stationary processes basically gather near the fitting curve of variance similarity relation, while all data points during the nonstationary processes deviate significantly from the fitting curve. However, the similarity of vertical velocity variance is superior to that of the temperature variance. These results are consistent to the conclusions of testing ergodicity for the different scale eddies described in Sections 4.2-4.4. The ergodicity of small-scale eddy is superior to that of the larger-scale eddy, and eddies of 10 min in the temporal scale has the best variance similarity function. These results also signify that when the eddy at the stationary random processes satisfies the ergodic conditions, then both the vertical velocity variance and temperature variance of eddies in the different temporal scales comply with MOST very well; but, as for eddies with poor ergodicity during nonstationary random processes, the variances deviate from MOS relations.

5 Conclusion

From the above results, we can draw the below preliminary conclusions:

1. The turbulence in ABL is an eddy structure. When the temporal scale of turbulent eddies in ABL is about 2 min, the corresponding spatial scale is about 120-240 m to be equivalent to ASL's height; when the temporal scale of turbulent eddies in ABL is about 10 min, the corresponding spatial scale is about 600-1200 m to be equivalent to the ABL's height. As the larger temporal and spatial scale for eddies, such as eddies of 30-60 min in the temporal scale, and the corresponding spatial scale is about 1800-3600 m. Spatial scale exceeds the ABL's height.
2. For the atmospheric turbulent eddies below the ABL's scale, i.e. the eddies below 1000 m in the spatial scale and 10 min in the temporal scale, the mean ergodic function Ero(A) and autocorrelation ergodic function Er(A) converge to 0, and they can satisfy the conditions of mean and autocorrelation ergodic theorem. However, for the atmospheric turbulent eddies above 2000-3000m in the spatial scale and above 30-60 min in the temporal scale, the mean ergodic function doesn't converge

to 0, thus cannot satisfy the conditions of mean ergodic theorem. Therefore, the turbulent eddies below the ABL's scale belong to the wide ergodic stationary random processes, but the turbulent eddies which are larger than ABL's scale belong to the non-ergodic random processes, or even the nonstationary random processes.

3. Due to above facts, when the stationary random process information of eddies below 10 min in the temporal scale and below 1000 m of ABL's height in the spatial scale can be captured, the atmospheric turbulence may satisfy the conditions of mean ergodic theorem. Therefore, an average of finite time can be used for substituting for the ensemble average of infinite time to calculate mean random variable as measuring atmospheric turbulence with the eddy correlation method. But for the turbulence of eddies above 30 min in temporal scale and above 2000 m in spatial scale magnitude, it cannot satisfy the conditions of mean ergodic theorem, so that the eddy correlation method cannot completely capture the information of nonstationary random processes. This will inevitably cause a high level of error due to the lack of low frequency component information of the large-scale eddies when the average of finite time is used to substitute for the ensemble average in observation.
4. Although the atmospheric temperature stratification has different effects on the stability of eddies in the different scales, the ergodicity is mainly related to the local stability of eddies, and its relation with the stratification stability of ABL is not significant.
5. The data series composed from seven stations compare with the observational data from a single station. The results show that the temporal and spatial scale of eddies to belong to the wide ergodic stationary random processes are extended from 10 min to below 30 min and from 1000 m to below 2000 m respectively. This signifies that the ergodic assumption is more likely to be satisfied well with multi-station observation data, and observational results produced by the eddy correlation method are much closer to the true values when calculating the turbulence average, variance or fluxes.
6. If the ergodic conditions of stationary random processes are more effectively satisfied, then the turbulence variance of eddies in the different temporal scales can comply with MOST very well; however, the turbulence variance of the

non-ergodic random processes deviates from MOS relations.

6 Discussion

1. Galanti (2004) proved that the turbulence which was temporally steady and spatially homogeneous is ergodic, but ‘partially turbulent flows’ such as the mixed layer, wake flow, jet flow, flow around and boundary layer flow may be non-ergodic turbulence. However, it has been proven through atmospheric observational data that the turbulence ergodicity is related to the scale of turbulent eddies. Since the large-scale eddies in ABL may be strongly influenced by the boundary disturbance, thus belong to ‘partial turbulence’; however, since the small-scale eddies in atmospheric turbulence may be not influenced by boundary disturbance, may be temporally steady and spatially homogeneous turbulence. So that the mean ergodic theorem and autocorrelation ergodic theorem for the turbulent eddies in small scale in ABL is applicative, but the large-scale eddies are non-ergodic.
2. The eddy correlation method for turbulence measurement is based on the ergodic assumption. A lack of ergodicity related to the presence of large-scale eddy transport can lead to a consider error of a tower flux measurement. This has already been pointed out by Mauder et al. (2007) or Foken et al. (2011). Therefore, we realize from the above conclusions that the large scale eddies may include non-ergodic random process components which exceed ABL’s height. The eddy correlation method for the measurement and calculation of turbulent variance and covariance may not capture the information of large-scale eddy exceeded ABL’s scale, thus resulting in large error. MOST is developed on the conditions of steady time and homogeneous surface. MOST’s conditions, steady time and homogeneous surface, are in line with the ergodic conditions, therefore the turbulence variance, even the turbulent fluxes of eddies in the different temporal scales may comply with MOST very well, if the ergodic conditions of stationary random processes are more effectively satisfied.
3. According to Kaimal and Wyngaard (1990), the atmospheric turbulence theory and observation method were feasible and led to success under ideal conditions including a short period, steady state and homogeneous underlying surface, and through observation in the 1950s-1970s, but these conditions are rare in reality. In the land surface processes and ecosystem, the turbulent flux observation in ASL is

a scientific issue in which commonly interest researchers in the fields of atmospheric science, ecology, geography science, etc. These observations must be implemented under conditions such as with complex terrain, heterogeneous surface, long period and unsteady state. It is necessary that more neoteric observational tools and theories will be applied with new perspectives in future research.

4. It is successful that the banausic ergodic theorem of stationary random processes is introduced from the mathematics into atmospheric sciences. It undoubtedly provides a profited tool for overcoming the challenges which encounter during the modern measurement of atmospheric turbulent flow. At least it offers a promising first step to diagnosticate directly the ergodic hypotheses for ASL's flows as a criterion. And that the necessary and sufficient conditions of ergodic theorem can introduce to the applicative scope of eddy correlation method and MOST, and seek potential reasons disable for using them in the ABL.
5. In the future, we shall keep up to study the ergodic problems for the atmospheric turbulence measurement under the conditions of complex terrain, heterogeneous surface and unsteady, long observational period, and to seek effective schemes. The above results indicate the atmospheric turbulent eddies below the scale of ABL can be captured by the eddy correlation method and comply with MOST very well. Perhaps MOST can be as the first order approximation to deal with the turbulence of eddies below ABL's scale satisfying the ergodic theorems, then to compensate the effects of eddies dissatisfying the ergodic theorems, which may be caused by the advection, local circulation, low frequency effect, etc under the complex terrain, heterogeneous surface. For example, we developed a turbulent theory of non-equilibrium thermodynamics (Hu, Y., 2007; Hu, Y., et al., 2009) to find the coupling effects of vertical velocity, which is caused by the advection, local circulation, and low frequency, on the vertical fluxes. The coupling effects of vertical velocity may be as a scheme to compensate the effects of eddies dissatisfying the ergodic theorems (Hu, Y., 2003; Chen, J., et al., 2007, 2013).
6. It is clear that such studies are preliminary, and many problems require further research. The attestation of more field experiments is necessary.

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References

Aubinet, M., Vesala, T., and Papale, D.: Eddy covariance, a practical guide to measurement and data analysis, Springer, Dordrecht, Heidelberg, London, New York, 438, 2012.

Birkhoff, G. D.: Proof of the ergodic theorem, Proc. Nat. Acad. Sci. USA. 18, 656-660, 1931.

Boltzmann, L.: Analytischer beweis des zweiten Haubtsatzes der mechanischen Wärmetheorie aus den Sätzen über das Gleichgewicht der lebendigen Kraft, Wiener Berichte , 63, 712-732, in WAI, paper 20, 1871.

Chang, S. S. and Huynh, G. D.: Analysis of sonic anemometer data from the CASES-99 field experiment. Army Research Laboratory, Adelphi, MD. 2002.

Chen, J., Hu Y., and Zhang L.: Principle of cross coupling between vertical heat turbulent transport and vertical velocity and determination of cross coupling coefficient, Adv. Atoms. Sci., 23 (4), 639-648, 2007.

Chen, J., Hu, Y., Lu, S., and Yu, Ye.: Experimental demonstration of the coupling effect of vertical velocity on latent heat flux, Sci. China. Ser. D-Earth Sci., 56, 1-9, 2013.

Eichinger, W. E., Parlange, M. B., Katul, G. G.: Lidar measurements of the dimensionless humidity gradient in the unstable ASL, Lakshmi, V., Albertson, J.

and Schaake, J., Koster, R. D., Duan, Q., Land Surface Hydrology, Meteorology, and Climate, American Geophysical Union, Washington, D. C. 7-13, 2001.

Foken, T., Wichura, B.: Tools for quality assessment of surface-based flux measurements. Agric For Meteorol., 78, 83-105, 1996.

Foken, T., Göckede, M., Mauder, M., Mahrt, L., Amiro, B. D., and Munger, J. W.: Post-field data quality control, in: Handbook of micrometeorology: a guide for surface flux measurement and analysis, Lee, X., Massman, W. J., and Law, B.: Kluwer, Dordrecht, 181-208, 2004.

Foken, T., Aubinet, M., Finnigan, J. J., Leclerc, M. Y., Mauder, M., Paw, U. K. T.: Results of a panel discussion about the energy balance closure correction for trace gases. Bull Am. Meteorol. Soc., 92(4), ES13-ES18, 2011.

Galanti, B. and Tsinober, A.: Is turbulence ergodic? Physics Letters A, 330, 173–18, 2004.

Higgins, C. W., Katul, G. G., Froidevaux, M., Simeonov, V. and Parlange, M. B.: Atmospheric surface layer flows ergodic? Geophy. Res. Let., 40, 3342-3346, 2013.

Hu, Y.: Convergence movement influence on the turbulent transportation in atmospheric boundary layer, Adv. Atoms. Sci., **20**(5), 794-798, 2003.

Hu, Y., Chen, J., Zuo, H.: Theorem of turbulent intensity and macroscopic mechanism of the turbulence development, Sci China Ser D-Earth Sci, **37**(2), 789-800, 2007.

Hu, Y., and Chen, J.: Nonequilibrium Thermodynamic Theory of Atmospheric Turbulence, In: Atmospheric Turbulence, Meteorological Modeling and Aerodynamics, Edited by Peter R. Lang and Frank S., Nova Science Publishers, Inc., 59-110, 2009.

Kaimal, J. C. and Wyngaard, J. C.: The Kansas and Minnesota experiments, Bound.

Lay. Meteor., 50, 31-47, 1990.

Kaimal, J. C. and Gaynor, J. E.: Another look at sonic thermometry, Bound. Lay. Meteor., 56, 401–410, 1991.

Katul, G. G., Hsieh, C. I.: A note on the flux-variance similarity relationships for heat and water vapor in the unstable atmospheric surface layer, Bound. Lay. Meteor., 90, 327–338, 1999.

Katul, G., Cava, D., Poggi, D., Albertson, J., and Mahrt, L.: Stationarity, homogeneity, and ergodicity in canopy turbulence, Handbook of micrometeorology a guide for surface flux measurement and analysis, Lee, X., Kluwer Academic Publishers, New York, 161–180, 2004.

Krengel, U.: Ergodic theorems, de Gruyter, Berlin, New York, 363, 1985.

Lennaert van, V., Shigeo, K., and Genta, K.: Periodic motion representing isotropic turbulence, Fluid Dyn. Res., 38, 19–46, 2006.

Li, X., Hu, F., Pu, Y., Al-Jiboori, M. H., Hu, Z., and Hong, Z.: Identification of coherent structures of turbulence at the atmospheric surface layer, Adv. Atoms. Sci., 19(4), 687-698, 2002.

Mauder, M., Desjardins, R. L., MacPherson, J.: Scale analysis of airborne flux measurements over heterogeneous terrain in a boreal ecosystem. J. Geophys. Res. 112, D13112, 2007.

Martano, P.: Estimation of surface roughness length and displacement height from single-level sonic anemometer data, J. Appl. Meteorol., 39(5), 708–715, 2002.

Mattingly, J. C.: On recent progress for the stochastic Navier Stokes equations, Journées équations aux dérivées partielles, Univ. Nantes, Nantes, Exp. No. XI, 1-52, 2003.

McMillen, R. T.: An eddy correlation technique with extended applicability to non simple terrain, Bound. Lay. Meteor., 43, 231-245, 1988.

Moore, C. J.: Frequency response corrections for eddy correlation systems, Bound. Lay. Meteor., 37, 17-35, 1986.

Neumann, J. V.: Proof of the quasi-ergodic hypothesis, Mathematics Proc. N. A. S., 18, 70-82, 1932.

Oncley, S. P., Friehe, C. A., John, C. L., Businger, J. A., Itsweire, E. C., Chang, S. S.: Surface-Layer Fluxes, Profiles, and Turbulence Measurements over Uniform Terrain under Near-Neutral Conditions, J. Atmos. Sci., 53 (7), 1029-1044, 1996.

Padro, J.: An investigation of flux-variance methods and universal functions applied to three land-use types in unstable conditions, Bound. Lay. Meteor., 66, 413–425, 1993.

Panofsky, H, A,, Lenschow, D, H,, and Wyngaard, J. C.: The characteristics of turbulent velocity components in the surface layer under unstable conditions. Bound. Lay. Meteor., 11, 355–361, 1977.

Papoulis, A. and Pillai, S. U.: Probability, random variables and stochastic processes. McGraw-Hill. New York. 666, 1991.

Poulos, G. S., Blumen, W., Fritts, D, C., Lundquist, J. K., Sun, J., Burns, S. P., Nappo, C., Banta, R., Newsom, R., Cuxart, J., Terradellas, E., and Balsley, Ben.: CASES-99: a comprehensive investigation of the stable nocturnal boundary layer. Bull. Amer. Meteor. Soc., 83, 555–581, 2002.

Schotanus, P., Nieuwstadt, F. T. M., and de Bruin, H. A. R.: Temperature measurement with a sonic anemometer and its application to heat and moisture fluxes, Bound. Lay. Meteor., 26, 81–93, 1983.

Stull, R. B.: An introduction to boundary layer meteorology. Kluwer Academic Publ. Dordrecht. 670, 1988.

Uffink, J.: Boltzmann'S work in statistical physics, Stanford encyclopedia of philosophy, Edward, N. Z., 2004.

Vickers, D., Mahrt, L.: Quality control and flux sampling problems for tower and aircraft data. J. Atmos. Oceanic. Technol., 14, 512-526, 1997.

Webb, E. K., Pearman, G. I., and Leuning, R.: Correction of the flux measurements for density effects due to heat and water vapor transfer, Q. J. R. Meteorol. Soc., 106, 85–100, 1980.

Wilczak, J. M., Oncley, S. P., Stage, S. A.: Sonic anemometer tilts correction algorithms. Bound. Lay. Meteor., 99(1), 127-150, 2001.

Wyngaard, J. C.: Turbulence in the atmosphere, getting to know turbulence, Cambridge University Press, 2010.

Zuo, H., Xiao X., Yang Q., Dong L., Chen J., Wang S.: On the atmospheric movement and the imbalance of observed and calculated energy in the surface layer, Sci. China. Ser. D-Earth Sci., 55(9), 1518-1532, 2012.

Te 1 Local Stability Parameter $(z\text{-}d)/L_c$ of the Eddies in Different Temporal Scales on August 25

Time / Eddy scale	3:00-4:00	7:00-8:00	14:00-15:00
≤2 min	0.59	0.52	-0.38
≤3 min	0.31	0.38	-0.44
≤5 min	0.28	0.16	-0.40
≤10 min	-0.01	0.15	-0.34
≤30 min	-0.04	-0.43	-0.27
≤60 min	-0.07	-1.29	-0.30

Te 2 Parameters of the Fitting Curve of MOS relation for Vertical Velocity Variance

	10 min		30 min		60 min	
	$z/L<0$	$z/L>0$	$z/L<0$	$z/L>0$	$z/L<0$	$z/L>0$
c_1	1.08	1.17	1.06	1.12	0.98	1.06
c_2	4.11	3.67	3.64	3.27	4.62	2.62
R	0.97	0.76	0.94	0.56	0.83	0.30
S	0.19	0.25	0.17	0.27	0.25	0.31

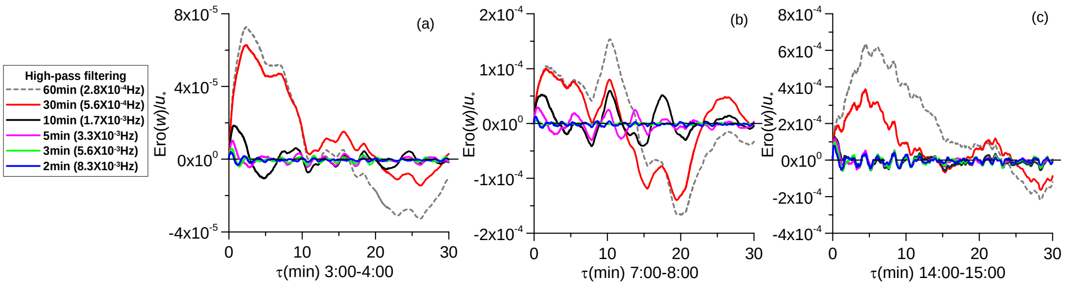

Fig. 1. Variation of mean ergodic function Ero(***w***) of vertical velocity measured at the height 3.08 m in NSPCE with relaxation time for the different scale eddies after High-pass filtering. Panels (a), (b) and (c) are the respective results of the three time frames. If their mean ergodic function is more approximate to zero, then the average of eddies in the corresponding temporal scale will more closely satisfy the ergodic conditions.

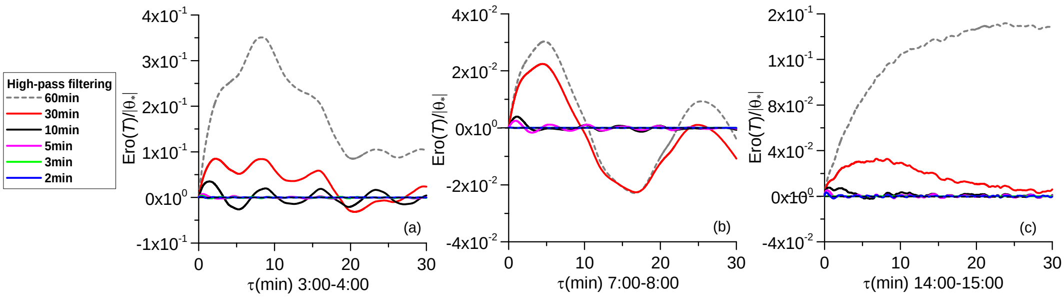

Fig. 2. Variation of mean ergodic function Ero(*T*) of the different scale eddies of temperature with relaxation time (other conditions are similar to Fig. 2, and the same applies to the following figures).

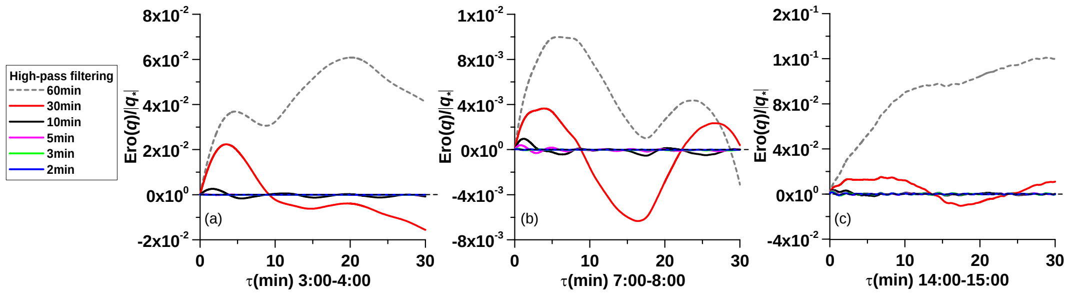

Fig. 3. Variation of mean ergodic function Ero(*q*) of the different scale eddies of humidity with relaxation time.

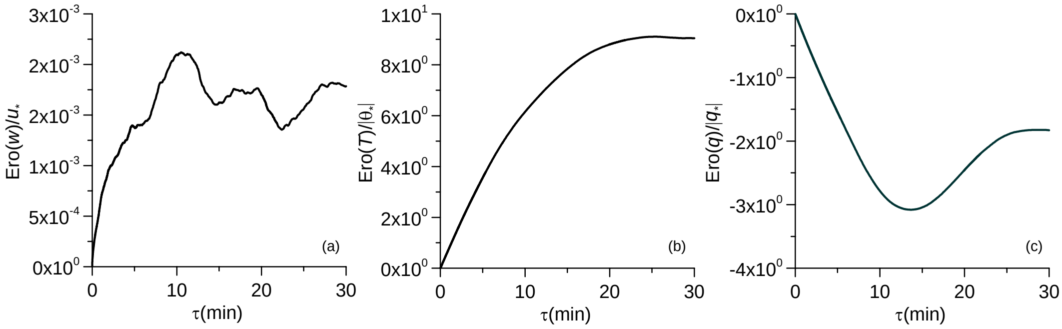

Fig. 4. Variation of mean ergodic function Ero(***w***) of the vertical velocity (a), temperature (b) and specific humidity (c) before filtering during midday 14:00-15:00 (CST) in NSPCE with relaxation time *τ*.

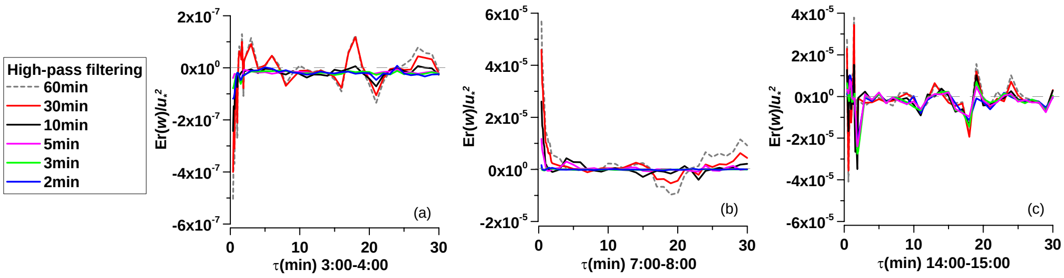

Fig. 5. Variation of the autocorrelation ergodic function of vertical velocity with relaxation time for different scale eddies.

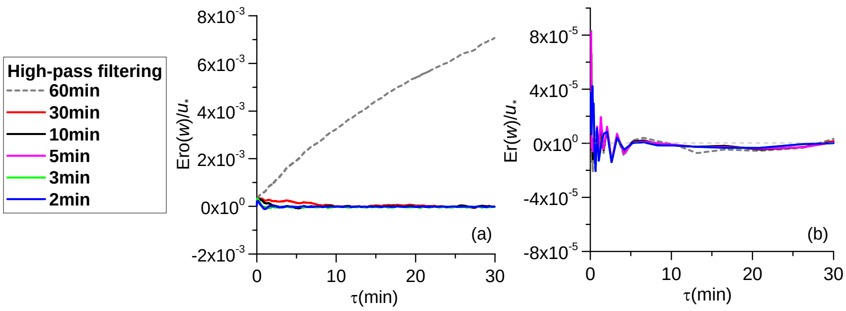

Fig. 6. Variation of mean ergodic function (a) and autocorrelation ergodic function (b) of the vertical velocity with relaxation time for the different scale eddies in CASES-99's seven stations.

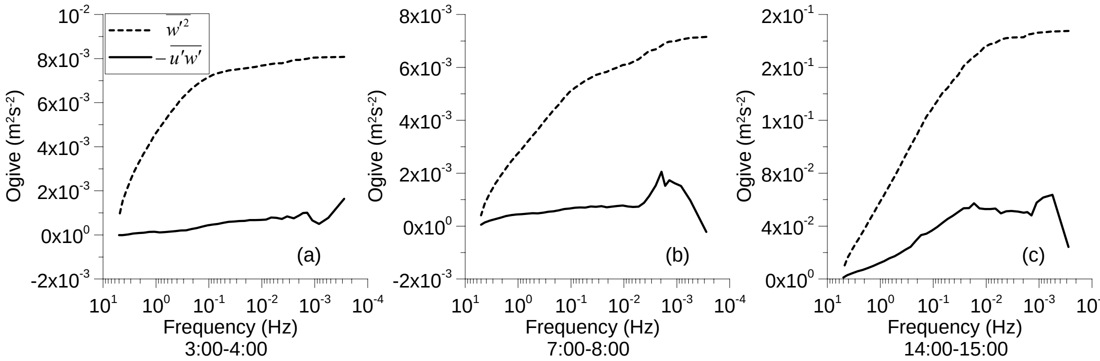

Fig. 7. Variation of Ogive functions of $\overline{w'^2}$ and $-\overline{u'w'}$ with frequency at the height 3.08 m in NSPCE for the three time frames.

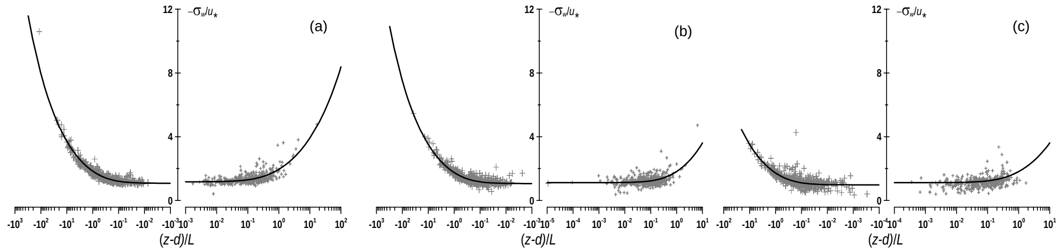

Figure 8. MOS relation of vertical velocity variances of the different scale eddies in NSPCE; Panels (a), (b) and (c) respectively represent the similarity of eddies of 10 min, 30 min and 60 min in the temporal scale.

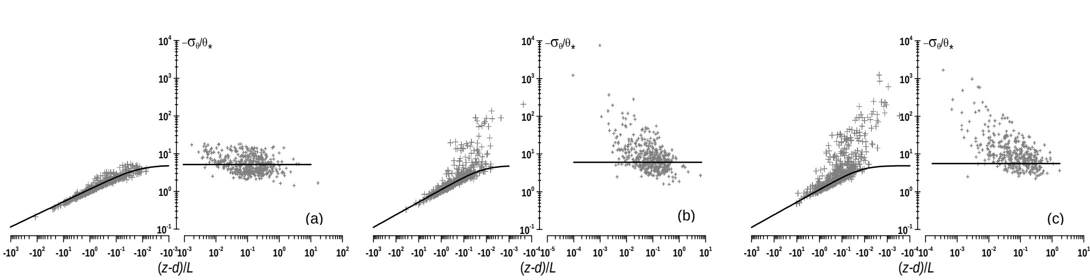

Figure 9. MOS relations of temperature variance of in different scale eddies of NSPCE; Panels (a), (b) and (c) respectively represent the similarity of the eddies of 10 min, 30 min and 60 min in the temporal scale.