Comments on the paper entitled

``Atmospheric waves as scaling, turbulent phenomena'' by

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This manuscript attempts to view "atmospheric waves" as (scaling) turbulent phenomena by proposing the so-called turbulence-wave propagator with proper scaling. This study should be interesting to a broad audience and will have broad impact on research activities in both turbulence and atmospheric wave dynamics, which have been studied with different approaches separately, e.g., nonlinear vs. linear approaches. However, although a general term ``atmospheric waves" is used in this study, the mathematical foundations are based on the dispersion of the classical wave equation, which can only represent special kinds of atmospheric waves such as the Kelvin wave and 2D (x,y) gravity waves. Therefore, please be advised to construct the turbulence-wave propagator using a more general dispersion relation and then to examine if different atmospheric waves can be viewed as (scaling) turbulent phenomena. Alternatively, the authors may consider to revise the manuscript as well as its title to focus on the specific type of atmospheric wave (i.e., the Kelvin wave). In recognition and appreciation for the interesting and challenging study, the reviewer recommends that the manuscript be accepted for publication after the following issues are addressed. General and specific comments and a note on two linear wave equations with more general dispersion relations are provided below.

General Comments:

i. The authors are advised to use a more general dispersion relation (as discussed near the end of this comment file) to construct the turbulence-wave propagator.

ii. Two types of linear wave solutions are (1) propagating wave solutions in the form of $e^{i\sigma t-imz}$ and (2) evanescent wave solutions in the form of $e^{i\sigma t-Mz}$. Here σ, m, and M are real umbers, and represent the frequency, wavenumber, and the reciprocal of the scale height, respectively. While the Kelvin wave propagates in the x direction, its meridional component (in the y direction) is evanescent. Thus, the "scaling" for the Kelvin wave is different from other types of waves which may propagate in both x and y directions (e.g., gravity waves or other types of waves). In addition, under certain atmospheric conditions (e.g., the occurrence of a critical level or critical latitude), the wavenumber in a specific direction may not be integers. Therefore, the existence of an isotropic scale really depends on type of wave. Several atmospheric waves with different dispersion relations are discussed in the note below.

iii. A (linear) dispersion relation is derived from a set of linearized governing equations that describe the spatial distribution and temporal evolution of a flow with multiple fields (e.g., wind speeds, temperature etc) and the correlation among the different fields. The latter is often used to distinguish different weather systems. For example, the phase relationship between the low pressure center and convergence/divergence of wind fields is used to illustrate the differences between a mixed Rossby-gravity (MRG) wave and a tropical-depression- (TD-) type disturbance. Therefore, the reviewer is wondering if and how the concept of the fractional turbulence-wave propagator can provide something equivalent or similar (e.g., via a set of nonlinear governing equations) that can help identify different physical processes for different weather systems?

iv. Linear wave theories have been developed to improve our understanding of the dynamics for terrain-induced or heating-induced mesoscale waves (e.g., Smith, 1979; Lin 1987; Lin 2007 and references therein), and for large-scale equatorial tropical waves (e.g., Matsuno 1996; Wheeler and Kiladis 1999). These studies have suggested an effective means of detecting atmospheric waves in real data. By analyzing global analysis data, Frank and Roundy (2006) and Schreck et al. (2012) have shown the strong relationship between tropical wave activities and tropical cyclone (TC) genesis. These tropical waves may be viewed as precursors to TC genesis. The association of TC genesis with different (linear) tropical waves has been illustrated with modeling studies (Shen et al., 2012; 2013 and references therein), leading to the hypothesis that the lead time of TC genesis prediction can be extended by improving the representation and evolution of (linear) tropical waves and their modulations on TC activities. In addition, the linear wave solutions have been used to verify the solutions of the numerical models. The aforementioned studies, just to name a few, have illustrated the usefulness of linear wave theories (for short-term weather simulations, at least). As the authors conclude that no linear theories are needed, it is important for the authors to provide strong justifications for the superiority of the proposed approach (with the turbulence wave propagator) with respect to the linear wave approaches.

Specific Comments

page 14798, line 18-19: please consider to expand the discussions on the linear atmospheric waves. Some references are provided below.

page 14800, line 17-18: The authors state ``*A key characteristic of linear theories is that they involve integer powers of the (space and time) differential operators*''. When atmospheric conditions are not "uniform," the governing equation such as Eq. (2.71) in the section 2.3 of Nappo (2002) can be solved only by the so-called W.K.B. method. Therefore, is the above statement still valid?

page 14801-14802, Eqs. (2) and (4): When other dispersion relations are used, can you always find the corresponding fractional propagators? if so, will the "anomalous" exponent H be the same or different? Is the value of H dependent of data type and length?

page 14805, line 25: please provide justifications for the scaling: $(k_x, k_y, \omega) \rightarrow \lambda(k_x, k_y, \omega)$ for the Kelvin wave (whose meridional component is evanescent) and the other types of waves as well.

page 14806-14807, line 25: The authors state: ``*Although the dispersion relation is independent of the propagator exponent H_{wav}; the exponent does determine the (power law) rate of decay of the forcing so that the value of H_{wav} will affect the transport of momentum and energy.*" The reviewer have two questions: (1) when different dispersion relations are used, is the above statement still valid? (2) Given a specific H_{wav}, how can the transport of momentum and energy be determined? Namely, what are the mathematical expressions for the transport of momentum and energy?

page 14807, line 10-12: as $\|k\| = (k_x^2 - a^2 k_y^2)^{1/2}$ is used, which is consistent with the characteristic of the Kelvin wave, the authors may want to make changes in Eq. (14) on page 14805.

page 14808, line 9: ``*Data were divided into five 277h (~12 day) blocks, and each block is calculated …*'' please provide justifications for the choice of 277h? In addition, is the spectral density analysis sensitive to the choice of this time scale?

page 14809, line 1: $H = H_{tur} + H_{wav}$ indicates that the total propagator exponent (H) is a linear superposition of H_{tur} and H_{wav}. Therefore, given the dispersion relation in this manuscript, the ``dynamics'' (e.g., energy transfer) of the turbulence and waves depend on H_{tur} and H_{wav}, respectively. However, how can this concept be generalized when other waves with dispersion relations are considered?

page 14809, Figure 3: was only the Kelvin wave analyzed?

page 14810, Eq. (19): if only the Kelvin wave was analyzed, Eq. (19) should be revised with $\|k\| = (k_x^2 - a^2 k_y^2)^{1/2}$.

page 14811, line 11-14 and Figure 4: the authors state: ``*A drawback of the method is that it does not distinguish maxima due to the turbulent contribution and from the (presumed) wave contribution and in the empirical case, the separation is not always evident.*'' The agreement between the theoretical dispersion curve (black) and empirically estimated one (blue) appears only at small wave numbers (i.e., for long waves). Does the black (or blue) line represent the Kelvin wave? If so, how can real k_y be possible for the Kelvin waves? As discussed in Eqs. (5) and (6) below, the black line in Figure 4 should be revised with the usage of $\|k\| = (k_x^2 - a^2 k_y^2)^{1/2}$.

page 14811, Figure 4: The Kevin wave (grey in Figure 3) appears between ω=$(4)^{-1}$ and ω =$(100)^{-1}$. However, selected values of ω are 2, 3, 5, and 10 h^{-1} in Figure 4. Please explain why these values are selected and then add discussions on the differences among different panels with different values of ω. In addition, what are the corresponding periods? 1/ω or 2π/ ω.

page 14813, line 7-10: the authors made the following strong statement: ``*The main conclusion is thus that strongly turbulent atmospheric dynamics are a priori compatible with the observed waves, that to understand them, that one needn't invoke the existence of large laminar regimes nor linear theories.*" It is difficult for the reviewer to agree on the above statement because (1) only the Kelvin wave, which has a very special dispersion relation, was analyzed; (2) only limited data sets (for two months) were analyzed. Additional comments are given in the general comments (iii) and (iv).

A Note on Linear Wave Equations:

Here, two linear wave equations with more general dispersion relations are presented. The two equations have been used to study mesoscale gravity waves and large-scale (equatorial) tropical waves.

(A) 3D linear wave equations on an f-plane

We begin with the following equation for a 3D (x,y,z) flow on an f-plane with the Boussinesq approximation (e.g., Eq. 3.2.1 of Lin 2007):

$$\frac{D}{Dt}\left\{\frac{D^2}{Dt^2}\nabla^2 w + f^2 w_{zz} - U_{zz}\frac{D}{Dt}w_x + fU_{zz}w_y + N^2\left(w_{xx}+w_{yy}\right) + 2fU_z w_{yz}\right\}$$
$$-2fU_z^2 w_{xy} - 2f^2 U_z w_{xx} = 0, \qquad (1)$$

where $\frac{D}{Dt} = \frac{\partial}{\partial t} + U(z)\frac{\partial}{\partial x}$ and $\nabla^2 w = w_{xx} + w_{yy} + w_{zz}$. U(z) represents the basic state zonal wind, and w is the vertical velocity perturbation. N^2 is the Brunt Vaisala frequency and f is the Coriolis force. Eq. (1) has been used to examine the solutions for potential flows, hydrostatic flows with (pure) gravity waves (with f=0), ageostrophic flows with inertia gravity waves (f≠0), and quasi-geostrpophic flow with Eady-type modes. In addition, it has been used to study the impact of the so-called critical level on wave dynamics (e.g. Shen and Lin, 199). The critical levels are defined as the levels where the phase speed (c) is equal to the basic wind (i.e., c=U) or c=U±f/k. σ is the frequency, k is the wavenumber and c =σ/k. With further simplifications, Eq (1) can be reduced to the so-called Taylor-Goldstein equation or the shallow water question. Therefore, Eq. (1) can provide a more general dispersion relation. Simplifications are given as follows.

(1) Taylor-Goldstein equation:

Assume a 2D (x,z), non-rotating (f=0) flow, Eq. (1) becomes:

$$\frac{D^2}{Dt^2}(w_{xx}+w_{zz}) - U_{zz}\frac{D}{Dt}w_x + N^2 w_{xx} = 0. \qquad (2)$$

(2) The shallow water equation on an f-plane:

Assume a 3D, uniform (no shear) hydrostatic flow, Eq. (1) is simplified to the following form:

$$\frac{D^2}{Dt^2}w_{zz} + f^2 w_{zz} + N^2\left(w_{xx} + w_{yy}\right) = 0. \qquad (3)$$

We further assume $\rho = \rho_o e^{-z/H_s}$ and $w = \frac{D}{Dt}\eta(t,x,y)e^{i\alpha z/H_s}$, here ρ is the density, H_s is the scale height of the density and H_s/α is the vertical wavelength. Eq. (3) becomes:

$$\frac{D}{Dt}\left\{\frac{D^2}{Dt^2}\eta + f^2\eta - \frac{gH_s}{\alpha^2}\left(\eta_{xx} + \eta_{yy}\right)\right\} = 0. \qquad (4)$$

Equation (4) has the same form as Eq. (3.9.2) of Pedlosky (1979), which is derived from a set of linearized shallow-water equations (e.g., Eqs. 3.6.3a-c of Pedlosky 1979). The above derivations suggest that given a wave mode with the vertical wavelength of H_s/α, its solutions are equivalent to those in the shallow water equations with an equivalent depth of H_s/α^2, denoted H_e ($=H_s/\alpha^2$).

(3) The dispersion relation for the Kelvin wave:

Based on the characteristics of the Kevin wave (e.g., section 3.9 of Pedlosky 1979), Eq. (4) gives the following two equations:

$$\eta_{yy} - \frac{f^2}{gH_e}\eta = 0, \qquad (5a)$$

and

$$\frac{D^2}{Dt^2}\eta - gH_e\eta_{xx} = 0. \qquad (5b)$$

Equation (5a) suggests an evanescent (not wavelike) mode in the y direction, which leads to $\eta \propto e^{-f/c_0|y|}$, here $c_o = \sqrt{gH_e}$. Eq. (5b) does not explicitly include the "f" term and resembles the classic wave equation. The corresponding dispersion relation is written as:

$$\sigma = Uk + \sqrt{gH_e} = Uk + C_o k, \qquad (6)$$

which has a much simpler form than the others in Eqs. 8-10.

(B) The shallow water equation on a beta plan

To study the large-scale tropical waves, the shallow-water equations proposed by Matsuno (1966) have been used (e.g., Wheeler and Kiladis 1999; Kiladis et al. 2009). Assuming the meridional velocity perturbation to be $v = \hat{v}(y)exp(i\sigma t - ikx)$, the linearized shallow water equation on a β-plane (f=βy) can be written as follows:

$$\frac{d^2\hat{v}}{dy^2} + \left(\frac{\sigma^2}{gH_e} - k^2 - \frac{k}{\sigma}\beta - \frac{\beta^2 y^2}{gH_e}\right)\hat{v} = 0. \qquad (7)$$

It was shown by Matsuno (1966) that the Eq. (7) gives the following dispersion relation

$$\frac{\sigma^2}{gH_e} - k^2 - \frac{k}{\sigma}\beta = \frac{\beta(2n+1)}{\sqrt{gH_e}}, \qquad n = 0,1,2,3 \ldots \qquad (8)$$

Equation (8) is a cubic equation and therefore has three roots in general. Among different types of waves, the dispersion relations for the following two different tropical waves are presented and compared with that for the Kelvin wave.

(1) Equatorial Rossby (ER) waves:

For very low frequency waves (with a small σ^2), Eq. (8) has only one root which is

$$\sigma = \frac{-\beta k}{k^2 + \beta(2n+1)/\sqrt{gH_e}}. \tag{9}$$

(2) Mixed Rossby-gravity (MRG) wave:

The dispersion relation of the MRG is obtained when n=0:

$$\sigma = \frac{k\sqrt{gH_e}}{2}\left[1 - \left(1 + \frac{4\beta}{k^2\sqrt{gH_e}}\right)^{1/2}\right]. \tag{10}$$

(3) Kelvin wave:

The dispersion relation of the Kelvin wave can be obtained from Eq. (8) under a special condition with n=-1:

$$\sigma = k\sqrt{gH_e}. \tag{11}$$

Equation (11) is the same as that in Eq. (6), which is much simpler than the others (e.g., Eqs. 9 and 10). Note that the meridional velocity perturbation for the Kelvin wave is identical to zero, i.e., $v \equiv 0$, which is a trivial solution to Eq. (7). Therefore, non-trivial solutions for other components are obtained by solving the original linearized governing equations (e.g., Equations 1-3 of Kiladis et al., 2009).

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