The activated fraction AF can be expressed as followed:

$$AF = 1\text{-}\exp(-J.A.t_{nuc}) \quad (1)$$

Where t_{nuc} is the residence time, A the surface area and J the nucleation rate.

For a similar AF and residence time (AF_1=AF_2 and t_{nuc1}=t_{nuc2}):

$$A_1J_1=A_2J_2 \quad (2)$$

The nucleation rate can be expressed as followed:

$$J = K.\exp\left(\frac{\Delta G(T).f_{het}(\alpha)}{kT}\right) \quad (3)$$

where K is the kinetic factor, f_{het} the geometric compatibility factor. K is assumed to be constant for the range of RH_i where ice nucleation happened. The Gibbs free energy ΔG can be expressed as follow:

$$\Delta G(T, S_i) = \frac{16\pi}{3}.\frac{\vartheta_{ice}^2(T).\sigma_{i/v}^3}{(kTlnS_i)^2} \quad (4)$$

With k=1.38.10^{-23} J/K, ϑ_{ice} the volume of water molecules in an ice embryo, $\sigma_{i/v}$ the surface tension at the ice/vapour interface, S_i the saturation ratio with respect to ice, k the Boltzmann constant.

By combining equations (1), (2), (3) and (4), we can obtain the following expression:

$$ln\frac{A_1}{A_2} = \frac{16\pi}{3}.\frac{\vartheta_{ice}^2(T).\sigma_{i/v}^3}{k^3T^3}.f_{het}.\frac{lnS_{i,2}^2-lnS_{i,1}^2}{lnS_{i,1}^2.lnS_{i,2}^2} \quad (5)$$

Any combination of $A_1=\pi D_1^2$ and $A_2= \pi D_1^2$, $S_{i,1}$ and $S_{i,2}$ for a given temperature can lead to a similar AF with adjustment of f_{het}.
In our study, D_1=130nm and D_2=180nm, $S_{i,1}$=1.58 and $S_{i,2}$=1.47 and T=238.15K

We derive f_{het}:

$$f_{het} = \frac{ln\frac{A_1}{A_2}}{\frac{16\pi}{3}.\frac{\vartheta_{ice}^2(T)\sigma_{i/v}^3}{k^3T^3}.\frac{lnS_{i,2}^2-lnS_{i,1}^2}{lnS_{i,1}^2.lnS_{i,2}^2}} \quad (6)$$

f_{het}= 0.00057 for our study.

We then derive the kinetic factor K from equation (3).
With the kinetic factor we can calculate the contact angles α_1 and α_2 with A_1 and A_2 their respective surface area:

$$f_{het_{1,2}} = -ln\left(\frac{J_{1,2}}{K}\right).\frac{kT}{\Delta G(T)} \quad (7)$$

$$\alpha_{1,2} = \frac{180°}{\pi} arccos\left(\sqrt[3]{-1 + 2f_{het_{1,2}} + 2\sqrt{-f_{het_{1,2}} + f^2{}_{het_{1,2}}}} + \frac{1}{\sqrt[3]{-1 + 2f_{het_{1,2}} + 2\sqrt{-f_{het_{1,2}} + f^2{}_{het_{1,2}}}}}\right)$$

Which gives us: $\mathbf{\alpha_1=30.1°}$ and $\mathbf{\alpha_2=27.6°}$