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Statistics of severe tornadoes and severe tornado outbreaks

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Abstract

The standard measures of the intensity of a tornado in the USA and many other countries are the Fujita and Enhanced Fujita scales. These scales are based on the damage that a tornado causes. Another measure of the strength of a tornado is its path length of touchdown, L. In this study we consider 4061 severe tornadoes (defined as $L \geq 10$ km) in the continental USA for the time period 1981–2010 (USA Storm Prediction Center Severe Weather Database). We find for individual severe tornadoes: (i) The noncumulative frequency-length statistics of severe tornado touchdown path lengths, $20 < L < 200$ km, is well approximated by an inverse power-law relationship with exponent near 3. (ii) There is a strong linear scaling between the number of severe tornadoes in a year and their total path lengths in that year. We then take the total path length of severe tornadoes in a day, L_{D}, as a measure of the strength of a 24-hour USA tornado outbreak. We find that: (i) On average, the number of days per year with at least one continental USA severe tornado (path length $L \geq 10$ km) has increased 16 % in the 30-year period 1981–2010. (ii) The daily numbers of severe tornadoes in a USA outbreak have a strong power-law relationship (exponent 0.87) on their daily total path lengths, L_{D}, over the range $20 < L_{\mathrm{D}} < 1000$ km dy^{-1}. (iii) The noncumulative frequency-length statistics of tornado outbreaks, $10 < L_{\mathrm{D}} < 1000$ km dy^{-1}, is well approximated by an inverse power-law relationship with exponent near 1.7. We believe that our robust scaling results provide evidence that touchdown path lengths can be used as quantitative measures of the systematic properties of severe tornadoes and severe tornado outbreaks.

1 Introduction

This paper introduces and tests hypotheses for quantifying the intensities of severe tornadoes and tornado outbreaks. Our approach is in analogy to the historic evolution of the qualitative (damage-based) Mercalli scale relative to the quantitative (displacement-based) Richter scale for earthquakes. The Fujita and Enhanced Fujita scales, currently used for tornadoes, are based qualitatively on damage, from which wind intensity and other quantitative measures are estimated. Ideally, tornado intensities would be based on the distribution of velocities in a tornado. However, as noted by Doswell et al. (2009), systematic and high-resolution Doppler remote sensing of wind velocities in tornadoes is not possible at this time.

In this paper, we will use the tornado path length L as a quantitative measure of tornado intensity and on the basis of our frequency-length statistics (shown in the next section), we will define a severe tornado as having $L \geq 10$ km. A detailed study of the statistical relationship between tornado path lengths L and Fujita scale intensities has been given by Brooks (2004). In this paper we extend his approach, to develop robust scaling relationships to aid in improving our understanding of tornado climatology.

The standard measure of the tornado intensity is the Fujita scale (Fujita, 1971, 1981; Fujita and Pearson, 1973) also referred to as the Fujita-Pearson scale. This scale was introduced in the 1970s as a measure of tornado intensity, with tornadoes rated on a scale of F0 to F5 based on the damage caused. In the United States, the enhanced Fujita scale replaced the Fujita scale for intensity assessment on 2 February 2007, using different and more specific criteria for assessment (Doswell et al., 2009). In terms of applications, the Fujita scale and the enhanced Fujita scale are considered equivalent, so that in the remainder of this paper we will refer to the Fujita scale.

It is of interest to compare probabilistic risk assessment for tornadoes with that of earthquakes. From 1880 until 1935 the Mercalli scale was used to determine the intensity of earthquakes. The Mercalli scale was based on damage, and is in direct analogy to the Fujita scale for tornadoes (Doswell et al., 2009). In 1935 the Mercalli scale was replaced by the Richter scale (Richter, 1935) as the accepted measure of earthquake intensity. The Richter scale utilized the displacement amplitudes obtained from regional seismographs to quantify the ground shaking responsible for damage and deaths. In 1979 seismograph displacements were used to directly determine the moment (radiated energy) of an earthquake (Hanks and Kanamori, 1979). Earthquake

moments are then converted to moment magnitudes because of the public acceptance of the Richter magnitude scale. The association between earthquake and tornado risk assessments has also been discussed by Schielicke and Névir (2011).

The principle purpose of this paper is to carry out a study of the statistics of tornado touchdown path lengths, L. In Sect. 2, we discuss the data. Because of data quality, we will consider tornadoes during the 30-year period 1981–2010 and only severe tornadoes, which we will define as having touchdown path lengths $L \geq 10$ km. Based on the work of Brooks (2004) and our studies, this lower limit approximately eliminates tornadoes F0 and F1, leaving only strong (F2 and F3) and violent (F4 and F5) tornadoes.

In Sect. 3, we consider the statistics of individual severe tornadoes ($L \geq 10$ km) during the period 1981–2010, including the statistics of severe tornado occurrence as a function of hour of day, day of the year, total number vs. path length per year, and probability of a given length L occurring. Then in Sect. 4, we extend our studies of individual severe tornadoes to the total path length of severe tornadoes in a day, L_D, which we take as a measure of the strength of a continental USA tornado outbreak in a one-day period. Doswell et al. (2006) have suggested that L_D is the preferred measure of the strength of a tornado outbreak. We relate the total number of severe tornadoes in an outbreak to L_D and find power-law scaling, then use L_D to observe trends over the last 30 years. Finally, in Sect. 5, we discuss our approach. We recognize the limitations of our approach, but obtain a sequence of well-defined scaling relationships similar to the established Gutenberg-Richter frequency-magnitude scaling for earthquakes. We believe our approach will augment the Fujita (and Enhanced Fujita) scale for the quantification of the tornado hazard.

2 Data

In this paper we consider the statistics of tornado occurrence in the continental United States. We use the National Weather Service (NWS) Storm Prediction Centre (SPC) database of tornadoes (McCarthy, 2003) for the time period 1951 to 2010 (NOAA,

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2011). For the 55 233 tornado records during this period, information includes (in most cases) tornado date, time, location (latitude, longitude, county, state), Fujita scale (or enhanced Fujita scale) value, injuries, fatalities, damage, and touchdown path length and width. A number of records were removed based on the listed values of tornado path length, L. In the original database, tornadoes that touched down in more than one state had a path length record for each state, and another one for the entire summed path length for the multiple states. Therefore 905 values (1.6 % of the original dataset records) were removed that were one part of a multi-state record (the multi-state record was left in place). Also removed from the original dataset were 57 values (0.1 % of the original dataset records) with lengths that were $L = 400, 300, 200, 100, 80, 50, 30, 25, 20, 15, 10, 8$ miles (the original units of the database), but where the starting and ending latitude and longitude coordinates were listed as being exactly the same (i.e., 0 miles traversed). It was assumed that these records were in error due to being exactly on multiples of one-hundred (or 10) and having zero path length based on touchdown starting/ending coordinates. The final database used here for 1950–2010 (all touchdown path lengths L), had a total of 54 271 unique tornado records.

We first consider the frequency-path length statistics for all tornadoes. In Fig. 1 we give the cumulative number of tornadoes per year N_c with touchdown path lengths greater than L, as a function of L. Mean values are given for six 10-year periods, 1951–2010. In Fig. 1a we consider all tornadoes of any path length L (54 271 values) and in Fig. 1b just those tornadoes with $L \geq 10$ km (7711 values). There is a clear visual difference between the three 10-year frequency-size distributions for 1951–1980 compared to the three 10-year frequency-size distributions during the period 1981–2010. Many fewer long path lengths were recorded in the later period. Schaefer et al. (2002), Brooks (2004) and Verbout et al. (2006) have previously noted this difference and suggest that the difference in completeness is related to the beginning of real-time touchdown surveys. Beginning in the early 1980s, a Warning Preparedness Meteorologist (WPM) was instituted at 52 Weather Service Forecast Offices; the WPM

was responsible for tornado surveys in their state (McCarthy, 2003). This contrasts with the earlier period (1951–1980), where tornado touchdown path lengths were primarily determined from newspaper accounts, which appear to have systematically over-stated the actual values (McCarthy, 2003). One explanation he gives for this effect was that several tornadoes with shorter path lengths were often combined to give a single long path length. Field surveys have certainly given more accurate data on tornado path lengths.

Based on the data shown in Fig. 1, we will consider in the remainder of this paper just those tornadoes that occurred during the period 1981–2010 (33 418 values, all touchdown path lengths L). It can be seen from Fig. 1 that the decadal frequency-length statistics for the three periods 1981–1990, 1991–2000, and 2001–2010 are reasonably self-consistent.

The emphasis in this paper is on the statistics of tornado path lengths as a measure of tornado intensity. Since the standard measure of tornado intensity is the Fujita scale, it is important to consider relations between the Fujita scale values and tornado path lengths. Brooks (2004) has studied in detail the statistical distribution of path lengths for F0 to F5 tornadoes. He carried out his study for all tornadoes from 1950–2001. In Fig. 2, we relate the statistical measures of tornado touchdown path lengths L as a function of Fujita scale for intensities F0 to F5, for all tornadoes 1981–2010. For each Fujita intensity, we give the mean touchdown path length (red diamonds), median (grey circles), and the 75th and 25th percentiles (upper and lower horizontal lines). For F2 to F5 (i.e., strong to violent) tornadoes, the best-fit linear trend line (thick red line) to the mean path length values is:

$$\log \bar{L}_{\mathrm{F}j} = 0.224 j + 0.667, \ j = 2, 3, 4, 5, \quad (1)$$

where $\bar{L}_{\mathrm{F}j}$ is the mean of all tornado path lengths L at a given Fujita scale value, Fj, j = 2, 3, 4, 5. Eq. (1) can be written as:

$$\frac{\bar{L}_{\mathrm{F}(j+1)}}{\bar{L}_{\mathrm{F}j}} = \frac{10^{0.224(j+1)+0.667}}{10^{0.224j+0.667}} = 10^{0.224} = 1.67, \ j = 2, 3, 4, 5, \quad (2)$$

That is, the mean path length of an F3 tornado is 1.67 times longer than the mean path length of an F2 tornado, and the mean path length of an F5 tornado is $1.67^3 = 4.6$ times longer than the mean path length of an F2 tornado. In Table 1 we compare our mean path lengths for the period 1981–2010 with those given by Brooks (2004) for the period 1950–2001, and find good agreement, despite the different time periods considered, and the differences in data completeness. We also include the early work of Fujita and Pearson (1973) where they gave a range of touchdown path lengths associated with specific Fujita scale values. Their values were based on a small sample of tornadoes and we believe over-estimate the path length values for each Fujita scale.

From Fig. 2 we see that reasonably good scaling of the mean touchdown path lengths (red diamonds) as a function of Fujita intensity is obtained for tornadoes F2 to F5. The deviation from this scaling for F0 and F1 tornadoes is likely due to limitations of the Fujita scale for weak tornadoes and/or measurement problems with determining path lengths for these weak tornadoes. For these reasons, we will limit further statistical studies in this paper to severe tornadoes, which we will define here as those that are F2 or larger (i.e., "strong" and "violent" tornadoes). Since our studies are based on tornado path lengths, we will define a severe tornado to be a tornado that has a touchdown path length $L \geq 10$ km. In terms of path lengths, we see from Fig. 2 and Table 1 that on average, the minimum touchdown path length value in our definition of severe tornadoes approximately coincides with F2 (strong) tornadoes, at $L = 11.6$ km, with F0 and F1 (weak) tornadoes having path lengths that significantly deviate from the scaling seen for strong (F2 and F3) and violent (F4 and F5) tornadoes.

In Table 2 we give the number of continental USA tornadoes with $L < 10$ km and $L \geq 10$ km as a function of Fujita intensity for the time period 1981–2010. The total number of "severe" tornadoes ($L \geq 10$ km) that we consider in this paper is 4061 (12 % of the database's tornadoes, 1981–2010), with 29 357 tornadoes ($L < 10$ km) omitted. We recognize that a substantial fraction of our severe tornadoes ($L \geq 10$ km) have designation F0 (i.e., 3 % of all F0 tornadoes) and F1 (15 % of all F1 tornadoes), and that a substantial fraction of the tornadoes we do not consider ($L < 10$ km) have designation

F2 (61 % of all F2 tornadoes) to F5 (7 % of all F5 tornadoes). We also note that from the results of Brooks (2004) and this paper (see Fig. 2, Table 1) there is a systematic increase in tornado path lengths as a function of increasing F value. However, there is large scatter. An important question is whether this scatter can be primarily associated with the damage assessments that give the F values or whether path lengths are simply not a good measure of tornado intensities.

In order to address this question we return to the comparison between the damage-based Mercalli scale for earthquakes and the Fujita scale for tornadoes. When a strong earthquake occurs, maps of Mercalli intensities are obtained. These intensities systematically decrease away from the earthquake epicenter, as expected. There are also local variations in values due to local variations in ground shaking intensity. However, in a strong earthquake, hundreds to thousands of Mercalli values are obtained, so that averaging can be carried out to obtain smoothed maps of intensity. These maps are considered useful even if instrumental earthquake magnitudes are available.

There is no question that Fujita scale evaluations of tornado intensities are very useful. However, results in this paper, along with other work, would indicate that tornado touchdown path lengths may also be a very useful measure of the intensity of both individual tornadoes and tornado outbreaks. We believe that the definition provided here of a severe tornado ($L \geq 10$ km), is easily quantifiable, and roughly corresponds with the Fujita intensities F2 to F5. We will now use this database of 4061 severe continental USA tornadoes ($L \geq 10$ km) that occurred over the time period 1981–2010.

3 Statistics of severe tornadoes, 1981–2010

In this section we carry out a systematic study of the statistics of severe tornadoes ($L \geq 10$ km) during the period 1981–2010. We first give the dependence of tornado occurrence on time of day, day of the year, and year. In Fig. 3, we give a histogram of times of occurrence of severe tornadoes, 1981–2010. We determine the probability of

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a severe tornado occurring in a given hour:

$$p(h)=\frac{N_h}{N_T} \quad (3)$$

where N_h is the number of severe tornadoes ($L \geq 10$ km) initiated during hour h CST (Central Standard Time), and N_T is the total number of tornadoes ($L \geq 10$ km) during the period 1981–2010. The dependence of p(h) on h is given in Fig. 3. There is an afternoon peak in activity h = 15:00 to 20:00 CST. Maximum activity is at h = 17:00 to 18:00 CST, with 12 % of all tornadoes initiated during this hour. The results are similar to those given by Kelly et al. (1978) for 17 659 tornadoes that occurred between 1950 and 1976.

In Fig. 4, we give the statistics of severe tornado occurrence as a function of day of the year (leap days, 29 February omitted). For each day of the year, 1 to 365, we give the number of years from 1981–2010 with at least one severe tornado $L \geq 10$ km. There is a peak from April to July (days 91 to 212). The highest peak activity was on day 151 (31 May), with on this day, 15 of the 30 years having at least one severe tornado.

We next turn to annual variability over the period considered. In Fig. 5, for each year t = 1981 to 2010, we give n the number of days per year in which one or more severe tornadoes ($L \geq 10$ km) occurred. The best-fit linear correlation of this data gives

$$n = 0.243\,t - 437 \quad (4)$$

On average, the number of days in a year with at least one severe tornado ($L \geq 10$ km) increased from n = 44 dy in 1981 to 51 dy in 2010. The standard deviation of the values about this trend line is 8.4 dy.

In Fig. 6, for all tornadoes ($L \geq 10$ km) we give the annual number N_Y and annual total path length L_Y, for each year t = 1981 to 2010. The best-fit linear trend for the annual number of tornadoes is given by:

$$N_Y = 2.903\,t - 5658 \quad (5)$$

In terms of this best-fit, the yearly number of tornadoes increased from, on average, N_Y = 93 tornadoes yr^{-1} in 1981 to 177 tornadoes yr^{-1} in 2010. The standard deviation of the values about this trend line is 41.5 tornadoes yr^{-1}. The best-fit linear trend for the annual total path length of severe tornadoes ($L \geq 10$ km) is given by:

$$L_Y = 45.58t - 87\,763 \quad (6)$$

In terms of the best-fit, the annual total path length of severe tornadoes ($L \geq 10$ km) increased from, on average, L_Y = 2530 km yr^{-1} in 1981 to 3850 km yr^{-1} in 2010. The standard deviation of the values about this trend line is L_Y = 1150 km yr^{-1}.

In Figs. 5 and 6, for severe tornadoes ($L \geq 10$ km), we have shown general increasing trends over the 30-year period (1981–2010) in the variables n (number of days per year with at least one severe tornado), N_Y (number of tornadoes per year), and L_Y (total path length per year). For n, the increase of values over the time period is 16 % (Fig. 5), and the standard deviation of these values around the trend line (8.4 dy) is 17 % of the mean of n (48.3 dy). Similarly (Fig. 6), the increase over the 30-year period for N_Y is 90 % and for L_Y 50 %, with corresponding percentage standard deviations (around the trend line) compared to the mean, equal to 36 % and 31 %. Although increasing trends are apparent, it is not clear that the increases are statistically significant, considering the large scatter in the data.

From Fig. 6 it is apparent that there is a strong correlation between N_Y and L_Y; when values of one variable are large (small), the same behaviour is seen in the other variable. This correlation is illustrated in Fig. 7, where for all severe tornadoes ($L \geq 10$ km) from 1981–2010, the yearly number N_Y is plotted as a function of the yearly total path length L_Y. The best-fit linear correlation is given (Fig. 7) by:

$$N_Y = 0.0422L_Y \quad (7)$$

with L_Y in km, and the intercept assumed to be 0. Setting N_Y = 1 tornado yr^{-1} in Eq. (7) gives the mean length of severe tornadoes ($L \geq 10$ km) during this 30-year period, $\bar{L}_Y$ = 24 km yr^{-1}. The results given in Fig. 7 are evidence of the scale invariant nature of tornadoes on a year to year basis. The number-length ratio is the same in years of few severe tornadoes and years with many severe tornadoes, i.e. the ratio is independent of the length considered.

We now consider the noncumulative frequency-length statistics of all severe tornadoes ($L \geq 10$ km) during the time period 1981–2010. Frequency densities are defined as:

$$f(L)=\frac{\delta N}{\delta L}, \tag{8}$$

where δN is the number of tornadoes with lengths between L and $L+\delta L$. In Fig. 8 we plot f(L) as a function of L, on logarithmic axes, and find a reasonably good power-law correlation:

$$f(L)=1.71\times10^{6}L^{-3.10}, \tag{9}$$

with L in km. The best-fit of the scaling relationship, Eq. (9), to the frequency densities, is between $20 \leq L \leq 200$ km, with a slight underfitting for $L < 20$ km. The implication of the inverse power-law scaling with exponent near 3, is that for the time period given and defined spatial region, a tornado with a factor of 10 longer path length has a probability of occurrence that is about 1000 less. For example, the probability of a tornado with an $L = 200$ km path length is 1000 times smaller than the probability of an $L = 20$ km path length.

The use of these frequency-size statistics to calculate the probability of given path length tornadoes occurring, implicitly assumes weak stationarity of the severe tornado time series. We acknowledge that there exists a yearly seasonality within the time series, and a clustering of values for tornadoes that occur with given atmospheric conditions. However, the clustering of values is in analogy to the aftershock sequence of earthquakes. We therefore have used here the tornado path length frequency-size statistics in a similar way that is currently used for earthquake hazard assessment, resulting in the conclusion that a tornado with a touchdown path length L_1 is 1000 times more probable than a tornado path length 10 times longer, $L_2 = 10L_1$.

4 Statistics of severe tornado outbreaks, 1981–2010

An important aspect of tornado climatology is the occurrence of tornado outbreaks. One definition of a tornado outbreak is the occurrence of multiple tornadoes within a particular synoptic-scale weather system (Glickman, 2000). The NWS SPC database of tornadoes used here does not explicitly categorize individual tornadoes as part of a specific tornado outbreak. In this paper, we follow the approach of Doswell et al. (2006) and will define a tornado outbreak to include all tornadoes in a day in the continental USA. However, consistent with our studies of individual severe tornadoes, we will consider a severe tornado outbreak to include only those tornadoes with path lengths $L \geq 10$ km.

Doswell et al. (2006) considered a variety of measures of the strength of a tornado outbreak based on daily records. They gave the highest weight to the total path length of all tornadoes during a day. In this paper, we will consider the statistics of the total path length, L_D, of all severe tornadoes ($L \geq 10$ km) in a day in the continental USA. In Fig. 9, for 1981–2010, for each day that has at least one severe tornado ($L \geq 10$ km), we give the daily total path length of tornadoes, L_D, for that day. The distribution appears to be relatively uniform over this period.

We now consider (Fig. 10) the correlation between N_D the total number of severe tornadoes ($L \geq 10$ km) in a day (i.e., a continental USA "outbreak") and $\overline{L_D}$ the mean of the daily total tornado path lengths for all days where N_D is the same value. We also consider the standard deviation of L_D for each N_D. For example, there are 74 days where $N_D = 4$ severe tornadoes occur during the day; the mean ± standard deviation of the total tornado daily path lengths L_D for those 74 occurrences is $\overline{L_D} = 91.9 \pm 37.1$ km. For very large values of N_D, because there are very few of them, we consider the mean of all L_D over multiple N_D. The best-fit linear correlation to N_D as a function of $\overline{L_D}$ is a power-law relationship:

$$N_D = 0.078\left(\overline{L_D}\right)^{0.874} \tag{10}$$

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over the range $20 < \overline{L_\mathrm{D}} < 1000\,\mathrm{km\,dy^{-1}}$. This power-law correlation is quite robust as it extends over almost two orders of magnitude. With a power-law exponent of 0.874, the correlation between the number of severe tornadoes in a daily USA outbreak, N_D, and the mean daily total tornado path length, $\overline{L_\mathrm{D}}$, is almost linear (i.e., exponent 1.0). We conclude that N_D and $\overline{L_\mathrm{D}}$ (calculated for all tornadoes $L \geq 10\,\mathrm{km}$) are equivalent measures of the strength of a USA severe tornado outbreak.

We next give the frequency-length statistics of daily USA tornado outbreaks for the time period 1981–2010. Similar to the definition of the frequency-density function $\mathrm{f}(L)$ given in Eq. (8), we plot $\mathrm{f}(L_\mathrm{D})$ vs. L_D in Fig. 11 on logarithmic axes, and find an excellent power-law correlation:

$$\mathrm{f}(L_\mathrm{D}) = 5180 L_\mathrm{D}^{-1.726} \tag{11}$$

with L_D in $\mathrm{km\,dy^{-1}}$. This power-law relationship is found to be robust over about two orders of magnitude, $10\,\mathrm{km\,dy^{-1}} < L_\mathrm{D} < 1000\,\mathrm{km\,dy^{-1}}$. Just as in the case of severe individual tornadoes, this power-law relationship can be used to estimate the relative probabilities of occurrence of severe tornado outbreaks. The probability of a $L_\mathrm{D} = 100\,\mathrm{km\,dy^{-1}}$ outbreak is a factor of about $10^{1.726} \approx 50$ less likely to occur than a $L_\mathrm{D} = 10\,\mathrm{km\,dy^{-1}}$ outbreak. Similarly, an $L_\mathrm{D} = 1000\,\mathrm{km\,dy^{-1}}$ outbreak is a factor of about 50 times less likely to occur than a $L_\mathrm{D} = 100\,\mathrm{km\,dy^{-1}}$ outbreak. It is interesting to note how this factor of 50 compares with the factor $10^{3.104} \approx 1300$ for individual severe tornadoes obtained from Eq. (9). Because the exponent in Eq. (11) is much less than that in Eq. (9), very severe tornado outbreaks are more likely to occur than very severe individual tornadoes.

5 Discussion and conclusions

In any study of the statistics of a natural hazard it is necessary to have a reliable database. In the case of tornadoes, an important question is what a database should

contain. The standard measure of tornado intensity is the damage-based Fujita scale. The only other widely available measure of tornado intensity is the path length of touchdown caused by a tornado. In Fig. 1, we have given the cumulative number of tornadoes per year with path lengths greater than L. The data are given for 10 year periods, between 1951 and 2010. The data during the three 10-year periods, 1981–2010, are relatively consistent and differ substantially from earlier periods. This difference can be attributed to systematic NWS tornado surveys introduced in the early 1980s. Based on Fig. 1's data, we restrict our statistical studies to the period 1981–2010.

The basic purpose of this paper has been to consider the statistics of tornado touchdown path lengths as a measure of tornado intensity. Since the standard measure of tornado intensity in the USA is the Fujita scale, we consider the variability of path lengths for a specified Fujita scale value. This dependence for our period of study, 1981–2010, was given in Fig. 2. Although there is a systematic increase in mean path length with increasing Fujita scale value, there is also a large variability. A reasonably good scaling of the mean touchdown path lengths as given in Eq. (1) was found for strong (F2, F3) and violent (F4, F5) tornadoes. The deviation from this scaling for weak (F0, F1) tornadoes is likely due to limitations of the Fujita scale for weak tornadoes and/or measurement problems with determining path lengths for these weak tornadoes. Since our studies utilize path lengths, we define a severe tornado to be one with a path length $L \geq 10\,\mathrm{km}$ and have restricted our studies to these tornadoes. From Fig. 2 and Table 1 we see that on average our definition ($L \geq 10\,\mathrm{km}$) corresponds to the size of F2 (strong) tornadoes, and where the lower-limit of the scaling relationship in Fig. 2 is found to hold.

In Fig. 5, we gave the yearly number of days n in which a severe ($L \geq 10\,\mathrm{km}$) tornado occurred and in Fig. 6, the yearly total number N_Y and yearly total path length L_Y of severe tornadoes. There are systematic increases of all three quantities (n, N_Y, L_Y) from 1981–2010, but also considerable scatter. Thus, we hesitate to attribute these increases to causes such as global warming or another external source. In Fig. 7, we plot the total number of severe tornadoes in a year, N_Y, as a function of the total path length of tornadoes in that year, L_Y. The strong linear correlation of N_Y with L_Y is evidence for a scale-invariant behaviour in the number length statistics, i.e. the ratio remains the same for all scales.

In order to study the occurrence probability of severe tornado path lengths we have given the dependence of the frequency density on path length in Fig. 8. Over the touchdown path length range $20 \leq L \leq 200$ km, we find good power-law scaling (Eq. 9) of the frequency density as a function of L. This scaling is valid over only a limited range of L but is still useful in forecasting tornado risk in terms of path length. For example, for $L = 20$ km we have $f(L) \approx 150\,\mathrm{km}^{-1}$ and for $L = 200$ km $f(L) \approx 0.15\,\mathrm{km}^{-1}$. Thus, if the touchdown path length L increases by a factor of 10, the probability of occurrence decreases by a factor of about 1000.

Tornado outbreaks are an important feature of tornado climatology. Ideally, a tornado outbreak would be associated with a particular synoptic-scale weather system. Although location information is available for each tornado path length, the association of specific tornadoes with a specified outbreak are still difficult to make in a systematic way. We follow the approach used by Doswell et al. (2006) who defines a tornado outbreak to be all tornadoes in a day in the continental USA. Consistent with our study of severe individual tornadoes with path lengths $L \geq 10$ km, we define a severe tornado outbreak to be all severe tornadoes ($L \geq 10$ km) during a day in the continental USA. As two measures of severe outbreak intensity, we utilize the number of severe tornadoes during a day, N_D, and the total path length of severe tornadoes during a day, L_D.

In Fig. 10, we find an excellent, near linear, correlation between N_D and $\overline{L_D}$. The ratio of the number of severe tornadoes to their total length, in any given year, is the same in years with many severe tornado outbreaks and in years with just a few severe tornado outbreaks. In Fig. 11, we give the frequency path length statistics of severe tornado outbreaks. We again find good agreement with a power-law distribution, given in Eq. (11). This scaling is useful in forecasting the risk of severe tornado outbreaks in terms of total path length, L_D. For example, for $L_D = 100\,\mathrm{km\,dy}^{-1}$ we have $f(L_D) \approx 2\,\mathrm{dy\,km}^{-1}$ and for $L_D = 1000\,\mathrm{km\,dy}^{-1}$, $f(L_D) \approx 0.04\,\mathrm{dy\,km}^{-1}$. Thus, if the outbreak path

length L_D increases by a factor of 10, the probability of occurrence decreases by a factor of about 50.

On the basis of our studies we believe it is desirable:

1. To restrict quantitative studies of tornadoes and tornado outbreaks to the time period subsequent to 1980.

2. To restrict tornado statistical studies based on touchdown path lengths, to path lengths $L \geq 10$ km. We term these severe tornadoes and they correspond approximately to F2 to F5 tornadoes on the Fujita scale.

3. To determine tornado frequency-size statistics utilizing touchdown path length statistics both for individual tornadoes and tornado outbreaks.

4. To use these tornado frequency-size statistics to estimate the tornado hazard. This is in direct analogy to the way (Schlelicke and Névir, 2011) that the frequency-size statistics for earthquakes are used to quantify the earthquake hazard.

In conclusion, we believe that our robust scaling results provide evidence that tornado touchdown path lengths can be used as quantitative measures of the systematic properties of severe tornadoes and severe tornado outbreaks.

List of Symbols.

Variable	Units	Description
δN	#	The number of tornadoes with lengths between L and $L+\delta L$.
f(L), f(L_D)	varies	Frequency density of L (see Eq. 8) or L_D.
h		Hour of the day.
j		Variable representing the Fujita scale value, where F0, F1, F2, F3, F4, F5 is given as Fj, $j = 0,1,2,\ldots,5$.
L	km	Individual tornado touchdown path length.
L_D	km dy^{-1}	Total touchdown path length of severe tornadoes ($L \geq 10$ km) in a day.
$\overline{L_\mathrm{D}}$	km dy^{-1}	Mean of the daily total path lengths of severe tornadoes ($L \geq 10$ km) over multiple days.
$\bar{L}_{\mathrm{F}j}$	km	Mean tornado path length for Fujita intensity Fj, $j = 0,1,2,\ldots,5$.
L_Y	km yr^{-1}	Total path length of severe tornadoes ($L \geq 10$ km) in a year.
n	dy, yr	Number of "days per year" or "years per day of the year", with at least one severe tornadoes ($L \geq 10$ km).
N_c	#	Cumulative number of continental USA tornadoes per year with path lengths greater than or equal to L.
N_D	# dy^{-1}	Total number of severe tornadoes ($L \geq 10$ km) in a day.
N_h	#	Total number of severe tornadoes ($L \geq 10$ km) initiated during hour, h.
N_T	#	Total number of values in the dataset considered.
N_Y	# yr^{-1}	Total number of severe tornadoes ($L \geq 10$ km) in a year.
p(h)		Probability of a severe tornado occurring for a given hour of the day, h.
t	yr	Time in years.

References

Brooks, H. E.: On the relationship of tornado path length and width to intensity, Weather Forecast., 19, 310–319, 2004.

Doswell, C. A., Edwards, R., Thompson, R. L., Hart, J. A., and Crosbie, K. C.: A simple and flexible method for ranking severe weather events, Weather Forecast., 21, 939–951, 2006.

Doswell, C. A., Brooks, H. E., and Dotzek, N.: On the implementation of the Enhanced Fujita scale in the USA, Atmos. Res., 93, 554–563, 2009.

Fujita, T. T.: Proposed characterization of tornadoes and hurricanes by area and intensity. SMRP Research Paper 91, Dept. Geophys. Sciences, Univ. of Chicago, 42 pp., 1971.

Fujita, T. T.: Tornadoes and downbursts in the context of generalized planetary scales, J. Atmos. Sci., 38, 1511–1534, 1981.

Fujita, T. T. and Pearson, A. D.: Results of FPP classification of 1971 and 1972 tornadoes, Preprints, Eighth Conf. on Severe Local Storms, Denver, CO, Amer. Meteor. Soc., 142–145, 1973.

Glickman, T. S. (eds.): Glossary of Meteorology, 2nd edn., Amer. Meteor. Soc., 782 pp., 2000.

Hanks, T. C. and Kanamori, H.: A moment magnitude scale, J. Geophys. Res., 84, 2348–2350, 1979.

Kelly, D. L., Schaefer, J. T., McNulty, R. P., Doswell, C. A., and Abbey Jr., R. F.: An augmented tornado climatology, Mon. Weather Rev., 106, 1172–1183, 1978.

McCarthy, D. W.: NWS tornado surveys and the impact on the national tornado database, First Symposium on F-Scale and Severe Weather Damage Assessment, Long Beach, CA, Amer. Meteor. Soc., preprint 3.2, 2003.

NOAA (National Oceanic and Atmospheric Administration): Storm Prediction Centre (SPC), Tornado, Hail, and Wind Database, available online at: www.spc.noaa.gov/wcm/, last access: 24 December 2011.

Richter, C.: An instrumental earthquake magnitude scale, Bull. Seism. Soc. Amer., 25, 1–32, 1935.

Schaefer, J. T., Schneider, R. S., and Kay, M. P.: The robustness of tornado hazard estimates, Third Symposium on Environmental Applications, Orlando, FL, Amer. Meteor. Soc., paper 4.1, 35–41, 2002.

Schielicke, L. and Névir, P.: Introduction of an atmospheric moment combining Eulerian and Lagrangian aspects of vortices: Application to tornadoes, Atmos. Res., 100, 357–365,

doi:10.1016/j.atmosres.2010.08.027, 2011.
Verbout, S. M., Brooks, H. E., Leslie, L. M., and Schultz, D. M.: Evolution of the U.S. tornado database: 1954–2003, Weather Forecast., 21, 86–93, 2006.

Table 1. Continental USA tornado touchdown path lengths L as a function of Fujita scale intensities Fj, $j = 0, 1, 2, \ldots, 5$. (i) The range of path lengths L (km) given by Fujita and Pearson (1973). (ii) Mean tornado path lengths $\bar{L}_{\mathrm{F}j}$ in the continental USA given by Brooks (2004) and in this paper (Fig. 2), with all path lengths of L considered.

Fujita Intensity	Fujita and Pearson (1973) Range of tornado path lengths L (km)	Brooks (2004) 1950–2001 Mean tornado path length $\bar{L}_{\mathrm{F}j}$ (km)	This paper 1981–2010 Mean tornado path length $\bar{L}_{\mathrm{F}j}$ (km)
F0	0.5–1.5	1.4	1.6
F1	1.6–5.0	4.7	5.2
F2	5.1–15.9	10.7	11.6
F3	16.0–50	22.5	24.5
F4	51–159	43.6	40.8
F5	160–500	54.6	54.6

Table 2. Number and percentage of continental USA tornado path lengths from 1981–2010, $L < 10$ km and $L \geq 10$ km (i.e., "severe" tornadoes as defined in this paper), as a function of Fujita scale intensities F. Data from NOAA (2011).

Fujita Intensity	Tornadoes with $L < 10$ km # (% in Fujita category)	"Severe" tornadoes with $L \geq 10$ km # (% in Fujita category)	All tornadoes ($L > 0$ km) # (% in Fujita category)
F0	18 763 (97 %)	500 (3 %)	19 263 (100 %)
F1	8375 (85 %)	1496 (15 %)	9871 (100 %)
F2	1917 (61 %)	1244 (39 %)	3161 (100 %)
F3	272 (30 %)	637 (70 %)	909 (100 %)
F4	29 (15 %)	170 (85 %)	199 (100 %)
F5	1 (7 %)	14 (93 %)	15 (100 %)
Total	29 357 (88 %)	4061 (12 %)	33 418 (100 %)

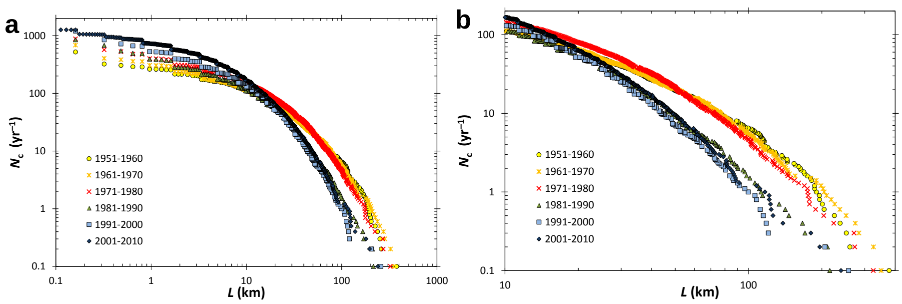

Fig. 1. Cumulative number of continental USA tornadoes per year N_c with path length greater than or equal to L are given as a function of L. Data are given for six 10-year periods from 1951–2010. **(a)** All tornadoes. **(b)** Data for $L \geq 10$ km, which we define to be severe tornadoes. Tornado path length data from NOAA (2011).

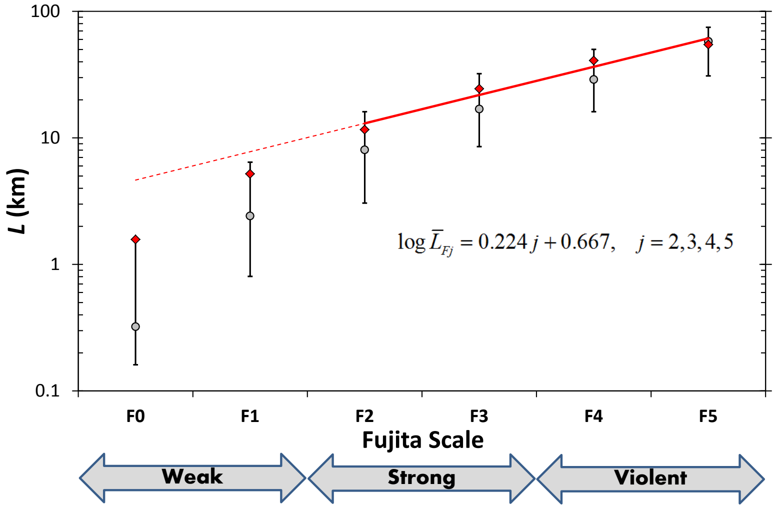

Fig. 2. Continental USA tornado touchdown path length statistics as a function of Fujita scale values F0, F1, ..., F5, for the time period 1981–2010, with all path lengths L considered. Included are the mean path lengths $\bar{L}_{Fj}$ (red diamonds) for each Fujita scale value (j = 0,1,2,...,5), median values (grey circles), and the 75th and 25th percentile (upper and lower horizontal lines). Also given (thick red line) is the best-fit to the mean values for strong (F2, F3) and violent (F4, F5) tornadoes (Eq. 1).

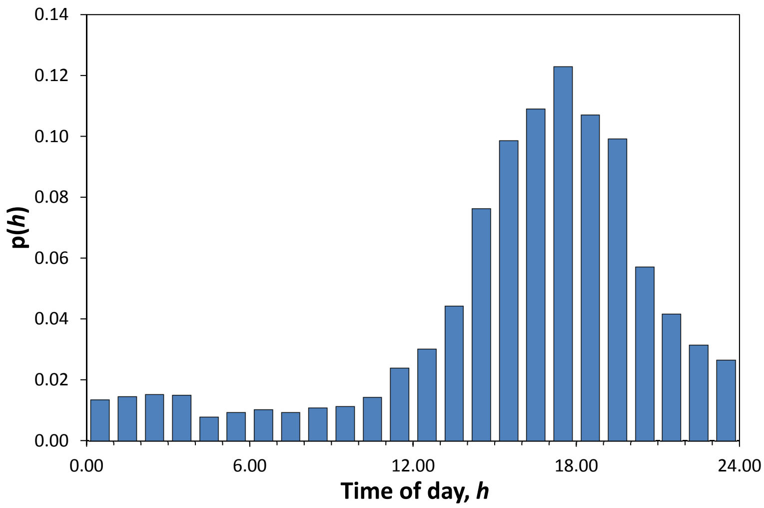

Fig. 3. Histogram of the distribution of continental USA severe tornadoes ($L \geq 10$ km) as a function of the hour of the day, h (Central Standard Time). The probabilities p(h) of a severe tornado occurring are given as a function of h for the time period 1981–2010.

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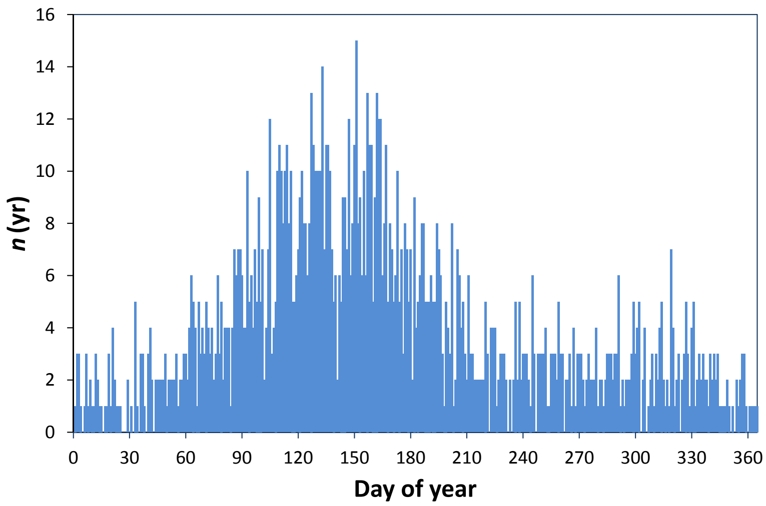

Fig. 4. Distribution of continental USA severe tornadoes ($L \geq 10$ km) as a function of day of the year. The number of years n with at least one severe tornado ($L \geq 10$ km) is given for each day of the year, 1 to 365 (leap day removed), for the 30-year period 1981–2010.

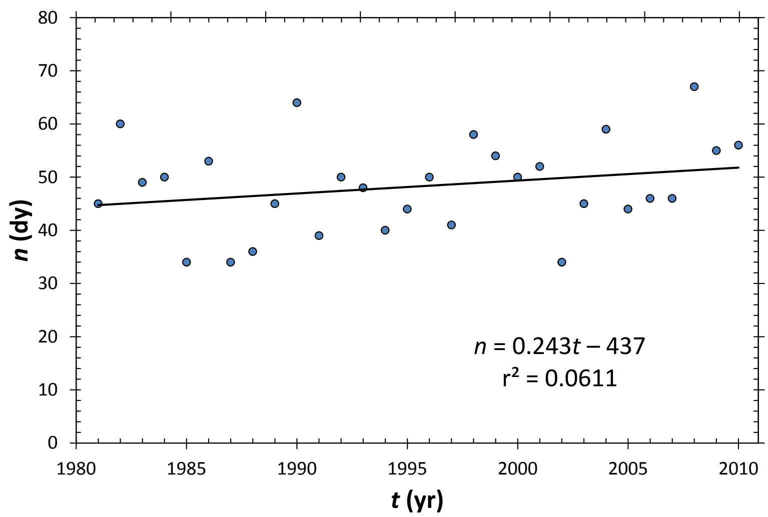

Fig. 5. Number of days per year n with at least one continental USA severe tornado with path length $L \geq 10$ km is given for the time period 1981–2010. The best-fit linear correlation is also given (Eq. 4).

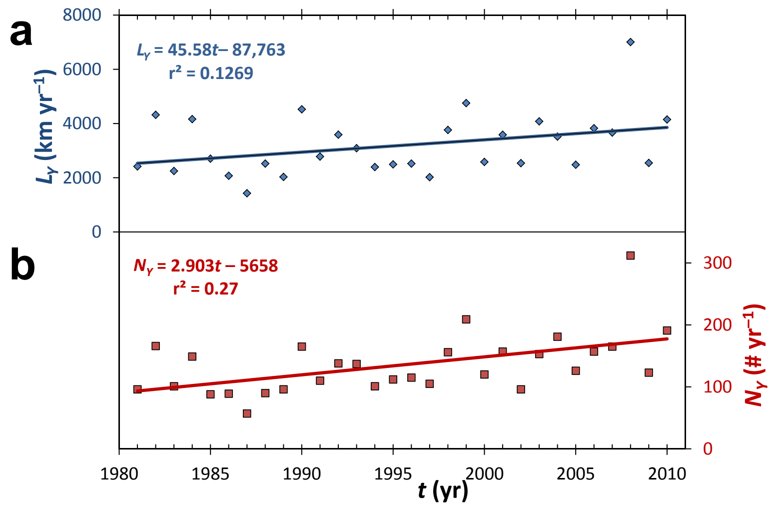

Fig. 6. For continental USA severe tornadoes ($L \geq 10$ km), **(a)** the number per year, N_{Y}, and **(b)** total path length per year, L_{Y}, are given for the time period 1981–2010. In both cases the best-fit linear correlation is shown (Eqs. 5 and 6).

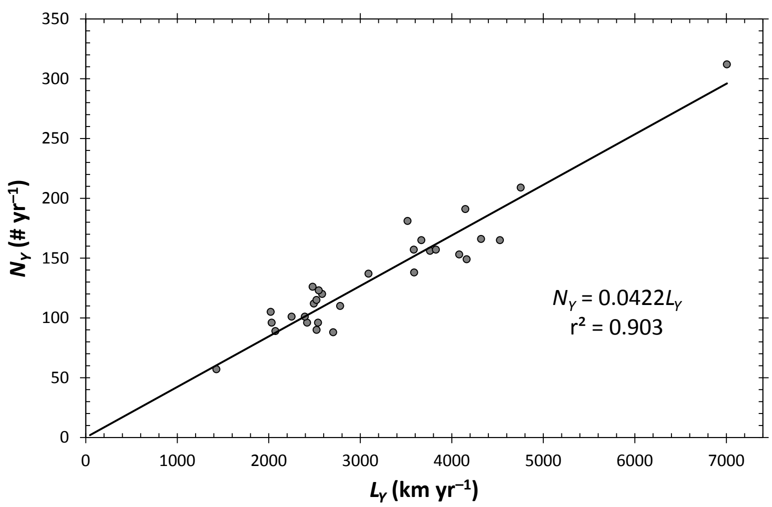

Fig. 7. For continental USA severe tornadoes ($L \geq 10$ km), 1981–2010, the number in a given year, N_{Y}, is given as a function of the total path length in that year, L_{Y}. The best-fit linear correlation of the data is also given (Eq. 7).

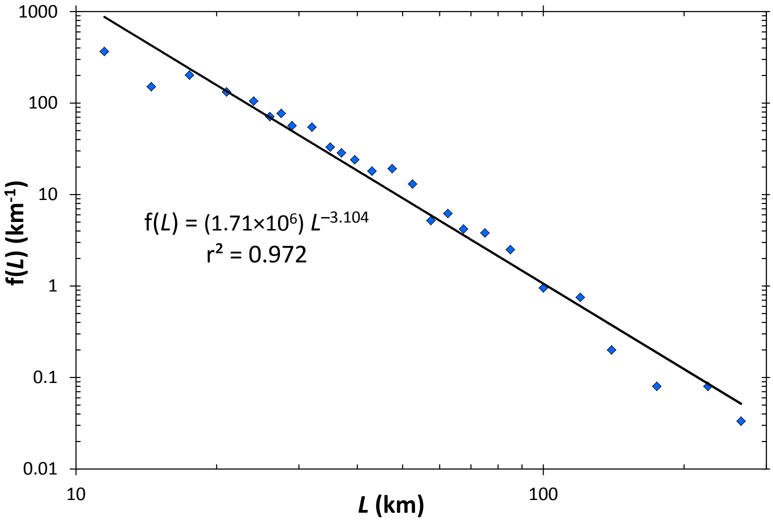

Fig. 8. For continental USA severe tornadoes ($L \geq 10$ km), 1981–2010, the frequency densities f(L) are given as a function of path length L. The best-fit power-law correlation of the data is also given (Eq. 9).

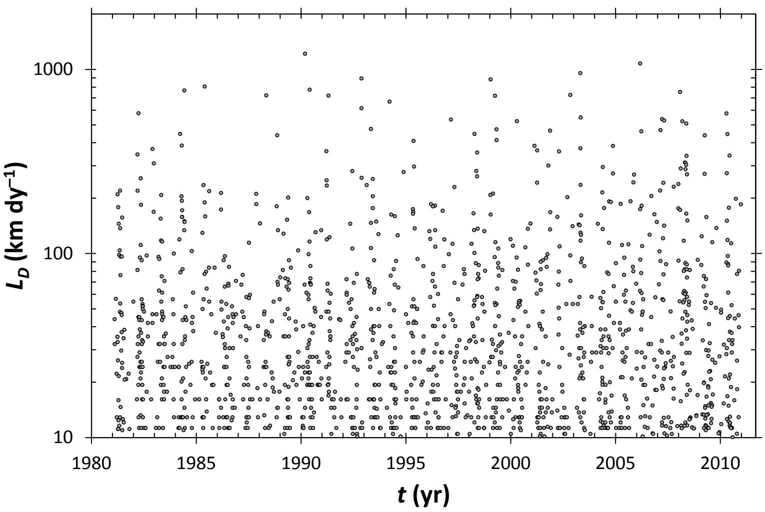

Fig. 9. For continental USA severe tornadoes ($L \geq 10$ km), the daily total path lengths, L_D, are given for the time period 1981–2010. Each L_D represents a quantitative measure of a USA daily "outbreak" of tornadoes.

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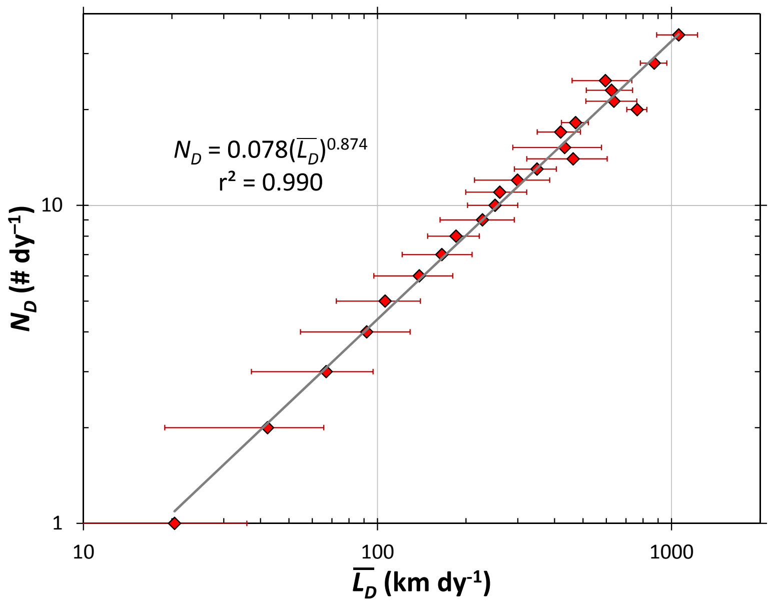

Fig. 10. The number of continental USA severe tornadoes ($L \geq 10$ km), 1981–2010, in USA daily "outbreaks" with N_D, is given as a function of the mean of the daily total path lengths $\overline{L}_D$. Horizontal error bars represent ±1 s.d. (standard deviation) of the L_D for a given N_D. The best-fit power-law correlation of the data is also given (Eq. 13).

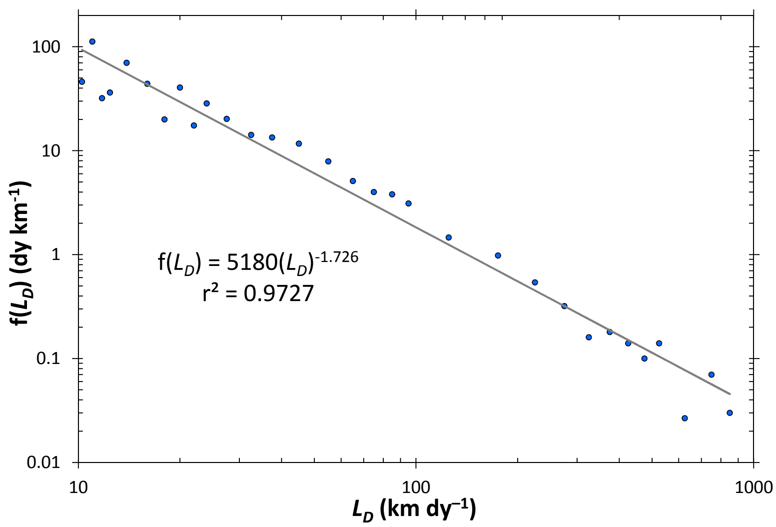

Fig. 11. The frequency-length statistics of continental USA daily tornado outbreaks during the period 1981–2010. The frequency densities $f(L_D)$ are given as a function of L_D, the total path length of all tornadoes during a USA daily outbreak. The best-fit power-law correlation of the data is also given (Eq. 14).

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