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Interactive comment on "Quasi-geostrophic turbulence and generalized scale invariance, a theoretical reply" *by* D. Schertzer et al.

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In their reply to Lindborg et al. (2010), Schertzer et al. (2011), STLT hereafter, considered only an asymmetry between the horizontal and the vertical. It is therefore useful, both for the derivation and the expression of the fractional vorticity equations (Eqs.30 of STLT), to decompose the fields and operators into horizontal and vertical components (with respective indices h and v), e.g. for the velocity field $\vec{u}$ and gradient operator $\vec{\nabla}$:

$$\vec{u} = \vec{u}_h + \vec{u}_v;\ \vec{\nabla} = \vec{\nabla}_h + \vec{\nabla}_v \tag{1}$$

rather than with respect to a three-dimensional basis $(\vec{e}_1, \vec{e}_2, \vec{e}_3)$, e.g. :

$$\vec{u} = (u^1, u^2, u^3);\ \vec{\nabla} = (\partial_1, \partial_2, \partial_3); \tag{2}$$

as done by STLT. With the help of the pullback transform T^*_λ (see Eqs. 21-22 of STLT) and of the scaling anisotropy exponent h ($0 \leqslant h \leqslant 1$, Eqs. 24-26 of STLT), one obtains the following scaling for the velocity field $\vec{u}$ and gradient operator $\vec{\nabla}$:

$$T^*_\lambda(\vec{u}) = \lambda^\gamma(\vec{u}_h + \lambda^h \vec{u}_v);\ T^*_\lambda(\vec{\nabla}) = \lambda(\vec{\nabla}_h + \lambda^{-h}\vec{\nabla}_v) \tag{3}$$

Due to the bilinearity of the vorticity with respect to the gradient operator and the velocity field, one readily obtains the following decomposition for the vorticity field:

$$\vec{\omega} = \vec{\omega}_v + \vec{\sigma} + \vec{\tau};\ \vec{\omega}_v \equiv \vec{\nabla}_h \times \vec{u}_h;\ \vec{\sigma} \equiv \vec{\nabla}_h \times \vec{u}_v;\ \vec{\tau} \equiv \vec{\nabla}_v \times \vec{u}_h \tag{4}$$

where $\vec{\omega}_h = \vec{\sigma} + \vec{\tau}$ is the horizontal vorticity. However, both $\vec{\sigma}$ and $\vec{\tau}$ are needed, because they exhibit a different scaling (as confirmed below). For comparison with equations of STLT, $\vec{\omega}_v$, $\vec{\sigma}$ and $\vec{\tau}$ can be expressed with respect to the basis $(\vec{e}_1, \vec{e}_2, \vec{e}_3)$:

$$\vec{\omega}_v = (\partial_1 u^2 - \partial_2 u^1)\vec{e}_3;\ \vec{\sigma} = \partial_2 u^3 \vec{e}_1 - \partial_1 u^3 \vec{e}_2;\ \vec{\tau} = -\partial_3 u^2 \vec{e}_1 + \partial_3 u^1 \vec{e}_2 \tag{5}$$

It is straightforward to obtain their respective scaling from Eqs.3- 4 :

$$T^*_\lambda(\vec{\omega}_v) = \lambda^{1+\gamma}\vec{\omega}_v;\ T^*_\lambda(\vec{\sigma}) = \lambda^{1+\gamma+h}\vec{\sigma};\ T^*_\lambda(\vec{\tau}) = \lambda^{1+\gamma-h}\vec{\tau} \tag{6}$$

The scaling of the Lagrangian derivative (Eq. 27 of STLT):

$$T^*_\lambda(D/Dt) = \lambda^{\gamma+1} D/Dt \tag{7}$$

yields the following scaling for the Lagrangian evolution of the vorticity:

$$T^*_\lambda(D\vec{\omega}/Dt) = \lambda^{2(1+\gamma)}(\lambda^h D\vec{\sigma}/Dt + \lambda^{-h} D\vec{\tau}/Dt + D\vec{\omega}_v/Dt) \tag{8}$$

On the other hand, the stretching vector has the following decomposition:

$$\vec{s} = ((\vec{\sigma} + \vec{\tau}) \cdot \vec{\nabla}_h + \vec{\omega}_v \cdot \vec{\nabla}_v)(\vec{u}_h + \vec{u}_v) \tag{9}$$

that yields the following scaling ($(\vec{\sigma} \cdot \vec{\nabla}_h)\vec{u}_v = 0$ due to the asymmetry of $\vec{\sigma}$) :

$$T^*_\lambda(\vec{s}) = \lambda^{2(1+\gamma)}[\lambda^h(\vec{\sigma} \cdot \vec{\nabla}_h)\vec{u}_h + \lambda^{-h}(\vec{\tau} \cdot \vec{\nabla}_h + \vec{\omega}_v \cdot \vec{\nabla}_v)(\vec{u}_h + \lambda^h \vec{u}_v)] \tag{10}$$

One therefore obtains the following dynamical equations from Eqs.8,10:

$$D\vec{\sigma}/Dt = (\vec{\sigma} \cdot \vec{\nabla}_h)\vec{u}_h \tag{11}$$

$$D\vec{\tau}/Dt = (\vec{\tau} \cdot \vec{\nabla}_h + \vec{\omega}_v \cdot \vec{\nabla}_v)\vec{u}_h \tag{12}$$

$$D\vec{\omega}_v/Dt = (\vec{\tau} \cdot \vec{\nabla}_h + \vec{\omega}_v \cdot \vec{\nabla}_v)\vec{u}_v \tag{13}$$

which are identical to Eqs. 30 of STLT, but already in a more compact form:

$$D\partial_2 u^3/Dt = (\partial_2 u^3 \partial_1 - \partial_1 u^3 \partial_2)u^1;\ -D\partial_1 u^3/Dt = (\partial_2 u^3 \partial_1 - \partial_1 u^3 \partial_2)u^2 \tag{14}$$

$$-D\partial_3 u^2/Dt = (\partial_1 u^2 \partial_3 - \partial_3 u^2 \partial_1)u^1;\ D\partial_3 u^1/Dt = (\partial_3 u^1 \partial_2 - \partial_2 u^1 \partial_3)u^2 \tag{15}$$

$$D(\partial_1 u^2 - \partial_2 u^1)/Dt = (\partial_1 u^2 \partial_3 - \partial_3 u^2 \partial_1 + \partial_3 u^3 \partial_2 u^3 - \partial_2 u^1 \partial_3)u^3 \tag{16}$$

Equations 11-13, together with the large scale boundary condition $\vec{\omega}_v \simeq 2\Omega$ (Ω being the Earth's angular velocity), have several interesting features to be investigated. This includes the fact that Eq.11 formally looks like a vorticity equation for $\vec{\sigma}$, although the latter is not (and could not be!) the curl of the horizontal velocity field $\vec{u}_h$ (see Eq.4). Nevertheless, it allows a nonlinear growth of this horizontal vorticity component by quadratic interactions with horizontal gradients of the horizontal velocity field.

Lindborg, E., Tung, K. K., Nastrom, G. D., Cho, J. Y. N., and Gage, K. S.: Comment on "Reinterpreting aircraft measurement in anisotropic scaling turbulence" by Lovejoy et al. (2009), Atmos. Chem. Phys., 10, 1401–1402, doi:10.5194/acp-10-1401-2010, 2010.

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