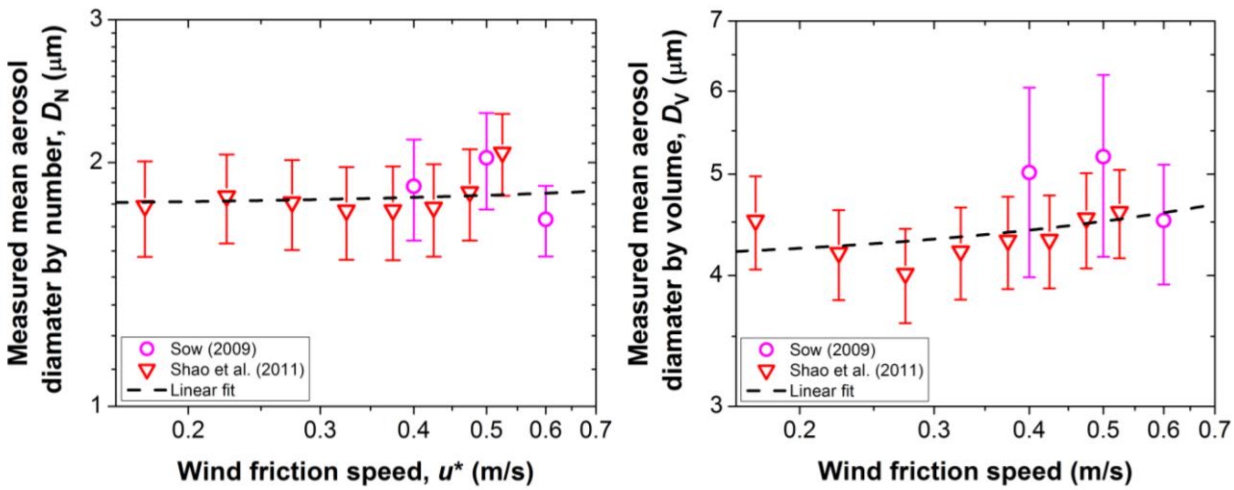
Supplementary material for "Does the size distribution of mineral dust aerosols depend on the wind speed at emission?"



Supplementary figure S1. As Figs. 2a and 2b, except with D_{low} = 0.6 μm and D_{up} = 8.4 μm, which spans the measured dust aerosol diameter range of the most recent data sets of Sow et al. (2009) and Shao et al. (2011). The dashed line denotes the linear fit to both these data sets. The trends of D_V and D_N with u* are statistically insignificant (i.e., the slope is within one standard error from zero) for both the individual data sets and both data sets combined.

Supplementary text

This supplementary document describes the methods used to calculate both the mean dust aerosol diameters by number (D_N) and volume (D_V) and their uncertainties, as well as the trend of D_N and D_V with $u*$ and its uncertainty.

As noted in the main text, the limits on the integration in Eq. (1) are determined by the highest lower limit on the measured aerosol diameters among data sets, which is 1.2 μm for the Gillette (1974) and Gillette et al. (1974) data sets, and the lowest upper limit, which is 8.4 μm for the Shao et al. (2011) data set. However, bins in other data sets that extend past these size limits must be corrected by truncating those bins. I do so by assuming that the sub-bin distribution follows the power law found by both Gillette et al. (1974) and Kok (2011) for dust aerosols in the ~2 – 10 μm diameter range (i.e., $dN/d\log D \sim D^{-2}$ and $dV/d\log D \sim D$). Specifically, we have that

$$N_i = N_i^{'} \frac{D_i^{'2}}{D_i^2}, \; V_i = V_i^{'} \frac{D_i}{D_i^{'}}, \tag{S1}$$

where $D_i^{'}$ is the geometrically averaged aerosol diameter for bin i, and D_i is the particle bin diameter after truncating bin i to either start at 1.2 μm, for a bin at the lower size range, or to end at 8.4 μm, for a bin at the upper size range. Correspondingly, $N_i^{'}$ and $V_i^{'}$ are respectively the measured number ($dN/d\log D$) and volume ($dV/d\log D$) of dust per logarithmically space bin i, and N_i and V_i are the corrected values after truncating the bins.

After truncating the bins and correcting N_i and V_i according to Eq. (S1), the mean aerosol diameter by number is calculated using Eq. (1) in the main text. Specifically,

$$D_N = \frac{\int_{D_{low}}^{D_{up}} D \frac{dN}{dD} dD}{\int_{D_{low}}^{D_{up}} \frac{dN}{dD} dD} = \frac{\sum_i \int_{D_{i-}}^{D_{i+}} \frac{dN}{d\log D} dD}{\ln(10) N_{tot}}, \tag{S2}$$

where i sums over all bins within the size range spanned by D_{low} and D_{up}, and D_{i-} and D_{i+} are the lower and upper size limits on bin i. Moreover, I used that

$$\frac{dN}{dD} = \frac{1}{D \ln 10} \frac{dN}{d\log D}, \tag{S3}$$

and defined N_{tot} as

$$N_{tot} = \int_{D_{low}}^{D_{up}} \frac{dN}{dD} dD. \tag{S4}$$

By again assuming that the sub-bin distribution follows $dN/d\log D \sim D^{-2}$ (Gillette et al., 1974; Kok, 2011) and using the notation of Eq. (S1), I obtain that

$$D_N = \frac{\sum_i \int_{D_{i-}}^{D_{i+}} N_i \frac{D_i^2}{D^2} dD}{\ln(10) N_{tot}} = \frac{\sum_i N_i \left(D_{i+} - D_{i-}\right)}{\ln(10) N_{tot}}. \tag{S5}$$

N_{tot} is now evaluated in a similar manner,

$$N_{\mathrm{tot}} = \int_{D_{\mathrm{low}}}^{D_{\mathrm{up}}} \frac{dN}{dD}\, dD = \sum_i \int_{D_{i-}}^{D_{i+}} \frac{N_i}{D \ln 10} \frac{D_i^2}{D^2}\, dD = \sum_i \frac{N_i}{\ln 10} \left(\frac{D_{i+}^2 - D_{i-}^2}{2 D_{i+} D_{i-}} \right), \tag{S6}$$

and combining this with Eq. (S5) then yields the final expression for D_{N}

$$D_{\mathrm{N}} = \frac{\sum_i N_i \left(D_{i+} - D_{i-} \right)}{\sum_j N_j \left(\frac{D_{j+}^2 - D_{j-}^2}{2 D_{j+} D_{j-}} \right)}. \tag{S7}$$

The mean aerosol diameter by volume can be evaluated using a similar procedure, which yields

$$D_{\mathrm{V}} = \frac{\sum_i V_i \left(\frac{D_{i+}^2 - D_{i-}^2}{2 D_i} \right)}{\sum_j V_j \left(\frac{D_{j+} - D_{j-}}{D_j} \right)}. \tag{S8}$$

Calculating the uncertainty on D_{N} and D_{V}

In order to assess whether the trend in the mean aerosol diameter with the friction speed u^* is statistically significant, it is necessary to determine the uncertainties on the calculations of D_{N} and D_{V}. Using standard error propagation (Bevington and Robinson, 2003), the errors in D_{N} and D_{V} due to uncertainties in the size distribution measurements N_i and V_i are:

$$\sigma_{D_{\mathrm{N}}}^2 = \sum_i \sigma_{N_i}^2 \left(\frac{\partial D_{\mathrm{N}}}{\partial N_i} \right)^2 \text{, and} \tag{S9}$$

$$\sigma_{D_{\mathrm{V}}}^2 = \sum_i \sigma_{V_i}^2 \left(\frac{\partial D_{\mathrm{V}}}{\partial V_i} \right)^2 , \tag{S10}$$

where we obtain from Eqs. (S7) and (S8) that

$$\frac{\partial D_{\mathrm{N}}}{\partial N_i} = -\frac{D_{i+} - D_{i-}}{\sum_j N_j \left(\frac{D_{j+}^2 - D_{j-}^2}{2 D_{j+} D_{j-}} \right)} - \frac{\left(\frac{D_{i+}^2 - D_{i-}^2}{2 D_{i+} D_{i-}} \right) \sum_j N_j \left(D_{j+} - D_{j-} \right)}{\left[\sum_j N_j \left(\frac{D_{j+}^2 - D_{j-}^2}{2 D_{j+} D_{j-}} \right) \right]^2} \text{, and} \tag{S11}$$

$$\frac{\partial D_{\mathrm{V}}}{\partial V_i} = -\frac{\left(\frac{D_{i+}^2 - D_{i-}^2}{2 D_i} \right)}{\sum_j V_j \left(\frac{D_{j+} - D_{j-}}{D_j} \right)} - \frac{\left(\frac{D_{i+} - D_{i-}}{D_i} \right) \sum_j V_j \left(\frac{D_{j+}^2 - D_{j-}^2}{2 D_j} \right)}{\left[\sum_j V_j \left(\frac{D_{j+} - D_{j-}}{D_j} \right) \right]^2}. \tag{S12}$$

Eqs. (S9) – (S12) thus allow the computation of the uncertainty in D_N and D_V from the uncertainties in the measurements of the number flux N_i and the volume flux V_i. Note, however, that these dust flux measurements are themselves computed from measurements of the number and volume concentration at two different height. Gillette et al. (1972, p. 978) showed that the number flux of dust aerosols can be approximated as

$$N_i = \frac{-Cu_1^2(n_2 - n_1)}{u_2 - u_1} \equiv -c(n_2 - n_1), \tag{S13}$$

where n_1 and n_2 are the measured number concentrations at the upper and lower measurement heights, u_1 and u_2 are the corresponding wind speeds during the measurement, and the proportionality constant C depends on the air density, the drag coefficient, and the bin size. A similar equation can be derived for the volume flux. Since not all studies reported the uncertainty on their dust flux measurements in a consistent manner, I describe below how the uncertainties in N_i and V_i were determined for each data set individually.

The study of Gillette et al. (1974) determined the error in N_i from propagation of the measurement uncertainty of the two dust concentration measurements (see Eq. (S13)) into N_i (see Figure 6 in Gillette et al., 1974). They determined this uncertainty on a dust concentration measurement through parallel operation of two identical sets of instruments (i.e., jet impactors) in the field (see their Table 1 and accompanying text). For the error analysis of the Gillette et al. (1974) data set, I thus used the reported errors on the fluxes in Figure 6 of Gillette et al. (1974).

Although the methodology of Gillette (1974) was similar to that of Gillette et al. (1974), the former study only reported dust concentration measurements at two separate heights, and did not compute the corresponding fluxes using Eq. (S13), let alone their uncertainties. As in Kok (2011), Eq. (S13) was thus used to calculate N_i for each particle size bin, soil, and value of $u*$ for which dust concentration measurements were reported in Figures 1–3 of Gillette (1974). I then calculated the corresponding uncertainty on N_i by using the relative errors on individual concentration measurements from Table 1 in *Gillette et al.* (1974), and propagating those into N_i using Eq. (S13). That is,

$$\sigma_{N_i}^2 = \sigma_{n_1}^2\left(\frac{\partial N_i}{\partial n_1}\right) + \sigma_{n_2}^2\left(\frac{\partial N_i}{\partial n_2}\right) = c^2\left(\sigma_{n_1}^2 + \sigma_{n_2}^2\right), \tag{S14}$$

such that the relative standard error is

$$\frac{\sigma_{N_i}}{N_i} = \frac{\sqrt{\sigma_{n_1}^2 + \sigma_{n_2}^2}}{n_1 - n_2} = r_i\,\frac{\sqrt{n_1^2 + n_2^2}}{n_1 - n_2}, \tag{S15}$$

where r_i is the relative error for the relevant particle size bin, taken from Table 1 of *Gillette et al.* (1974). Equations similar to Eqs. (S13) – (S15) were used to determine the uncertainty on measurements of V_i.

The error analyses for the other data sets used in this article are more straightforward. Although no uncertainties were reported in *Alfaro et al.* (1998), the highly similar study of *Alfaro et al.* (1997) note a relative uncertainty of ~5% for their measurements (p. 11,244). I thus assume that this relative uncertainty also applies to the study of Alfaro et al. (1998), which uses the same methodology. The *Sow et al.* (2009) study does report the uncertainties on their N_i measurements (see their Fig. 9). In contrast, Shao et al. (2011) does not report measurement uncertainties. However, since both Shao et al. and Sow et al. used an optical particle counter, it seems reasonable to assume that the relative uncertainty of the Shao et al. measurements are similar to those of the Sow et al. measurements. That is, I assumed a relative uncertainty of 55% for the Shao et al. measurements.

Calculating the trend of D_N and D_V with $u*$ and its uncertainty

The trend of D_N with $u*$ and its uncertainty are calculated using standard linear least squares analysis (p. 98-115, Bevington and Robinson, 2003). That is,

$$a = \frac{1}{\Delta}\left(\sum_i \frac{u_i^{*2}}{\sigma^2_{D_N,i}} \sum_i \frac{D_{N,i}}{\sigma^2_{D_N,i}} - \sum_i \frac{u_i^*}{\sigma^2_{D_N,i}} \sum_i \frac{D_{N,i} u_i^*}{\sigma^2_{D_N,i}}\right), \tag{S16}$$

$$b = \frac{1}{\Delta}\left(\sum_i \frac{1}{\sigma^2_{D_N,i}} \sum_i \frac{D_{N,i} u_i^*}{\sigma^2_{D_N,i}} - \sum_i \frac{u_i^{*2}}{\sigma^2_{D_N,i}} \sum_i \frac{D_{N,i}}{\sigma^2_{D_N,i}}\right), \tag{S17}$$

$$\sigma_a^2 = \frac{1}{\Delta}\sum_i \frac{u_i^*}{\sigma^2_{D_N,i}}, \text{ and} \tag{S18}$$

$$\sigma_b^2 = \frac{1}{\Delta}\sum_i \frac{1}{\sigma^2_{D_N,i}}, \tag{S19}$$

where i sums over all measurements of D_N and their uncertainty, and

$$\Delta = \sum_i \frac{1}{\sigma^2_{D_N,i}} \sum_i \frac{u_i^{*2}}{\sigma^2_{D_N,i}} - \left(\sum_i \frac{u_i^*}{\sigma^2_{D_N,i}}\right)^2, \tag{S20}$$

and where the trend is defined by

$$y = a + bu*. \tag{S21}$$

The uncertainty at a given point on the fitted line can be derived from error propagation of Eq. (S21),

$$\sigma_y = \sqrt{\sigma_a^2\left(\frac{\partial y}{\partial a}\right)^2 + \sigma_b^2\left(\frac{\partial y}{\partial b}\right)^2 + 2\sigma_{ab}^2\left(\frac{\partial y}{\partial a}\right)\left(\frac{\partial y}{\partial b}\right)} \tag{S22}$$

$$= \sqrt{\sigma_a^2 + \sigma_b^2 u*^2 + 2\sigma_{ab}^2 u*},$$

where the covariance σ_{ab}^2 is defined as (p. 123, Bevington and Robinson, 2003)

$$\sigma_{ab}^2 = \sum_i \sigma^2_{D_N,i} \frac{\partial a}{\partial D_{N,i}} \frac{\partial b}{\partial D_{N,i}}. \tag{S23}$$

The partial derivatives in Eq. (S23) quantify the dependence of the parameters a and b to the value $D_{N,i}$ of each individual measurement. These partial derivatives are defined as (p. 109, Bevington and Robinson, 2003)

$$\frac{\partial a}{\partial D_{\mathrm{N},j}} = \frac{1}{\Delta\sigma^2_{D_\mathrm{N},j}}\left(\sum_i \frac{u_i^{*2}}{\sigma^2_{D_\mathrm{N},i}} - u_j^*\sum_i \frac{u_i^*}{\sigma^2_{D_\mathrm{N},i}}\right), \text{ and} \tag{S24}$$

$$\frac{\partial b}{\partial D_{\mathrm{N},j}} = \frac{1}{\Delta\sigma^2_{D_\mathrm{N},j}}\left(u_j^*\sum_i \frac{1}{\sigma^2_{D_\mathrm{N},i}} - \sum_i \frac{u_i^*}{\sigma^2_{D_\mathrm{N},i}}\right). \tag{S25}$$

Note that the above equations do not account for uncertainties in the measurement of u^*, which are generally less than 10 % (Namikas et al., 2003) and are thus not expected to substantially affect the trends of D_V and D_N with u^*.

Equations similar to Eqs. (S16) – (S25) are used to determine the linear fit of D_V with u^* and its uncertainty.

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