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Technical Note: Simple analytical relationships between Ångström coefficients of aerosol extinction, scattering, absorption, and single scattering albedo

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Abstract

Ångström coefficients are commonly used to parameterize the slow wavelength dependence of aerosol scattering, absorption, and extinction coefficients and single scattering albedo. Here we introduce simple analytical relationships between these coefficients that establish a framework for intercomparison between theory and experimental results from different instruments and platforms and allow for closure studies and improved physical understanding.

1 Introduction

The Ångström coefficient AC was originally introduced as a wavelength-independent constant in a power law to describe wavelength-dependent extinction (or optical depth) of light by aerosols (Ångström, 1929). Since then, it has found additional extensive use in characterizing the "slow" wavelength dependence of scattering, absorption, and single scattering albedo (SSA) (Russell et al., 2010; Fischer et al., 2010; Virkkula et al., 2005; Flowers et al., 2010). ACs are generally not considered appropriate for "fast" wavelength dependences as encountered, for example, in the fast oscillations of scattering coefficients for individual, non-absorbing, spherical particles (Eversole et al., 1993). Symbols used in this paper are summarized in Table 1.

While simple analytical relationships between extinction, scattering, and absorption coefficients and SSA exist (e.g., Moosmüller et al., 2009), we are not aware of corresponding relationships for ACs. Such relationships are useful to compare ACs obtained from extinction, scattering, and absorption, including the ground truthing of remote sensing and satellite measurements. For example, aerosol optical depth (path-integrated extinction) can be obtained from ground-based and satellite remote sensing at multiple wavelengths yielding extinction Ångström coefficients EACs. Simple analytical relationships between EACs, scattering Ångström coefficients SACs, and absorption Ångström coefficients AACs, will help attributing the EACs to the underlying

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physical phenomena, namely scattering and absorption and analyzing closure between the different Ångström coefficients. In addition, SSA is the key parameter for obtaining the sign and magnitude of aerosol radiative forcing. SSA can be obtained at multiple wavelengths from in-situ measurements (Lewis et al., 2008; Virkkula et al., 2005; Flowers et al., 2010), ground-based remote sensing measurements (Dubovik et al., 1998; Dubovik and King, 2000), and potentially from satellite measurements (Mishchenko et al., 2007; Zhu et al., 2011). Relating the SSA Ångström coefficient SSAAC to the underlying SAC, AAC, and EAC will help with data interpretation and closure and physical understanding.

Ångström coefficients can be used to express the dependence of any parameter $p(\lambda)$ on wavelength λ, provided $p(\lambda)$ can be approximated by a power-law function of wavelength. Conventionally, the two-wavelength AC is used to give the ratio of $p(\lambda)$ at two wavelengths λ_1 and λ_2 as function of the ratio of these wavelengths as (Moosmüller et al., 2009)

$$\frac{p(\lambda_1)}{p(\lambda_2)} = \left(\frac{\lambda_1}{\lambda_2}\right)^{-\mathrm{AC}}. \tag{1a}$$

The AC can be written explicitly as

$$\mathrm{AC}(\lambda_1,\lambda_2) = -\frac{\ln(p(\lambda_1)) - \ln(p(\lambda_2))}{\ln(\lambda_1) - \ln(\lambda_2)}, \tag{1b}$$

which is the negative slope of $p(\lambda)$ between wavelengths λ_1 and λ_2 on a log-log plot (Moosmüller et al., 2011). Such a slope obtained at a single wavelength λ defines the single-wavelength AC(λ) as (Moosmüller et al., 2011)

$$\mathrm{AC}(\lambda) = -\frac{\lambda}{p(\lambda)}\frac{dp(\lambda)}{d\lambda}. \tag{1c}$$

In aerosol optics, ACs are of interest for scattering, absorption, and extinction coefficients and for the single scattering albedo (SSA) ϖ. The relationships between these ACs can be investigated using Eq. (1c).

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2 Extinction Ångström coefficient EAC

The extinction coefficient γ is defined as the sum of absorption coefficient α and scattering coefficient β as (e.g., Moosmüller et al., 2009),

$$\gamma(\lambda) = \alpha(\lambda) + \beta(\lambda). \tag{2a}$$

Using Eq. (1c), the extinction Ångström coefficient EAC can be written as

$$\mathrm{EAC}(\lambda) = -\frac{\lambda}{\gamma(\lambda)}\frac{d\gamma(\lambda)}{d\lambda} = -\frac{\lambda}{\alpha(\lambda)+\beta(\lambda)}\frac{d(\alpha(\lambda)+\beta(\lambda))}{d\lambda} = -\frac{\lambda}{\alpha(\lambda)+\beta(\lambda)}\left(\frac{d\alpha(\lambda)}{d\lambda}+\frac{d\beta(\lambda)}{d\lambda}\right). \tag{2b}$$

Rewriting Eq. (1c) for the absorption Ångström coefficient AAC yields (Moosmüller et al., 2011)

$$\frac{d\alpha(\lambda)}{d\lambda} = -\frac{\alpha(\lambda)}{\lambda}\mathrm{AAC}(\lambda), \tag{2c}$$

with an equivalent expression for the scattering Ångström coefficient SAC

$$\frac{d\beta(\lambda)}{d\lambda} = -\frac{\beta(\lambda)}{\lambda}\mathrm{SAC}(\lambda). \tag{2d}$$

Substituting Eqs. (2c, d) into Eq. (2b) yields

$$\begin{aligned}\mathrm{EAC}(\lambda) &= \frac{\lambda}{\alpha(\lambda)+\beta(\lambda)}\left(\frac{\alpha(\lambda)}{\lambda}\mathrm{AAC}(\lambda)+\frac{\beta(\lambda)}{\lambda}\mathrm{SAC}(\lambda)\right)\\ &= \frac{\beta(\lambda)}{\beta(\lambda)+\alpha(\lambda)}\left(\frac{\alpha(\lambda)}{\beta(\lambda)}\mathrm{AAC}(\lambda)+\mathrm{SAC}(\lambda)\right)\end{aligned}. \tag{2e}$$

With the single scattering albedo (SSA) $\omega(\lambda)$ defined as

$$\omega(\lambda) = \frac{\beta(\lambda)}{\beta(\lambda)+\alpha(\lambda)}, \tag{2f}$$

and the single scattering co-albedo (SSCA) $\varpi(\lambda)$ defined as

$$\varpi(\lambda) = 1-\omega(\lambda) = \frac{\alpha(\lambda)}{\alpha(\lambda)+\beta(\lambda)}, \tag{2g}$$

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Eq. (2e) can be written as

$$\mathrm{EAC}(\lambda) = \omega(\lambda)\left[\left(\frac{1}{\omega(\lambda)} - 1\right)\mathrm{AAC}(\lambda) + \mathrm{SAC}(\lambda)\right] = \varpi(\lambda)\,\mathrm{AAC}(\lambda) + \omega(\lambda)\,SAC(\lambda), \tag{2h}$$

where the extinction Ångström coefficient EAC is the sum of the SSCA(ϖ)-weighted absorption Ångström coefficient AAC, and the SSA(ω)-weighted scattering Ångström coefficient SAC. As expected, the EAC is dominated by the AAC for mostly absorbing (i.e., black, $\varpi \approx 1$) particles and by the SAC for mostly scattering (i.e., white, $\omega \approx 1$) particles.

Simple examples for specific cases include (a) If $\mathrm{AAC}(\lambda) = \mathrm{SAC}(\lambda)$, the EAC can be written as $\mathrm{EAC}(\lambda) = \mathrm{AAC}(\lambda) = \mathrm{SAC}(\lambda)$ and (b) If AAC and SAC are constants, independent of wavelength λ, the wavelength dependence of EAC is determined by that of the SSA $\omega(\lambda)$ and by the difference between SAC and AAC as shown in a variation of Eq. (2h) as $\mathrm{EAC}(\lambda) = \mathrm{AAC} + \omega(\lambda)[\mathrm{SAC} - \mathrm{AAC}]$.

3 Generalization to two-wavelength Ångström coefficients

As Eq. (2h) is written for single-wavelength Ångström coefficients and albedos, it is not a priori clear how to use it for two-wavelength Ångström coefficients, especially as no two-wavelength definitions of SSA and SSCA are in common use. However, using two-wavelength SSA and SSCA in Eq. (2h) and the two-wavelength definitions of AAC, SAC, and EAC given in Eq. (1b), the two-wavelength SSA $\omega(\lambda_1,\lambda_2)$ can be defined as

$$\omega(\lambda_1,\lambda_2) = \frac{\mathrm{EAC}(\lambda_1,\lambda_2) - \mathrm{AAC}(\lambda_1,\lambda_2)}{\mathrm{SAC}(\lambda_1,\lambda_2) - \mathrm{AAC}(\lambda_1,\lambda_2)} = \left[1 - \frac{\ln[\omega(\lambda_1)] - \ln[\omega(\lambda_2)]}{\ln[1-\omega(\lambda_1)] - \ln[1-\omega(\lambda_2)]}\right]^{-1}. \tag{3a}$$

Equation (3a) is well defined for the entire parameter space of $0 \le \omega(\lambda_1) \le 1$ and $0 \le \omega(\lambda_2) \le 1$. It essentially gives the two-wavelength SSA $\omega(\lambda_1,\, \lambda_2)$ as logarithmically

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weighted "average" of $\omega(\lambda_1)$ and $\omega(\lambda_2)$ and the two-wavelength SSCA $\varpi(\lambda_1,\lambda_2)$ can be defined equivalent to the single-wavelength definition of Eq. (2g) as

$$\varpi(\lambda_1,\lambda_2) = 1-\omega(\lambda_1,\lambda_2). \tag{3b}$$

Note that the definition of the two-wavelength SSA $\omega(\lambda_1,\lambda_2)$ has a couple of symmetries, namely
(a) symmetry around the line $\omega(\lambda_1) = \omega(\lambda_2)$, for which $\omega(\lambda_1,\lambda_2) = \omega(\lambda_1) = \omega(\lambda_2)$, yielding the symmetry $\omega(\lambda_1,\lambda_2) = \omega(\lambda_2,\lambda_1)$ and (b) symmetry around the line $\omega(\lambda_1)+\omega(\lambda_2) = 1$, for which $\omega(\lambda_1,\lambda_2) = 0.5$, yielding the symmetry $\omega(\lambda_1,\lambda_2) = 1 - \omega(1-\lambda_1,1-\lambda_2)$. Related symmetries exist for SSCA.

Using Eqs. (3a, b), as definitions of two-wavelength SSA and SSCA, Eqs. (2h), (4d), and (5) can be used for two-wavelength Ångström coefficients by replacing all occurrences of (λ) with (λ_1,λ_2).

4 Single scattering albedo (SSA) Ångström coefficient SSAAC

Inserting the definition of single scattering albedo (SSA) ω from Eq. (2f), into the general definition of AC given by Eq. (1c) and using the quotient rule, the SSA Ångström coefficient SSAAC can be expressed as

$$\begin{aligned}\mathrm{SSAAC}(\lambda) &= -\frac{\lambda}{\omega(\lambda)}\frac{d\omega(\lambda)}{d\lambda} = -\frac{\lambda}{\omega(\lambda)}\frac{d\frac{\beta(\lambda)}{\beta(\lambda)+\alpha(\lambda)}}{d\lambda} \\ &= -\frac{\lambda}{\omega(\lambda)}\left[\frac{\frac{d\beta(\lambda)}{d\lambda}}{\beta(\lambda)+\alpha(\lambda)} - \beta(\lambda)\frac{\frac{d\beta(\lambda)}{d\lambda}+\frac{d\alpha(\lambda)}{d\lambda}}{(\beta(\lambda)+\alpha(\lambda))^2}\right].\end{aligned} \tag{4a}$$

In Eq. (4a), $d\beta/d\lambda$ and $d\alpha/d\lambda$ can be replaced using Eqs. (2c) and (2d), respectively, yielding

$$\mathrm{SSAAC}(\lambda) = -\frac{\lambda}{\omega(\lambda)(\beta(\lambda)+\alpha(\lambda))}\left[-\frac{\beta(\lambda)}{\lambda}\mathrm{SAC}(\lambda)+\frac{\beta(\lambda)}{\beta(\lambda)+\alpha(\lambda)}\left(\frac{\beta(\lambda)}{\lambda}\mathrm{SAC}(\lambda)+\frac{\alpha(\lambda)}{\lambda}\mathrm{AAC}(\lambda)\right)\right]. \tag{4b}$$

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Some further rearrangement and use of the definition of SSA in Eq. (2f) yields

$$\begin{aligned}\mathrm{SSAAC}(\lambda) &= -[\mathrm{SAC}(\lambda)(\omega(\lambda)-1)+(1-\omega(\lambda))\mathrm{AAC}(\lambda)] \\ &= \omega(\lambda)(\mathrm{AAC}(\lambda)-\mathrm{SAC}(\lambda))+(\mathrm{SAC}(\lambda)-\mathrm{AAC}(\lambda))\end{aligned} \tag{4c}$$

and finally reduces to the form

$$\mathrm{SSAAC}(\lambda) = \varpi(\lambda)(\mathrm{SAC}(\lambda)-\mathrm{AAC}(\lambda)). \tag{4d}$$

The SSAAC equals the difference between SAC and AAC multiplied with the SSCA ϖ. Note that for two-wavelength Ångström coefficients, the two-wavelength definition of SSCA in Eqs. (3a, b) must be used.

5 Combinations of equations

Additional useful equations for SSAAC can be obtained by combining Eq. (4d) with Eq. (2h) yielding

$$\mathrm{SSAAC}(\lambda) = \mathrm{SAC}(\lambda)-\mathrm{EAC}(\lambda). \tag{5a}$$

and

$$\mathrm{SSAAC}(\lambda) = (\mathrm{EAC}(\lambda)-\mathrm{AAC}(\lambda))\frac{\varpi(\lambda)}{\omega(\lambda)}. \tag{5b}$$

In addition, with SSA and SSCA Ångström coefficients SSAAC and SSCAAC defined by Eq. (1c), their relationship can be directly obtained as

$$\mathrm{SSCAAC}(\lambda) = \frac{-\omega(\lambda)}{\varpi(\lambda)}\mathrm{SSAAC}(\lambda) \tag{5c}$$

and combing Eq. (5c) with Eq. (5b) yields

$$\mathrm{SSCAAC}(\lambda) = \mathrm{AAC}(\lambda)-\mathrm{EAC}(\lambda). \tag{5d}$$

Again Eqs. (5) can be used with two-wavelength Ångström coefficients, if the two-wavelength definitions of SSA and SSCA in Eqs. (3a, b) are employed.

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6 Conclusions

Simple analytical relations have been developed connecting ACs for aerosol extinction, scattering, and absorption coefficients, and single scattering albedo. These relationships will be useful for performing comparisons and closure between different measurements of the wavelength-dependent aerosol optical properties parameterized in terms of ACs. They will be of special interest for ground-truthing of the wavelength dependence obtained from satellite aerosol optical depth measurements with that from ground-based and airborne measurements of scattering and absorption coefficients. Future work will include applications of these relationships to existing data of wavelength dependent aerosol optics measurements.

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Table 1. Symbols used.

Name	Symbol
Absorption coefficient	α
Absorption Ångström coefficient	AAC
Ångström coefficient	AC
Extinction Ångström coefficient	EAC
Extinction coefficient	γ
Parameter	p
Scattering Ångström coefficient	SAC
Scattering coefficient	β
Single scattering albedo (SSA) Ångström coefficient	SSAAC
Single scattering albedo (SSA)	ω
Single scattering co-albedo (SSCA) Ångström coefficient	SSCAAC
Single scattering co-albedo (SSCA)	ϖ
Wavelength	λ

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