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Interactive comment on "Why anisotropic turbulence matters: another reply" *by* S. Lovejoy et al.

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Lovejoy et al. (2009) have shown by adopting a new correction method for flight observation data that a universal scaling law consistent with an anisotropic turbulence theory is established for much wider scale than hitherto. Lindborg et al. (2010), in turn, argue that their data correction method is fundamentally flawed. Lovejoy et al.'s new contribution here constitutes their further efforts to answer these criticisms.

Lindborg et al. (2010)'s criticism consists of the two major parts: 1) inconsistency of Lovejoy et al.'s data correction method with a simple scale-estimate based argument, and 2) inconsistency of the anisotropy theory with the established two-dimensional turbulence theory in large scale limit for geophysical flows.

The present comment is concerned with the second aspect, leaving the debate on the first aspect, albeit more central in Lovejoy et al. (2009), deemed to be too technical for me, to the others. The focus is on possible methodologies for reconciling the virtual inconsistency of the standard geophysical turbulence theory and their anisotropy turbulence theory. I more specifically ask a question: how a dynamically more consistent theory can be developed along this line of argument?

According to a widely accepted perception, atmospheric flows are divided into two major regimes: quasi-two dimensional flows in large-scale limit and fully three-dimensional flows in smaller scale limit. They are furthermore expected to be described by two-dimensional and three-dimensional turbulence theories, respectively.

On the other hand, the anisotropic turbulence theory argues that such a sharp transition of the dimensionality does not exist, but the whole atmospheric flows are characterized by a single fractal dimension somewhere between 2 and 3. However, a major weakness of this theory is a lack of such a model system that describes an evolution of this system with a fractal dimension in deterministic manner.

In the standard theory, the large-scale two-dimensional limit is well described by a quasi-geostrophic system. The small-scale three-dimensional limit is well described by a system under nonhydrostatic anelastic approximation. What will be an equivalent system to describe an anisotropic turbulent flow with a fractal dimension?

The quasi-geostrophic system would be a better starting point for further discussions, because this system is derived under a well-defined asymptotic expansion method for a limit of small Rossby number. Many discussions on the limitations of this system exist. Various proposals also exist for improving this approximation by taking into account higher-order effects in Rossby-number expansion. Semi-geostrophy is the most notable such example (Hoskins 1975).

However, as far as I could follow the literature, there is no theory to indicate that the system gradually transform from two dimensionality to three dimensionality, say, by gradually increasing the Rossby number. Will it be possible to construct such a theory? If it is possible, how can it be done?

One possibility could be to formally develop a system based on an infinite series of asymptotic Rossby-number expansion. Will we recover a fully three-dimensional nonhydrostatic flow under such an expansion? My intuition says "no".

This is another difficulty from a point of view of traditional geophysical fluid dynamics for accepting this anisotropic turbulence theory: as it stands for now, the quasi-geostrophic and the nonhydrostatic anelastic systems are so separated in phase space that it appears to be not feasible to obtain one from another by any continuous transformation. This perspective better supports a traditional view with a sudden transition of dimensionality rather than the anisotropic turbulence point of view based on fractal dimensionality.

A weakness of the standard theory may well be pointed out. As far as I am personally aware of, there is no much work done on the dynamic regimes for large Rossby numbers with an inverse of the Rossby number as a parameter for an asymptotic expansion. The hardest problem least investigated is the transition of the dynamical regime from those for the Rossby number less than one to those for the Rossby number larger than one.

This problem of the transition of the dynamic regimes over the Rossby-number order one can be seen in analogy with development of an asymptotic solution over a turning point (cf., Olver 1974). Here, the turning point means a point that a function turns from an oscillatory form to an evanescent (exponentially-decaying) form. Fair to say, the development of a general asymptotic expansion theory is the muddiest around this point. A procedure is simply developed by introducing a genetic function for, say, a first-order transition point, called the Airy function, and all the general asymptotic solution is fit into the Airy function. The Airy function is specifically designed to represent a generic behavior at the turning point.

If this is a good analogy for what would happen with the geophysical flows by crossing the point with the Rossby number one, and furthermore if the traditional point of view with two- and three-dimensional turbulence is correct at both sides of the transition point, then the problem of asymptotic expansion reduces to that of identifying a generic partial-differential equation system that describes the two- and three- dimensional flows at both sides of the transition point, in analogy with the Airy function.

Over this transition point, we would see a "gradual" transition from two- to three-dimensional flows, in the same sense as the Airy function describes a "gradual" transition from an oscillatory solution to an evanescent solution. The main theoretical question would be the width of such a transition zone. If the anisotropic turbulence theory is correct, this transition zone is in fact infinitely wide in logarithmic scale of Rossby number. If the traditional theory is correct, such a transition zone should be relatively narrow and probably of the order of unity in logarithmic scale of Rossby number.

Many years ago (Yano and Sommeria 1997), I solved a similar, but much simpler problem with wave dynamics. In this case, I was dealing with the problem of a continuous transition of inertia-gravity waves to Rossby waves over a scale of "negative" Rossby radius of deformation (or the equivalent depth) for "unstably" stratified geophysical flows. In this case, after a proper nondimensionalization, the inertia-gravity wave frequency is given as the order one, the Rossby-wave frequency as the order of nondimensional beta parameter. A dispersion equation for the transition zone (their Eq. 28) is obtained by assuming a frequency of the order of the nondimensional beta to the power of 1/3. This dispersion relationship recovers those for inertia-gravity and Rossby waves to the limits of the frequency to one and to the nondimensional beta, respectively.

Under a similar argument, we might be able to derive a system with a fractal dimension over the trasition zone over the Rossby number equal to one. Unfortuntately, such an analysis is still to be performed.

In summary, in spite of appealing nature of the anisotropic turbulence theory that potentially unifies the atmospheric flows of all scales, as it stands for now, it remains a purely statistical theory without a counterpart dynamical model for describing the system in deterministic manner. Such a system should have a capacity of continuously transforming from a quasi-geostrophy to nonhydrostatic anelasticity. My naive feeling is that an elaborated use of a renormalization group theory might potentially lead to a necessary theoretical breakthrough, but I should not be too speculative.

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