We thank the reviewer for the thoughtful comments that help to clarify the presentation of our results. In the blue text below we address the reviewer's comments in a point-by-point fashion. For completeness, we have repeated the reviewer's comments here (in black).

Response to reviewer #3

**Summary:** A simple kinematic framework for the interaction of tropical cyclones with environmental air is presented under the assumptions of a layer-wise two-dimensional and steady flow. The authors argue that the kinematic structure is responsible for intensity modulation in both an idealized experiment and a point vortex model subjected to background flow. This paper is scientifically significant and will motivate future research.

**General Comments:** The authors provide overall well written and interesting arguments that "dividing streamlines" emanating from a stagnation point are the key structures which govern the interaction of dry or moist environmental air with the core. While the paper is well written, there are several terms that are used synonymously or are not clearly defined. In some cases, the terminology is not consistent with many previous studies. These will be addressed further in the general and specific comments.

**Steady Flow:** The authors have made the assumption of a steady flow in a storm- relative reference frame. Since this is the first paper since WMF84 to address the kinematic structures related to streamlines and stagnation points, the assumption of a steady flow is a reasonable assumption for introducing these new ideas. However, there are some places where additional care should be taken or additional details given about the implications of this assumption. The authors hint at the requirement of including time-dependence by showing the times required for trajectories to enter the downdraft region, and by noting the time scale of intensity modulation due to downdrafts. The relationship between these time scales and the time scales involved in the numerical computations of streamlines is likely important when one considers the implications of these results in the case where the flow is not assumed to be steady, yet has not been addressed in this paper. A brief discussion of this relationship could validate the assumptions of a steady flow and strengthen the results of this paper.

In response to the reviewer's comment we have extended our discussion of the steady-flow assumption in the Introduction. We now further emphasize that the assumption of steady flow is reasonable for a time-scale of 1 day. This time scale is consistent also with the time for which we have calculated back trajectories of the steady flow (section 2.3, Fig. 6).

The last sentence on page 28062 has been replaced by the following paragraph:

"During the first 24 h after shear is imposed, the TC in the experiment both

weakens and then re-intensifies by approx. 15 ms<sup>-1</sup>. The initial vertical shear flow that is imposed on the TC is found to be steady in the environment. Although the axisymmetrization of the weak potential vorticity gradient associated with the environmental flow may lead to some modification of this flow in the vicinity of the TC, we show in Section 2.1.3 that the flow topology remains essentially the same over the 1-day time period under consideration. Barring interaction with an upper-level trough or a strong jet stream, e.g. as during extratropical transition, changes of the large-scale flow in which TCs are embedded in the real atmosphere are usually modest on the time-scale of 1 day also. Thus, we believe that the steady-flow approximation provides scientific value for studying TC-environment interaction in the real atmosphere."

**Stagnation Points:** The transient nature of stagnation points is well known and is apparent in this paper. A point that could be made more clear is whether the sensitivity of translation pints both in number and location is due to the time averaging of the velocity fields or the time averaging of the background flow.

Multiple stagnation points (SP) at 8 km height are found at individual times also (2 SP at 3 h, 2 SP at 5 h, 3 SP at 7 h, 4 SP at 8h). Hence, the occurrence of multiple SP is not a result of the time averaging in this case. We have added a short discussion on multiple stagnation points (see our response to your third-last comment below) in the manuscript after the discussion of Fig. 3.

The term separatrix or separatrices is used many times in this paper, but the meaning is not clear. The meaning of separatrix is a boundary separating two types of solutions of a differential equation, and generally means an enclosed structure. In the context of streamlines, a separatrix is formed by enclosed streamlines. On P28065 L15, P28067 L17, and many other places in the text, separatrix refers to a single spiraling streamline, which does not divide different solution behavior, as all solutions are spiraling inward. In the unstable direction, the manifold of the hyperbolic stagnation point which leads to a sink or limit cycle is not distinguished from any other streamlines. The manifolds are distinguished, and form separatrices only because nearby trajectories behave differently by splitting as they approach the stagnation point. Backward trajectories, as in Figure 6, would show different behavior only as they approached the stagnation point, depending on which side of the manifold they originated.

Figure 7 (a) represents a separatrix, while the manifolds in 7 (b) and (c) are not separatrices for the purpose of air entering the core. In the latter cases, the stable manifold branch which comes close to the center may be part of a separatrix, since solutions either enter the core or they don't, depending on the which side of the stable manifold they originate.

I recommend changing P28065 L15 to indicate that manifolds are not the

same as separatrices. The use of the separatrix at other places in the text should be carefully checked, and can likely be removed in most cases.

We agree that the use of these terms in our manuscript was not always consistent with the strict sense in which the reviewer is using these terms. We now avoid this terminology completely. In particular, the statement on P28065 L15 has been deleted. We now use the term (stable or unstable) manifold, or dividing streamline as defined at the beginning of section 1.2.

**Specific Comments:** P28062 L2: "dividing streamline" If the flow is steady, all streamlines are dividing. The particular streamline emanating from a hyperbolic fixed point is apparently what is meant here. The convention of "unstable manifold" is well established in many previous studies including Ide Et Al. 2002, which you reference. I would suggest that the authors either define dividing streamline specifically where it is first mentioned in the text to mean this particular streamline, or simply use the unstable manifold convention.

We have followed the suggestion of the reviewer and have defined the term 'dividing streamline' when we first use this term (beginning of Section 1.2). For the purposes of this paper, we define the dividing streamline as the (closed) manifold that emanates from the hyperbolic fixed point of a vortex embedded in a background flow. The term 'dividing streamline' applies only to the steady, non-divergent case and is used just once in the revised paper.

P28062 L17: "the stagnation point" is central to the results of the paper, and should be defined explicitly.

We agree with the reviewer. The stagnation point is now defined at the beginning of section 1.2.

P28063 L1 The authors state "In the vicinity of the eyewall, both assumptions break down." While this statement is true, some of the results of this paper including the idea of a limit cycle far away from the stagnation point are highly dependent on this assumption. Is this limit cycle purely an artifact of a steady flow, or of the averaging of the velocities, or is it apparent in the unsteady flow as well?

We appreciate the reviewer raising these important issues. The limit cycle is a robust feature of the flow topology derived from instantaneous snapshots of the storm-relative flow at individual times (hourly from 3 h to 8 h). It is thus highly unlikely that the limit cycle is an artifact of the time averaging of the flow. We have added this information in the manuscript as a footnote on page 28067.

Isopleths of  $\theta_e$  are nearly congruent to the limit cycle. Highest  $\theta_e$  values are contained within the limit cycle (e.g., see Figures 4a and 4b). Since  $\theta_e$  above the surface layer tends to be a materially conserved quantity in the absence of strong diffusion and mixing processes,  $\theta_e$  is an approximate invariant of the

time-dependent, three-dimensional flow. On the basis of these considerations it is therefore highly likely that a flow boundary in the full flow field does indeed exist.

P28063 L17 The term Lagrangian coherent structure is more general than the time-dependent analogs of the manifolds which are computed in this study, and generally means any structure that can be tracked through the time-dependent flow. Perhaps saying that these LCSs are finite-time manifolds of the time-dependent flow would remove any confusion as to how a LCS is defined.

As the reviewer suggests, we now inform the reader that "These LCS are the finite-time manifolds of the time-dependent flow." (P 28063 L12)

P 28064 L10 Is the average translation speed time averaged or spatially averaged?

We actually meant to use the term 'approximate'. The translation speed of the TC varies little with time and so a  $5 \text{ ms}^{-1}$  value suffices for this analysis.

P 28065 L11 and P 28070 L 18 The velocities are averaged over a 6 hour period. However, the manifolds are streamlines computed by starting at the stagnation point. Figure 6 indicates that trajectories along the streamline near the fixed point require more than 1 day to reach the downdraft region. This appears to be an assumption that the flow is steady for the integration time of trajectories, which is as long as the 24 hour time of intensity modulation, not only the 6 hours of averaging. To be able to infer information about the core region from a distinguished streamline originating at a stagnation point, the assumption of a steady flow is as long as the integration time required to produce the streamline.

As discussed above, the flow outside the inner core is approximately steady over a 24 h time period. We use a 6 h time average to represent this flow. When we introduce the 6 h time average (page 28065), we are now explicit about these facts and replace the last two sentences of the first paragraph in section 2.1 by the following:

"As discussed in the introduction, for the purposes of this investigation the flow outside the inner core in the numerical experiment can be meaningfully approximated as steady for O(1 day). This steady flow is represented here by a 6 h time average over the period from 3 h - 8 h after the shear was imposed."

We also remind the reader on how we represent the steady flow on page 28070, line 18 by adding the following sentence:

"Note that this steady flow is represented here by a 6 h time average."

P28067 L5 The stagnation point is located at 1000 km, while the storm relative flow is averaged from 200 km to 1000 km. Are the values at 200 km and 1000 km similar?

The asymmetric storm-relative flow between 200 km and 1000 km is essentially uniform (see Fig. A below) and hence similar.

P28069 L5-12 The multiple stagnation points appear very close to each other, so the flow toward the limit cycle is more related to the streamlines than the stagnation points.

We are not sure if we understand the reviewer's point here. After the discussion of Fig. 3, we have now provided some additional information about the number of stagnation points and their impact on the flow topology. On page 28067 we specifically say the following:

"It is interesting to note that there are multiple stagnation points at 8 km. These multiple stagnation points are not an artifact of the time averaging of the velocity field but are found at individual times also (not shown). The occurrence of multiple stagnation points here does not imply a change of the system-scale flow topology. From each stagnation point, one unstable manifold spirals inwards and merges into the limit cycle."

We hope that this discussion addresses the reviewer's comment.

P28079 L17-21 I don't think italics are necessary.

Agreed, italics have been removed.

P28099 L7. Both branches of the unstable manifold extend away, but one leaves the domain. Only one branch of the stable manifold can be seen. What does the other manifold branch do if it doesn't extend away? Is the time scale just very slow in the stable direction at that location?

We very much thank the reviewer for carefully checking our figures. The reviewer's comment made us revisit our method and we soon discovered a small coding error in the manifold calculation. The main consequence of this error was that both branches of the stable manifold were inadvertently calculated to be essentially the same manifold; they were thus not distinct from one another in Fig. 3b of the original manuscript.

The coding error resulted in a shift of the meridional wind component by half a grid point relative to the zonal component. Thus, the wind field used to integrate the manifolds was not strictly consistent with the wind field used to identify the stagnation points, and it is only near a hyperbolic point that such an inconsistency yields a visible error in the flow topology.

Using the corrected code, the manifolds have been recalculated and Figs. 3 and 4 have been revised. The corrected version of Fig. 3 is shown below as Fig. B. At all four vertical levels (2 km, 5 km, 8km, and 10 km), the corrected manifolds are essentially the same as before, save two exceptions. First, one branch of the stable manifold at 5 km now extends away (as it should do!). Second, at 8 km, one branch of the unstable manifold again extends correctly away from the southernmost stagnation point. This latter unstable manifold now parallels the unstable manifold that extends away from the westernmost stagnation point.

None of our conclusions or physical interpretations are affected.



Figure A: Asymmetric storm-relative flow (arrows) within 1000 km of the TC center at 2 km height for the same data as used in the manuscript (averaged from 3 h – 8 h, after imposition of the environmental vertical shear flow). The horizontal scale depicted is in km. The manifolds are overlaid for orientation. Outside of approx. 200 km the asymmetric storm-relative flow is essentially uniform from west-southwest.



Figure B: Corrected version of Fig. 3 of the original manuscript. One branch of the stable manifold at 5 km height now extends correctly away from the stagnation point. One branch of the unstable manifold now extends correctly away from the southern-most stagnation point at 8 km. Save these two exceptions, the corrected flow topology is essentially the same as before.