We thank Brian Tang for his thoughtful comments on our manuscript. Below, the blue text addresses the reviewer's comments, which are repeated here for convenience in black.

The authors investigate the flow topology of a numerical simulation of a sheared tropical cyclone and a simple kinematic model of a point vortex with a mass sink. Under the assumption steady, horizontal flow, the flow topology indicates sectors in which the environmental air can intrude in to the vortex. Weak tropical cyclones in sufficiently strong environmental relative flow (vertical wind shear) can suffer significant incursions of environmental air in to the core of the circulation. On the other hand, strong tropical cyclones are quite resistant to environmental intrusions of dry air, with the exception of the outer rainbands.
The scientific significance and quality along with the presentation quality of the work is already good. There are a few areas in the manuscript that can benefit from further explanation and clarification.

**Assumption of Steady Flow**
One of the main results of this paper is that strong tropical cyclones (major hurricanes) require tremendous amounts of vertical wind shear in order for the separatrix structure to allow environmental air to reach the eyewall. The author's briefly mention in section one that the assumption of steadiness breaks down in the vicinity of the eyewall, where one observes a stew of asymmetric, transient motions. However, this point can easily be lost, and it would be nice to reemphasize in the concluding section that transient motions can and probably do play an important role in bridging the environment and storm inner-core. There is indirect evidence of this in Fig. 5 where one still sees large downdrafts inside the limit cycle. Dry air is still getting in to some extent, but not by the steady (time-averaged), horizontal flow.

We think that the reviewer's statement "… transient motions … play an important role in bridging the environment and storm inner-core." needs clarification. It is one of the main results of this study that the *horizontal, steady* environmental flow brings dry environmental air *close* to the TC's inner core (Fig. 4). In that sense, transient motion is not necessary to "bridge the environment and the inner core".
To address the reviewer's comment we have modified the end of the first paragraph in the conclusions as follows:
"From the perspective of the steady and layer-wise horizontal flow model the eyewall is well protected from the intrusion of environmental air. In order for the environmental air to intrude into the inner core convection, time-dependent and/ or vertical motions, which are prevalent in the TC inner-core, are needed."

**Vertical Motions**
In section 2.1.2, it is stated there is a high degree of congruence between the flow topology and the distribution of θe. Since the framework presented here doesn't include vertical motions, how important are they in determining the

distribution of θe? The vertical wind shear induces mesoscale ascent and subsidence on the downshear and upshear side, respectively. Moreover, convection itself strongly affects the distribution of θe, as evidenced by the stationary band complex. Additionally, at low levels, one has Ekman effects that also introduce wavenumber one asymmetries in θe. Hence, although the analysis suggests the separatrices do not permit significant mixing between the environment and inner core, important vertical or slantwise motions may not be captured here.

In the introduction of our manuscript we highlight the importance of vertical motions to ingest low-$\theta_e$ air into the TC circulation (pg. 28060, line 18 ff.). Furthermore, the discussion in the second paragraph of section 4 makes clear that vertical advection of high-$\theta_e$ air from out of the TC's boundary layer into the free troposphere is necessary to form the moist envelope.
It is one of the main results of this study, however, that there is a high degree of congruence between the theta_e isopleths and the manifolds of the steady, layer-wise horizontal flow. Since $\theta_e$ tends to be a materially conserved quantity above the surface layer, $\theta_e$ is an approximate invariant of the time-dependent, three-dimensional flow. On the basis of these considerations we argue that the steady, horizontal flow governs the shape and location of the moist envelope and the distribution of environmental air around TCs to first order (Figs. 4a and 4b).
At upper-levels, (at 10 km, Fig. 4d) we note that the convection associated with the SBC provides a source of high-$\theta_e$ air at this level (pg. 28068, line 9).
In view of the above comments we do not think that changes to the manuscript are necessary.

**Source Region of Environmental Air**
In section 2.3, the authors state that "the source region of the environmental air that feeds the downdraft area exhibits a pronounced asymmetric, azimuthal wave number one structure." Does one see this in the Cram et al. (2007) back trajectory analysis of their numerical simulation of Hurricane Bonnie (1998)?

Cram et al. did not specifically consider airstreams that feed into downdraft regions at low levels. Rather, they focused on the mixing of low-$\theta_e$ air into the eyewall convection at midlevels based on 5 h back trajectories confined to 100 km radius. For these reasons, we do not think that our results can be compared directly to Cram et al's result. Our results are nonetheless consistent with Cram et al.'s results in the sense that these authors found a distinct azimuthal asymmetry also: Their trajectories enter the eyewall preferably in the upshear semicircle.

**Limit Cycle**
The limit cycle seems like a very important feature. Would it be possible to analytically solve for the radius of the limit cycle using the idealized point vortex/divergence model? Since the value of the streamfunction at the stagnation point can be solved for, it seems one could then determine the

radius of the streamfunction at $\phi = 2\pi n$, where n is an integer. By taking the limit as $n \to \infty$ on the complex manifold, one could possibly determine the radius of the limit cycle, but I have not worked this out myself. This way, one can achieve an inner and outer radial bound on the possible penetration of environmental air by the steady, horizontal flow. Also, upon making some assumption about the divergence, D, a plot similar to Fig. 9 could be made.

In the divergent point vortex model the putative "limit cycle" emerges as a closed streamline just inside of the mass sink. The unstable manifold spiralling inwards, however, ends in the mass sink. Hence, the "limit cycle" in the divergent point vortex model is distinct from the unstable manifold. That is, the streamfunction value of the limit cycle is distinct from that of the unstable manifold. Thus, an analytical solution for the divergent point vortex model as outlined by the reviewer cannot be obtained.

From Gauss' theorem it is immediately clear that the net divergence within a closed streamline vanishes. The putative "limit cycle" in the point vortex model with asymmetric mass sink consists of the closed streamline that is located radially just inside of the mass sink. The radius of that closed streamline is therefore dependent on the location of the mass sink. For the situation depicted in Fig. 7c, the smallest radius of that closed streamline can be approximated by the following two steps.
First, the putative "limit cycle" is approximated by the closed streamline in the non-divergent model that passes through the location of the mass sink in the asymmetric divergent model. The value of the streamfunction in the non-divergent model at this location can be evaluated using Eq. 10 with $D = 0$. Using this value and assuming the closest approach of the dividing streamline is at $\phi = 0$ (south), Eq. 10 can be solved for the smallest radius. Substituting in the values from Fig. 7, the smallest radius is found at approx. 80 km.

We have not provided a regime diagram for the "limit cycle" similar to Fig. 9. Streamlines have to orbit around the center to get close to the closed streamline reminiscent of the limit cycle in the divergent point vortex model. We have explicitly excluded orbiting streamlines from our calculation of the radial inflow rate (Eq. 36). Note that the regime diagram Fig. 9 depicts the closest approach of air parcels *before* they start spiralling around the center. Air parcels along orbiting streamlines move inwards very slowly. It seems unreasonable to assume that the flow is approximately steady on this longer time scale. Air parcels that spiral through the rain band region towards the eyewall are likely to interact considerably with rain bands before reaching the eyewall. It can be assumed that this interaction moistens the environmental air considerably before eyewall interaction takes place (see last paragraph on pg. 28086).

**Downdrafts as a Mass Sink at Midlevels**
Another possible mass sink at midlevels is the formation of a large area of downdrafts, especially as shown in RMN. How might an organized, banded downdraft might affect the flow topology in such a way as to create a

feedback that allows even more environmental air to be ingested in toward the center of the tropical cyclone?

Our simple model is not designed to represent downdraft formation in detail. Speculation on a feedback between downdrafts and the flow topology is beyond the scope of our simple model.

**Operational Implications**
There are some operational implications regarding sampling of storms that I think would be useful to place in the concluding section if the authors so desire. Namely, is it important to get the the environmental flow and structure of the cyclone correct so that the flow topology can be correctly deduced. Additionally, the thermodynamic properties of the source region within the dividing streamline are important to sample correctly.

Based on our current results from idealized modelling and theoretical considerations we agree that an atmospheric sampling strategy as suggested by the reviewer could be helpful to estimate the degree of environmental air intrusion into the TC circulation. At the current time we refrain from giving a speculative recommendation for sampling strategies. The examination of the flow topology and its importance for the evolution of TCs in the real atmosphere awaits future research.

**Tropical Cyclone Size**
Although the intensity is primarily addressed here, the size of a tropical cyclone also seems to be an important parameter in this framework (through the circulation, $\Gamma$). Since tropical cyclones of similar intensities can have vastly different sizes, how would the flow topology of a small storm compare with a larger one? It seems the authors may be able to use their results to explain why smaller tropical cyclones, which can be quite intense, seem to be much more susceptible to vertical wind shear and environmental intrusions of dry air.

We thank the reviewer for this suggestion. Based on our results we can indeed argue that the ability of a vortex to thermodynamically isolate itself from the environment increases with the size of the vortex.

At the end of section 4.1 (pg 28085) we have added the following brief discussion:
"A similar point can be made for vortex size. A broad TC with a relatively large radius of gale force winds has evidently a larger circulation than a TC with the same maximum intensity but a smaller radius of gale force winds. Thus, it can be expected that the ability of a TC to thermodynamically isolate itself from the environment increases with TC size."
Furthermore, the last 2 sentences of the conclusions have been modified to include vortex size.

**Other Minor Points**

Fig. 2. The wind profile of the Cat1 point vortex is actually not flat between 0-50 km. I recommend eliminating this portion of the curve inside 50 km.

Good point. We have eliminated the curve inside of 50 km.

Section 2.1.1, second paragraph. It's a bit tough to visualize the environmental storm-relative flow from the in-text description. Adding vectors of the storm-relative flow to Fig. 3 at each level would aid the reader in assessing how the geometry of the flow topology changes with the direction and magnitude of the environmental flow.

Thank you for the suggestion. Wind vectors indicating the environmental flow have been added.

Section order. I would consider rearranging the article so that sections 3 and 4 come before section 2. The point vortex model serves are a nice didactic example to the more complex simulation. Additionally, some terms, like the "stagnation point" are clearly defined in section 3, but used before in section 2, which leads to some confusion initially.

The section order suggested by the reviewer is a reasonable alternative that was considered by us also before the submission of the manuscript. Because the results of section 2 help provide a solid motivation and justification for the simplified model we decided in favour of the current order. As the reviewer correctly notes there were some terms that were undefined before section 3. In section 1.2 we now give a definition of the stagnation point, the dividing streamline, and manifolds. These definitions should help avoid any potential confusion of the reader in section 2.

Inner and outer separatrix. I'm confused by this terminology on pg. 28080. Perhaps labeling Fig. 7 or defining it better in the text would help. Also, why is the width of the opening of the separatrices, d, important? Does larger d guarantee a more direct path of environmental air to the inner-core?

Below Eq. 26 we now define the (formerly called) inner "separatrix" as the unstable manifold and the outer "separatrix" as the stable manifold. An accompanying footnote has been added to clarify that for net *divergence*, the *stable* manifold spirals inwards (and would thus be the "inner separatrix"). Note that we now have replaced the term separatrix by the manifold terminology throughout the text (see response to reviewer #4).

The opening d is proportional to the divergence D (Eq. 28). For the convergent case ($D < 0$) the closest approach of the unstable manifold is found closer to the center (by approx. d/2) than in the non-divergent case. We use this closest approach as an estimate of the propensity of the TC to interact with the environment (section 4.1). We are unclear what the reviewer means by the phrase "more direct path". Equation 29 shows that a larger d allows a *closer* approach of the environmental air to the inner core. The appendix examines the relative importance of the divergence and the environmental relative flow in determining the closest approach of the unstable manifold.

**Technical Corrections**

Pg. 28080, Line 17. Comma missing between "dividing streamline" and "the presence."

Pg. 28089, Line 6. "Extend" should be "extends."

Thank you. We have implemented these corrections.