**Atmospheric
Chemistry
and Physics
Discussions**

# *Interactive comment on* "A minimum bulk microphysics" *by* J.-I. Yano and D. Bouniol

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We much appreciate very critical comments by the two anonymous referees.

A major modification we propose in revising the present manuscript is to add a case that considers an effect of precipitating ice in a very crude manner. In order to account for this effect, we simply reduce the terminal velocity of precipitating condensate from 5 m/s to 1 m/s below the freezing point (cf., Fig. 2 of the present manuscript). We show that this very simple modification permits the system to develop an extensive stratiform cloud.

The following response consists of the three parts: 1) general response to the two anonymous referees (especially concerning the motivation of the present study); 2) the proposed extensive text to be included in the revised, final manuscript in response to the comments of the two referees; 3) a final short remark about the typos in the text.

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The main goal of the response below is so that the "raison d'etre" of the present article is much clarified.

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1) General response to the two anonymous referees:

The original title has been slightly misleading, as the anonymous referee 1 suggests. We propose a new title "A minimum bulk microphysics and its implementation into NAM-SCA".

Though it is not well emphasized in the present manuscript, an important secondary point of the present manuscript is to report that this minimum microphysics has been successfully implemented into the NAM-SCA model. This implementation is considered an accomplishment by itself, because even with a radical truncation (compression rate up to 0.1), we demonstrate that the NAM-SCA model can run with reasonable cloud physics. More importantly, we have accomplished a higher model compression rate than the dry case considered in the original work (Yano et al., 2010).

As fully discussed in the original article (Yano et al., 2010), the main goal of NAM-SCA is to reduce the numerical complexity of an "explicit" model (CRM, LES) into a level comparable to a parameterization. For this purpose, drastic reduction in the number of finite volume elements is attempted so that the model can be highly "compressed. In developing a moist version here, the physics must also be highly "compressed". This is the reason that the "minimum microphysics" must be implemented into NAM-SCA.

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In revision, in responding to a request by the anonymous referee 2, a very simple approach for representing a major effect of ice clouds is proposed. As emphasized in Sec. 2.c of Grabowski (1998) as well as demonstrated by Fig. 2 of the present manuscript, a major difference of ice precipitating particles (snow) to liquid precipitating particles (rain) is the lower terminal velocity for the former. The last experiment with

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no threshold in Kessler scheme (Fig. 9) is repeated by simply reducing the terminal velocity to 1 m/sec below freezing point. The value of the snow terminal velocity is chosen by examining those for the snow plotted in Fig. 2. The modification helps to develop a stratiform cloud more extensively, more than the case with the Berry-Kessler scheme shown in Fig. 8(d). The result will be shown in the final manuscript (Fig. 1).

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We spend a whole section (Sec. 4) on the autoconversion scheme. We find such a focus has been necessary because it is not possible to formulate this process by a formal bulk formulation, at least in a simple manner. However, the main aim of the present manuscript does not reside there, against what the anonymous referee 1 suggests. The purpose of the present paper is rather to examine the structure of the bulk microphysics as a whole. For this reason, most importantly, the basic idea of the bulk formulation is carefully introduced in Sec. 3.1 as well as in the beginning of Sec. 3.2. The first author finds such a very basic elucidation of the principle of bulk microphysics important.

We are open to admit that not all the materials presented in the manuscript are totally original, but we believe that all these materials are presented in our particular context originally. Especially, the formulation summarized in Sec. 2.2 can be considered our major original contribution.

Note that the proposed scheme is even simpler than the simple scheme already proposed by Grabowski (1998). In fact, we have applied the principle of the bulk formulation more strictly than the case of Grabowski, pointing out some minor defects in his derivation, as carefully presented in Sec. 3.2. Only after the rigorous derivation is done, we perform further simplifications in Sec. 3.3 by carefully examining the derived exact result.

The obtained final minimum formulation summarized in Sec. 2.2 is very simple, and may even look trivial, as the anonymous referee 2 points out. However, it is important

to recognize that it is derived by strict application of the bulk principle and a subsequent careful examination of each term for approximations. We believe the derivation is not trivial, and is original.

Here, note that a very deductive methodology adopted here is more important than the result itself. Microphysics schemes are rarely simplified in this manner as long as our literature survey shows.

In our formulation, the level of complexity handled for the autoconversion is the same level as that of both Kessler (1965, 1969) and Berry (1968).

Main beauty of our phenomenological consideration is that it well elucidates a simple fact that the formulations by both Kessler and Berry are at the same level of complexity. Notice that Grabowski (1998) adopts Berry's scheme for his autoconversion description. Here, in the revised text, we refer to Cotton and Anthes (1989) for limits of this type of phenomenological approaches for the autoconversion.

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2) The proposed extensive text to be included in the revised, final manuscript in response to the comments of the two referees:

We much appreciate the encouragements of both anonymous referees to perform much more extensive literature survey. In response we propose to include the following text in the appropriate places of the revised manuscript. We believe that this historical review helps enormously for pointing to importance of perusing the minimum microphysics.

We will also add Cotton and Anthes (1989) as a general reference.

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Approaches for cloud microphysics modelling: A short historical review:

Wide range of approaches is possible in order to study the cloud microphysics pro-

cesses. The following review is hardly exhaustive, but solely intended to suggest the state of art of cloud microphysics modelling from a historical perspective in order to better identify the place of the present minimum bulk formulation proposal in a wider context of the microphysical investigations.

Clearly, a very rigorous approach is to treat the time evolution of the condensed particle number explicitly by solving the stochastic collection (coalescence) equation, as originally proposed by Smoluchowski (1916: see also Ch. 15 of Pruppacher and Klett 1997). First full numerical integrations of this system are performed by Berry (1967), Berry and Reinhardt (1974a, b).

Note that Berry's (1968) autoconversion scheme is formulated by a curve fitting of the result by Berry (1967). A similar curve "fitting approach" is more recently adopted by Khairoutdinov and Kogan (2000), but based on a use of stochastic collection equation in a three-dimensional LES, in order to develop a double-moment scheme (see below). A limit of the curve fitting approach must be recognized here: the developed model is strictly valid only for the case that the stochastic collection equation is run. For example, in the case of Khairoutdinov and Kogan (2000), their developed scheme is valid only for drizzling boundary-layer top straticumli.

A formal manner for developing a simpler microphysical description from the stochastic collection equation is to divide the whole distribution into the smaller part (cloud) and the larger part (rain). An analytically integrable simple kernel is, then, introduced. It leads to a set of equations for the number density and the mixing ratio for both cloud and rain water (and also for ice more generally), for example. Such a closed description is originally proposed by Berry and Reinhardt (1974c, d), and more recently adopted by Seifert and Beheng (2001), for example. These particular studies lead to the so-called double moment scheme (see also Koenig and Murray 1976, Ferrier 1994, Meyers et al. 1997, Milbrandt and Yau 2005, Morrison et al. 2005). The same general approach can also lead to either bulk microphysics or a higher-order moment description.

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Alternatively, the stochastic collection equation itself can be adopted as a microphysics scheme by dividing the condensate particle distribution into a finite number of bins (bin microphysics).

Earlier, Takeda (1971) already applies this method for warm cloud physics in order to investigate the interaction of cloud dynamics with the vertical wind shear. Due to ambitious nautre of this work, the limit of applying such a highly sophisticated microphysics to the cloud dynamics is also well revealed. The focus of the study is clearly on the role of wind shear in the cloud dynamics, and not much attention is paid on the detailed microphysics implemented into the model. A similar study could have been performed with much simpler microphysics leading to similar conclusions.

Arnason and Greenfield (1972), and Clark (1973) also consider the interaction of warm cloud physics and the dynamics by taking a bin approach under a two-dimensional model configuration.

The next step is taken by Takahashi (1976) and Beheng (1978), who attempt a simulation of of hail formation process by this approach. As examples of the most recent, updated bin mircophysics, we refer to Caro et al. (2002), and Khain et al. (2004). Recently, this approach is adopted in more cloud-resolving modelling (e.g., Khain and Lynn 2009) especially for investigating cloud-aerosol interactions.

However, in order to simulate the aerosol evolution satisfactory, associated extensive chemistry (e.g., aerosol formation processes) must also be fully taken into account (e.g., Pringle et al., 2010). In this respect, bin microphysics is only an entry point for complex aerosol chemical, which requires very intensive investigations in its own light.

A natural attempt is to simplify the bin microphysics by examining its formulation structure, as we attempt the same for the bulk microphysics. Such an attempt is made by Grabowski et al. (2010), for example.

Note that, on the other hand, the stochastic collectin equation is, in general, only valid

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when the number density of microphysical particles are reasonably high per volume. The validity can be carefully exmained by taking a more fundamental probablistic description of the cloud particle population. Such a pioneering work is performed by Gillespie (1972). In this manner, more and more fundamental investigations are required in order to better establish the existing microphysical approaches. They naturally lead to more open questions.

Clearly the present approach (minimum bulk microphysics) takes an opposite extreme by taking the simplest possible formulation among the all possible hierarchy of approaches. Recall that importance of hierarchy of the model approaches is emphasized by Held (2005).

We should realize that a level of complexity of the model required for a study more than often relies on a subjective judgement of a researcher. We both need an extensive hierarchy of models as well as an objective decision making process (e.g., Bayesian principle: Jaynes 1985) in order to choose the most optimized complexity of a model for a given study.

We should also realize that it is much easier to pursue a more and more complex model, by arguing for the complex microphysical processes with intricate interactions with aerosols, chemistry, etc. It is always hard to exclude a given process in concern as insignificant without testing this process. The most likely end effect, under considerations along this line, would be to take the most complex model possible within the limit of available computer resources. It is much harder to justify a simplified model with many "important processes" neglected. However, it must also be realized that a more detailed model is not necessarily more reliable, but due to various uncertainties in microphysical parameters, the results often contain more uncertainties.

On the other hand, the simplest approach often suffices when the focus of the studies is not on the microphysical processes, but rather in its interactions with the dynamics (recall the discussions on Takeda, 1971 above). A need for such a simplified formu-

lation becomes even clearer when considering complex mixed-phase microphysics on top of warm cloud physics considered here, as discussed in below.

In the following three sections, we furthermore consider 1) the issues of autoconversion process as a further generalization of the phenomenological formulation presented in the manuscript: 2) issues of inclusion of ice processes, which are totally neglected in the present manuscript, and more specifically 3) the issues of mixed phase microphysics.

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Further generalization of the autoconversion scheme:

A fairly general discussion on a phenomenological description of the autoconversion process has been developed in the present manuscript. However, the discussion has not been general enough to cover even all existing literature on the autoconversion description at the same level of simplification.

Most importantly, all the physical parameters as well as the threshold are assumed to be constant of the system. More generally, all of them should be functions of an environment. For example, the proportionality constant in the phenomenological autoconversion formula, Eq. (4.2), is assumed to be the environment dependent by Tripoli and Cotton (1980) and Khairoutdinov and Kogan (2000). On the other hand, Ooyama (2001) proposes to take the autoconversion onset threshold to be proportional to the air density.

The switch function proposed by Eq. (4.3c) is not even general enough. For example, Orville and Kopp (1977) propose to add an extra threshold, say, $x_c$, to this equation with the exponent, $\xi = 1$, with the switch function sets zeros below this second threshold.

It is also not a general consensus that the autoconversion formula must be a continuous function of the cloud-water mixing ratio. For example, Tripoli and Cotton (1980) propose to use the formula (4.2) without shifting the origin in the monotonic function, $f$, by

setting, $x* = x$ in combination with Kessler's stepfunction switch.

Inclusion of the ice processes:

Bulk microphysics including the ice processes are already attempted in early history of microphysics scheme development (e.g., Weinstein 1970, Ogura and Takahashi 1971 Wisner et al. 1972, Orville and Kopp 1977, Lin et al., 1983). Among those earlier studies, Lin et al. provide the most complete formulation under the bulk framework.

In developing bulk ice microphysics, the first challenge to be encountered is a description of the primary ice formation process. In warm microphysics, the saturation condition (against a plane surface) is considered to be a reasonable approximation as a criterion for the formation of liquid-phase cloud particles. This rather simple criterion works relatively well, because liquid-phase condensation nuclei are abundant, and almost as soon as the saturation condition (in respect to the plain surface) is satisfied, a cloud particle can be more or less immediately formed on a condensation nuclei. The saturation rarely exceed 1

The situation becomes completely different below the freezing point, because the ice condensation nuclei are relatively rare. For this reason, the deposition growth is much slower than condensation, and as a result, supersaturation over ice can be often as large as 70to slow ice formation processes, liquid-phase particles also remain in liquid phase for a relatively long time without freezing along the ascent. This condition persists until the temperature reaches below -36C. Below this temperature, all the liquid particles freeze by a process called homogeneous nucleation.

Thus, a simple formulation for ice particle (ice-phase cloud particles) formation process is not easy. For this reason, Weinstein (1970), as one of the first attempts to include ice process, only considers freezing heating at a threshold temperature (-8C by default), but even the change of the terminal velocity is not taken into account. Ogura and Takahashi (1971), Wisner (1972) do not include cloud ice at all, either, in order to avoid the issue of the primary ice formation.

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An easy way to override this difficulty is to invoke the so-called Bergeron, of Wegner-Bergern-Findeisen process (Wegner 1911, Bergeron 1935, Findeisen 1939). This process is realized when the atmosphere is undersaturated against liquid but supersaturated against ice. When cloud liquid and ice co-exist under this situation, cloud liquid evaporate into water vapor, and the evaporated water vapor, in turn, deposits on to the cloud ice. As a result, cloud ice can grow in expense of cloud liquid.

A phenomenological, simple formulation (e.g., Koenig 1971) can relatively easily be developed that transforms the cloud liquid into the cloud ice directly without explicitly invoking the water-vapor phase process. For example, Orville and Kopp (1977) solely rely on this process for the cloud ice (snow) formation.

However, the phase space that the Bergeron process can be strictly applied (i.e., undersaturated against liquid but supersaturated against ice) is very limited as pointed out by Korolev (2007). For this reason, in most of the situations, we need to consider vapor deposition on ice more explicitly rather than invoking a simplified Bergeron theory.

To a good extent, it has been considered that the key for simplifying the ice bulk microphysics description is to avoid an explicit treatment of primary ice formation process. The most popular approach along this line is to take a weighted average of the saturated water vapor in respect to liquid and ice (e.g., Squires and Turner 1962, Wisner et al., 1972, Lord et al., 1984). Proportion of ice to total condensate can be assumed to be a linear function of temperature between a prescribed maximum and minimum temperatures. Above the prescribed maximum temperature, no ice exists. Below the prescribed minimum temperature, the all condensate is in ice form.

Grabowski (1998) further proposes to simplify the treatment by even turning off heating by ice fusion (and also cooling by ice melting). As a result, the only difference of ice phase from the liquid phase is in their terminal velocity, as shown in Fig. 2 of the present manuscript. In the final manuscript, along this line, we simply account this effect by changing the terminal velocity from the standard value, 5 m/s, to a smaller

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value, 1 m/s below the freezing point. Note that the choice of 1 m/s is valid for wide range of snow sizes as shown in Fig. 2. This clearly oversimplified inclusion of ice effect produces much realistic stratiform clouds than the other cases, as will be shown in the final manuscript.

On the other hand, Ooyama (1990) argues that this interpolation procedure can be more elegantly achieved by taking the entropy instead of the water-vapor saturation. A smooth version of entropy by taking an interpolation between those for the liquid and the ice phase. By invoking the Kirchhoff equation, the saturated water vapor can be defined unambiguously from the defined smooth entropy.

Mixed phase microphysics:

Mixed phase microphysics are extremely complex. One of the difficulties is in defining the ice number concentration. This is difficult in multitude of reasons. Primarily ice formation process (heterogeneous nucleation) is already involved, as discussed just above. However, additionally there are multitude of secondary ice formation processes on that our knowledge is far from complete (cf., Cantrell and Heymsfield 2005). It may not be necessary to mention that the first author has just recently pointed out an importance of relatively less emphasized ice multiplication process due to the ice-ice collision (Yano and Phillips 2011). However, it makes the point clear how complex the mixed phase microphysics is. Less is yet clear, how to make it even simpler.

The first author strongly believes that careful sensitivity studies under an idealized set-up are required in order to identify which process is critical in what circumstances. For example, Yano and Phillips (2011) elucidate importance of the ice-ice collision as a secondary ice formation process by adopting an idealized zero-dimensional model. Under this zero-dimensional model, it is shown that the system is characterized by a single nondimensional parameter measuring the relative efficiency of the ice-multiplication process. When this parameter is larger than unity, an explosive ice multiplication is expected. Under this theoretical framework, we can easily identify whether this ice-

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multiplication process is effective (i.e., explosive) with a given cloud condition.

Similar process studies can be performed for the other mixed-phase ice physics processes. Such systematic study would ultimately lead to simple mixed phase microphysics. It is emphasized that a drastic simplification of the microphysics is the key in order to perform such a careful sensitivity analysis.

Disentangling of the complex mixed phase microphysics appears to be still a long way to go. However, the minimum bulk microphysics developed for the warm rain process in the present study is expected to provide an important prototype in order to develop a similar "minimum mixed phase microphysics".

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3) A final short remark about the typos in the text:

Probably needless to say, however, all the typos pointed by the anonymous referee 1 have been corrected accordingly.

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**Fig. 1.**

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