Atmos. Chem. Phys. Discuss., 10, C10926–C10941, 2010
www.atmos-chem-phys-discuss.net/10/C10926/2010/

Atmospheric
Chemistry
and Physics
Discussions

Interactive comment on "Where do winds come from? A new theory on how water vapor condensation influences atmospheric pressure and dynamics" *by* A. M. Makarieva et al.

A. M. Makarieva et al.

ammakarieva@gmail.com

Received and published: 10 December 2010

Reply to Dr. Gavin Schmidt on Bryan & Rotunno (2009)

1 Introduction

In our submitted article (hereafter M10) we claim that the dynamic effects of condensation-associated mass removal have never been considered adequately in meteorological theory. Yet we recognise several modelling studies that claim to have fully accounted for the mass removal effect. These have concluded that any conden-

sation influence is small compared to heat driven atmospheric pressure gradients and therefore relatively unimportant. This is a point we intend to address here.

Since the terms *theory* and *model* are often used interchangeably[1] and the meanings too easily confused, many climate researchers appear convinced that available model results have been sufficient to judge the magnitude of the mass sink effect[2]. Recently, an interesting discussion of the condensation mass sink at Jeff Id's blog was prompted by a summary from Dr. Kerry Emanuel who stated that[3]:

The bottom line is that while the effect should be included in any model that claims to conserve mass, it is not quantitatively large.

In his fuller argument Dr. Emanuel referred, for support, to the work of Dr. George Bryan. In the discussion that followed Dr. Gavin Schmidt made several comments referring to particular works of Dr. Bryan and colleagues (Bryan and Rotunno, 2009 [hereafter BR09]; Bryan and Fritsch, 2002 [hereafter BF02]) in the context of our work. Dr. Schmidt wrote[4]:

*It is beyond question that standard hurricane models (as used in routine forecasts by GFDL, NHC etc.) do produce hurricanes in the right circumstances. While not perfect (and therefore potentially improvable by consideration of the precip mass loss terms you are proposing – and has been suggested by Bryan et al), these models are an undeniable testament that your mechanism is *not* essential to the existence of hurricanes, and that one can get a very good approximation just from using the latent heat terms.*

---

[1] http://judithcurry.com/2010/12/01/climate-model-verification-and-validation/

[2] http://judithcurry.com/2010/10/23/water-vapor-mischief/#comment-10878

[3] http://noconsensus.wordpress.com/2010/10/26/weight-of-water-and-wind-hurricane- pros-weigh-in/

[4] http://noconsensus.wordpress.com/2010/10/26/weight-of-water-and-wind-hurricane-pros-weigh-in/#comment-39668

Here we shall briefly examine the BR09 model and show that how it accounts for the precipitation mass sink is incorrect. We show that the problem of neglecting the mass sink is not just a relatively minor violation of mass conservation – but leads to a violation of the law of energy conservation. We also discuss how current models reproduce observed wind speeds despite their neglect of condensation.

2 The methods to determine condensation rate in BR09 and BF02

With introducing a new variable, condensation mass sink S, to the previously closed system of equations of hydrodynamics, an additional equation is required to keep the system closed. Such an additional constraint cannot be derived from numerical integration of models that build on the originally closed system with $S = 0$. In the BF09 model there are seven equations for seven variables that are listed on p. 1771. However, the dimensionless condensation rate denoted as $\dot{q}_{cond}$ in BF09 is not listed among these variables despite its presence in the system of equations. There is no equation for $\dot{q}_{cond}$. It is noted that "the method to determine $\dot{q}_{cond}$ was described by Bryan and Fritsch (2002)".

BF02 describe this method as follows (p. 2920, note that in BF02 $\dot{q}_{cond}$ is denoted as $\dot{r}_{cond}$):

To account for phase changes, the model uses a saturation adjustment technique, similar to that proposed by Soong and Ogura (1973). In this technique, the equations are advanced forward in two steps: a dynamical step and a microphysical step. In the dynamical step, the model equations are integrated forward with all terms involving phase changes neglected. Then, the microphysics step is applied, in which only the terms involving phase changes are included. This technique is identical to that used by Klemp and Wilhelmson (1978).

Pressure tendencies due to phase changes are included in the microphysics step.

Since changes in pressure affect the saturation vapor pressure, an iterative scheme had to be developed for condensation. In this iterative scheme, equations (21)-(24) are advanced forward using a guess for $\dot{r}_{cond}$. The new values of θ and π are then used to calculate a new value of saturation mixing ratio (r_{vs}), which is then used to calculate a new guess for $\dot{r}_{cond}$. This cycle is repeated until the newest value of θ converges (to within machine accuracy) to the previous value. During each iteration, the value of $\dot{r}_{cond}$ is determined by the following equation from Rutledge and Hobbs (1983):

$$\dot{r}_{cond} \equiv \frac{r_v - r_{vs}}{\Delta t(1 + \frac{L_v^2 r_{vs}}{c_p R_v T^2})} \qquad \text{(Eq. 27 of BF02)}$$

where Δt is the time step. The iterative technique usually converges in 4-6 iterations.

In this context Dr. Schmidt commented[5]:

*So, I would like to ... ask again, why you do not think that the treatment of $\dot{q}_{cond}$ *in the dynamics* by B&R is satisfactory to you. I am not asking why $\dot{q}_{cond}$ is changing, but on the effect of a given change in $\dot{q}_{cond}$ has on the dynamics.*

Here the problem lies in finding a physically justfyable formulation of condensation rate to feed into the dynamics equations. In our previous comment (Makarieva et al., 2010, Eq. (1))[6] we showed that the magnitude of the horizontal pressure gradient is highly sensitive to the formulation of condensation rate. If condensation rate is incorrectly formulated under assumptions that are inconsistent with the assumptions for the dynamics equations, the model output will be flawed.

Despite being key for analysis of a mass sink, the physical meaning of Eq. (27) in BF02 is never explored. Instead, references are provided to numerical techniques designed in several earlier studies. Those studies were not concerned with investigating the

---

[5] http://noconsensus.wordpress.com/2010/10/26/weight-of-water-and-wind-hurricane-pros-weigh-in/#comment-39752

[6] http://www.atmos-chem-phys-discuss.net/10/C10922/2010/

mass sink dynamics but sought a reasonable approximate estimate for the rate of latent heat release. These expressions were adopted for the BF02 and BR09 models without evaluation of their suitability for the studies in question. Let us explore the physical meaning of Eq. (27) of BF02 here.

This formula can be written as follows[7], Fig. 1:

$$\delta\gamma_n = \frac{\delta\gamma}{1+\mu\gamma_s\xi^2}, \ \ \xi \equiv \frac{L}{RT}, \ \ \mu \equiv \frac{R}{c_p}, \gamma \equiv \frac{p_v}{p_d} \approx \frac{p_v}{p}, \tag{1}$$

where L, R and c_p are molar heat of vaporization, universal gas constant and heat capacity at constant pressure, respectively, $\delta\gamma \equiv \gamma_s - \gamma$, $\delta\gamma_n = \gamma_s^{'} - \gamma$, γ is the oversaturated mixing ratio of water vapor in the initial oversaturated state at temperature T, γ_s is the saturated mixing ratio corresponding to this temperature, $\gamma_s^{'}$ is the saturated mixing ratio in the final state at temperature $T^{'}$.

Condensation of oversaturated vapor can be viewed as two simultaneous processes: condensation of a certain amount of vapor, $\delta\gamma_n$, with release of latent heat $-L\delta\gamma_n > 0$, and evaporation of a certain amount of vapor, $\delta\gamma_s = \delta\gamma_n - \delta\gamma > 0$, with absorption of the released latent heat. The first law of thermodynamics describing this process (if it occurs adiabatically) will read, see Eqs. (7) and (1) in M10:

$$\frac{\delta T}{T} - \mu\frac{\delta p}{p} + \mu\xi\delta\gamma_n + \mu\xi\delta\gamma_s = 0, \ \ \delta T \equiv T^{'} - T. \tag{2}$$

Temperature change δT is related to pressure and saturated mixing ratios in the two states by the Clausius-Clapeyron law, see Eq. (8) in M10:

$$\frac{\delta T}{T} = \frac{1}{\xi}\left(\frac{\delta\gamma_s}{\gamma_s} + \frac{\delta p}{p}\right). \tag{3}$$

---

[7] In the notations of BF02, $\delta\gamma_n \equiv -\dot{r}_{cond}\Delta t$, $\delta\gamma \equiv -(r_v - r_{vs})$, $\gamma_s \equiv r_{vs}$, $\gamma \equiv r_v$.

Putting Eq. (3) into (2) and using $\delta\gamma_s = \delta\gamma_n - \delta\gamma$ we obtain:

$$\delta\gamma_n = \delta\gamma\frac{1+\mu\gamma_s\xi^2}{1+2\mu\gamma_s\xi^2} + \gamma_s(\xi\mu - 1)\frac{dp}{p}. \tag{4}$$

One can see that Eq. (4) does not coincide with Eq. (1) even at constant p (adiabatic condensation at constant volume).

Equation (1) can be obtained using the following (incorrect, as we shall see) chain of arguments. One has to assume that at first the oversaturated state characterized by γ goes to the final state with $\gamma_s^{'}$ with release of latent heat equal to $-L\delta\gamma_n > 0$ at constant pressure. Then the release of heat warms the gas at constant pressure, $dp = 0$, which means that $-L\delta\gamma_n = c_p\delta T$. Here $\delta T = T^{'} - T$, where $T^{'}$ is the temperature of the final state. One then relates δT to the final state mixing ratio $\gamma_s^{'}$ by the Clausius-Clapeyron law (3) with $dp = 0$. From $-L\delta\gamma_n = c_p\delta T$ and $\delta T/T = (1/\xi)(\delta\gamma_s/\gamma_s)$ one obtains Eq. (1).

This logic is overviewed in the paper by Yau and Austin (1979) that is referred to in the paper by Rutledge and Hobbs (1983):

> *Modification of supersaturation mixing ratio through the release of latent heat in a manner proposed by Asai (1965) is adopted. The saturation excess $\delta M = q_v - q_s$ is calculated using Teten's formula [as given in Asai (1965)] to obtain q_s. A part of δM, δM_1, is condensed releasing latent heat which causes an increase of temperature; in turn, this permits the residue, $\delta M_2 = \delta M - \delta M_1$, to remain in vapor form.*

Commentators who positioned themselves as knowledgeable in the subject[8] confirmed that it is indeed following the above logic that condensation rate is routinely estimated. But it is clear from Eqs. (2) and (3) that such logic and the resulting equation lead to a

---

[8] See, e.g., http://noconsensus.wordpress.com/2010/10/26/weight-of-water-and-wind-hurricane-pros-weigh-in/#comment-40156 and other comments there by commentator Jim D.

violation of the first law of thermodynamics. Indeed, it does not account for the latent heat $L\delta\gamma_s$ that is needed to obtain the final saturated state $\gamma_s^{'}$ from the initial state γ_s. Moreover, adiabatic phase transitions at constant pressure (as well as at constant molar volume) are thermodynamically prohibited, see Section 2 in M10. Therefore, evaporation of the amount $\delta\gamma_s$ of vapor cannot occur under such conditions. We thus conclude that the account of mass sink in the method used by BF02 is not in agreement with the first law of thermodynamics.

3 Neglected energy

The consequences for energy conservation of an incorrect formulation of condensation rate is easily illustrated by consideration of moist adiabatic ascent in an atmospheric column that is in approximate hydrostatic equilibrium. In such a case we have three equations: (1) 1st law of thermodynamics; 2) the Clapeyron-Clausius law and (3) hydrostatic equilibrium. The system of these equations can be solved to obtain vertical profiles of temperature lapse rate $\Gamma \equiv -dT/dz$, relative partial pressure of saturated vapor $\gamma \equiv p_v/p$ and pressure p, see Eqs. (22)-(23) of M10.

Condensation changes the amount of gas in the column and disturbs the equilibrium, Fig. 2. Suppose that an air parcel corresponding to a given amount of moist saturated air, e.g., 1 mol, is displaced upwards by a small distance dz. The three equations will dictate how the pressure, temperature, and vapor mixing ratio will change. They will also dictate the change of its molar volume. But due to loss of mass this parcel will occupy a smaller volume than it would in the absence of condensation. Indeed, the actual volume of the parcel was equal to the molar volume at height z, but has become less than molar volume at $z+dz$. With the vertical pressure distribution fixed, this could be visualized as a horizontal shrinking of the parcel as it ascends, Fig. 2.

Apparently, this causes a disequilibrium in the pressure distribution. To avoid this and

satisfy the average condition of hydrostatic equilibrium for an inherently non-hydrostatic process, one must add dry air (or moist air) to fill the void. Such a formalism implies that every act of condensation is accompanied by an immediate re-distribution of dry air to maintain the equilibrium. We emphasize that there is no alternative solution: if the column where the moist saturated air is ascending is observed to be in equilibrium, such a compensatory process must be occurring. The error has been to neglect the energy changes associated with condensation.

To date, the shortcomings of omitting the condensation mass sink has been discussed in terms of violating the mass conservation only. For example, Thuburn (2008) wrote[9]:

> *Currently most if not all atmospheric models fail to make proper allowance for the change in mass of an air parcel when water vapour condenses and precipitates out. A typical formulation in terms of virtual temperature implicitly replaces the condensed water vapor by an equal volume of dry air. This approximation can lead to noticeable forecast errors in surface pressure during heavy precipitation, for example. However, the approximation will not lead to a systematic long term drift in the atmospheric mass in climate simulations provided there is no long term drift in the mean water content of the atmosphere.*

However, the problem cannot be reduced to violation of mass conservation alone. The problem is that condensation disturbs equilibrium pressure distribution and thus creates a pressure gradient force. Thus, condensation is associated with a release of potential energy. Adding air to the column to sustain the equilibrium conditions involves work. Let us compare two steady-state hydrostatic equilibrium columns, one with and one without precipitation. In the first case one must account for work that is associated with adding air to the volume where condensation has taken place. This work is proportional to precipitation rate. Adding gas to the atmosphere via evaporation is associated

---

[9] http://judithcurry.com/2010/12/01/climate-model-verification-and-validation/

with work performed on the atmosphere – the point is that the other gases are compressed to make room for the incoming vapor. Condensation later releases this work and makes it available for conversion to mechanical energy of winds. In Section 4.1 of M10 it is shown that the global average rate of this potential energy release coincides with the intensity of the general circulation[10]. Therefore, lack of a proper account of the precipitation mass sink is equivalent to neglecting a major energy term in atmospheric dynamics.

The condition that hydrostatic equilibrium is maintained despite the presence of a non-hydrostatic process is a strong constraint. It should be properly taken into account in all theoretical formulations of atmospheric processes. The first law of thermodynamics is conventionally written for a given amount of gas (e.g., per 1 g or 1 mol). The change of vapor mass is taken into account in terms of change of the mixing ratio $\gamma_d \equiv p_v/p_d$ rather than in terms of relative partial pressure $\gamma \equiv p_v/p$. The physical meaning of using γ_d in the first law of thermodynamics in a laboratory system consists in the argument that changes in the dry air density can serve as the reference to discriminate condensation-related changes of vapor from those that affect all (condensable and non-condensable) gases equally. But this principle cannot be used to describe processes in the circulating atmosphere that is subject to different physical constraints than laboratory systems. The condition that dry air is instantaneously re-distributed to keep the column in equilibrium is mathematically equivalent to the presence of a source of dry air equal to the mass sink of vapor. Therefore, in hydrostatic equilibrium the change of mixing ratio $d\gamma_d$ does not describe condensation rate. Instead of dry air, the reference one must use is the total moist air that remains in equilibrium and, consequently, $d\gamma$ instead of $d\gamma_d$.

The hydrostatic distribution of dry air is explicitly assumed when deriving formulae to describe condensation rate. An example is Das (1969, p. 406) who is referred to by Klemp and Wilhelmson (1983) to whom Bryan and Fritsch (2002) refer in turn as one

---

[10] Makarieva and Gorshkov (2009) showed that the same potential energy is sufficient to power hurricanes.

authority for their method for estimating condensation rate. As we showed in our previous comment (Makarieva et al., 2010)[11] using mixing ratio γ_d instead of γ to describe condensation rate is equivalent to considering dry air to be in hydrostatic equilibrium and produces zero horizontal gradients. In the model of BF02 and BF09, which considers hydrostatic equilibrium of *moist air* as the basic state, calculations of condensation rate are based assuming *dry air* to be in equilibrium. This error and its implications remain hidden due to the lack of explicit theoretical analyses. It also leads to erroneous conclusions about the magnitude of the mass sink's influence on atmospheric dynamics. (Note that due to small $\gamma \ll 1$ the assumption that dry air is in equilibrium causes only a minor relative change in the dry air distribution compared to the assumption that moist air as a whole is in equilibrium. However, in absolute terms this change corresponds to large vertical accelerations and a total disequilibrium pressure of the order of p_v. Likewise, small changes in condensation rate formulations (in terms of γ_d versus γ) lead to profound changes in the resulting horizontal pressure gradients.)

Next we shall show how an approximate agreement with observations has been reached by the BR09 model despite it neglecting the energy term associated with the mass sink.

4 Empirical fitting of models

In the absence of a mass sink, the conventionally considered circulation driver is differential heating. Heat is somewhere added to gas, its temperature rises and a spatial pressure gradient is formed. However, every gas volume not only receives heat from a local source (e.g., latent heat release from condensation), but also loses/gains heat via thermal conductivity. If thermal conductivity is sufficiently large, it erases the dynamic effect of differential heating nearly completely. Thermal conductivity represents the in-

---

[11] http://www.atmos-chem-phys-discuss.net/10/C10922/2010/

evitable losses that are associated with any process where heat could be converted to work.

So, if one considers a theory that attempts to explain the circulation power based on consideration of (latent) heat release, such a theory must evaluate the losses to heat conductivity from the basic physical principles. If, after such an account of thermal conductivity, mechanical power produced by the differential heating remains sufficiently large (i.e., associated with a sufficiently large pressure gradient), one can conclude that the theory has explained the circulation.

Thermal conductivity in the atmosphere is associated with turbulence and is a function of wind speeds. Therefore, one should be able to estimate eddy viscosity ν in the hurricane. Using the same approach as Rotunno and Emanuel (1987), BR09 derive eddy viscosity ν_v for horizontal turbulence from dimensional considerations as a function of a horizontal length scale l_h (Eq. (17) in BR09). Thus, if one knew l_h from basic physical principles and, using the BR09 model of that ignores mass removal, were able to obtain a realistic hurricane velocity, one could confidently assert that hurricanes appear to be driven by latent heat alone while the mass sink effect must be negligible. But this is not the case. In reality no information about l_h exists. BR09 recognise this explicitly:

There is no quantitative theoretical guidance for how to set l_h *and* l_v *in an axisymmetric model. RE87 [Rotunno and Emanuel 1987] used* l_h *= 3000 m and* l_v *= 200 m in their simulations, which they determined by trial and error, and by subjective evaluation of model output. ...*

One might wonder whether we can determine values for l_h *and* l_v *that yield reasonably realistic hurricanes as compared to observations. Based on the estimated observed maximum intensity of 70 m/s, as well as comparisons of maximum radial inflow to observations, it seems that* l_h *= 1500 m and* l_v *= 100 m are appropriate.*

In other words, the key numerical parameter that determines hurricane intensity is taken to be precisely such that the latent heat release (more correctly, its spatial gradient related to vertical velocities) produces a realistic radial pressure gradient and realistic maximum velocity. No theoretical arguments explain why it should be 1500 m – it is rather the best fit to the data if *we assume* that heat is the primary source of pressure gradients driving the system. Despite its incorrect account of the precipitation mass sink, the model BR09 gives relatively realistic behaviours because it has been fitted to do so. It does not constitute a hurricane theory and does not in any scientific sense "prove" that hurricanes are driven by latent heat release.

Conceptually similar parameterization schemes are required in global circulation models. Yet theories of atmospheric circulation experience difficulties in reproducing (from the basic physical principles) even the mean intensity of circulation and other basic parameters. Characterized broadly, the problem with the current temperature gradient theory is that the realistic differential heating does not produce realistic wind velocities (the velocities are too low, see, e.g., Held and Hou, 1980; Fang and Tung, 1999; Schneider, 2006). (For us this is not surprising, as an essential mechanism has been neglected and omitted.) In the meantime, global circulation models have been reproducing the observed pressure and velocity fields rather precisely already from the onset of climate modelling in late 50s of the last century. That models outperform their foundational theories is remarkable but less than it may initially seem given the extensive reliance on empirical fitting and evolution-like selection and modification of models that do "seem to fit".

5 Conclusions

Here we have argued that the account of precipitation mass sink in BF02 and BF09 is incorrect for two reasons: (1) the formula used by BF02 to calculate condensation rate does not conform to the laws of thermodynamics and (2) the conclusion that the impact of the mass sink on dynamics is negligible is based on implicit assumption of dry air being in equilibrium when calculating condensation rate. We also showed that the agreement with observations in the model of BF09 is achieved by direct empirical fitting. To our knowledge, all current models that are said to incorporate the precipitation mass sink do so using formulations of condensation rate similar to those used in the models of BF02 and BF09. We thus conclude that none of such models can be used to investigate the importance of the true mass sink. We repeat our call to the climate community for a serious investigation of the dynamic consequences of vapor mass non-conservation in the atmosphere.

Finally, let us once again emphasize the conceptual difference of the mass sink dynamics versus differential heating dynamics in terms of the challenges these dynamics pose to a climate theorist. When heat is released, one must first estimate the efficiency with which heat can be converted to work. This efficiency can be close to zero (all heat is lost via thermal conductivity). The mass sink is associated with work from the very beginning – adding mass to the column is only possible via performing work on the column. Therefore, the intensity of the mass sink tells us about the intensity at which mechanical work is produced in the atmosphere directly, without any need to account for thermal conductivity (and, hence, atmospheric turbulence).

References

Bryan, G. H. and Fritsch, J. M., A benchmark simulation for moist nonhydrostatic numerical models, Mon. Wea. Rev., 130, 2917–2928, 2002.

Bryan, G. H. and Rotunno, R., The maximum intensity of tropical cyclones in axisymmetric numerical model simulations, Mon. Wea. Rev., 137, 1770–1789, 2009.

Das, P., The thermodynamic equation in cumulus dynamics, J. Atm. Sci., 26, 399–407, 1969.

Fang, M. and Tung, K. K.: Time-dependent nonlinear Hadley circulation, Journal of the Atmospheric Sciences, 56, 1797–1807, 1999.

Held, I. M. and Hou, A. Y.: Nonlinear axially symmetric circulations in a nearly inviscid atmosphere, J. Atm. Sci., 37, 515–533, 1980.

Klemp, J. B., and Wilhelmson, R. B., The simulation of three-dimensional convective storm dynamics. J. Atmos. Sci., 35, 1070–1096, 1983.

Makarieva, A. M. and Gorshkov, V. G.: Condensation-induced kinematics and dynamics of cyclones, hurricanes and tornadoes, Phys. Lett. A, 373, 4201–4205, 2009.

Makarieva, A. M., Gorshkov, V. G., Interactive comment on "Where do winds come from? A new theory on how water vapor condensation influences atmospheric pressure and dynamics" by A. M. Makarieva et al., Atmos. Chem. Phys. Disc., 10, C10922–C10925, 2010, www.atmos-chem-phys-discuss.net/10/C10922/2010/.

Rotunno, R., and Emanuel, K. A., An air-sea interaction theory for tropical cyclones. Part II: Evolutionary study using a nonhydrostatic axisymmetric numerical model, J. Atm. Sci., 44, 542–561, 1987.

Rutledge, S. A., and Hobbs, P. V., The mesoscale and microscale structure and organization of clouds and precipitation in mid-latitude cyclones. VIII: A model for the “seeder-feeder” process in warm-frontal rainbands, J. Atm. Sci., 40, 1185–1206, 1983.

Schneider, T.: The general circulation of the atmosphere, Annu. Rev. Earth Planet. Sci., 34, 655–688, 2006.

Soong, S.-T., and Y. Ogura, A comparison between axisymmetric and slab-symmetric cumulus cloud models. J. Atmos. Sci., 30, 879–893, 1973.

Thuburn, J., Some conservation issues for the dynamical cores of NWP and climate models, J. Comp. Phys., 227, 3715–3730, 2008.

Interactive comment on Atmos. Chem. Phys. Discuss., 10, 24015, 2010.

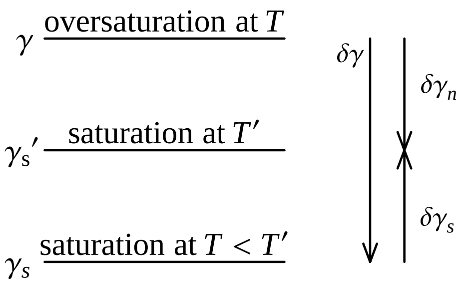

Fig. 1. Processes considered in the iteration scheme of BF02, Eq. (27). Note that here the gamma symbols all denote the vapor mixing ratio, not its relative partial pressure as in M10.

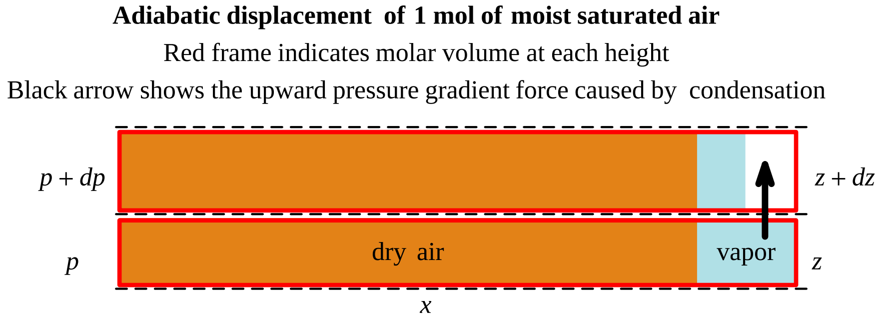

Fig. 2. Formation of a disequilibrium pressure gradient due to mass loss when one mol of moist saturated air is displaced upwards along a moist adiabat calculated for hydrostatic equilibrium of moist air.