



Supplement of

A new method to derive water vapor supersaturation at cumulus cloud base from in situ measurements

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Supplementary Figures

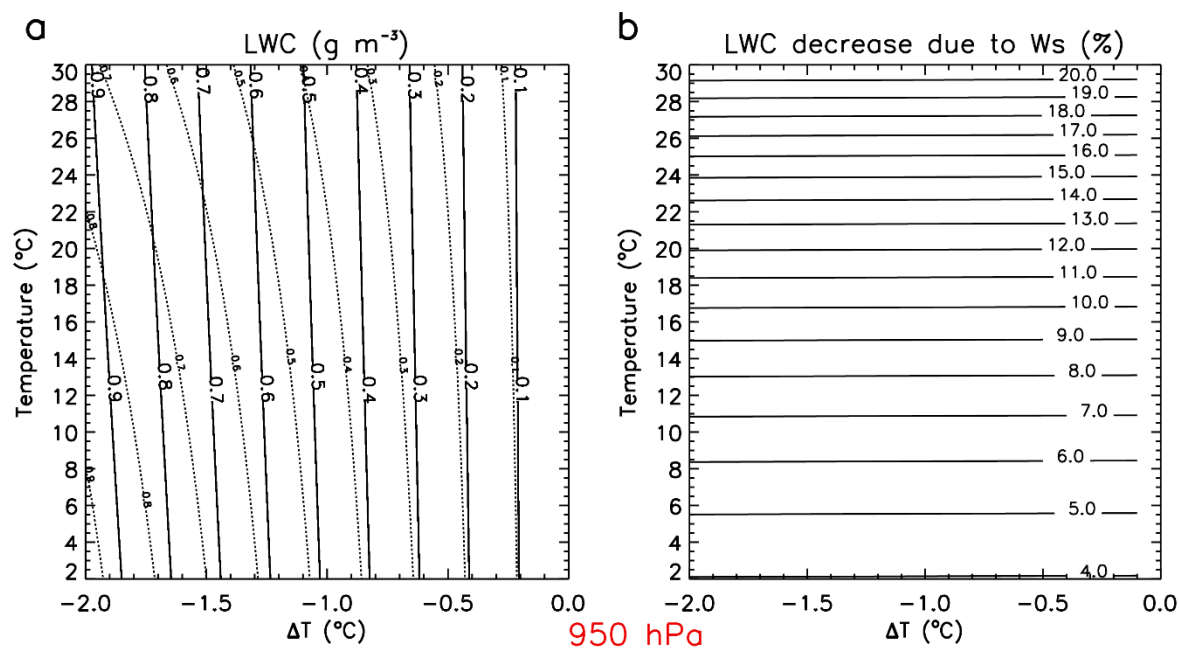


Figure S1. (a) Liquid water content (LWC) and its reduction due to vapor expansion work (W_s) at cloud base for a pressure of 950 hPa, shown as a function of cooling (ΔT) (x-axis) and initial temperature (T_0) (y-axis). LWC (g m^{-3}) calculated assuming all the sensible enthalpy change ($Q = dH$) contributes to condensational growth of droplets (solid contours), with the corresponding LWC including the effect of W_s shown as dotted contours. (b) similar to a) for fractional decrease in LWC (%) due to the inclusion of W_s .

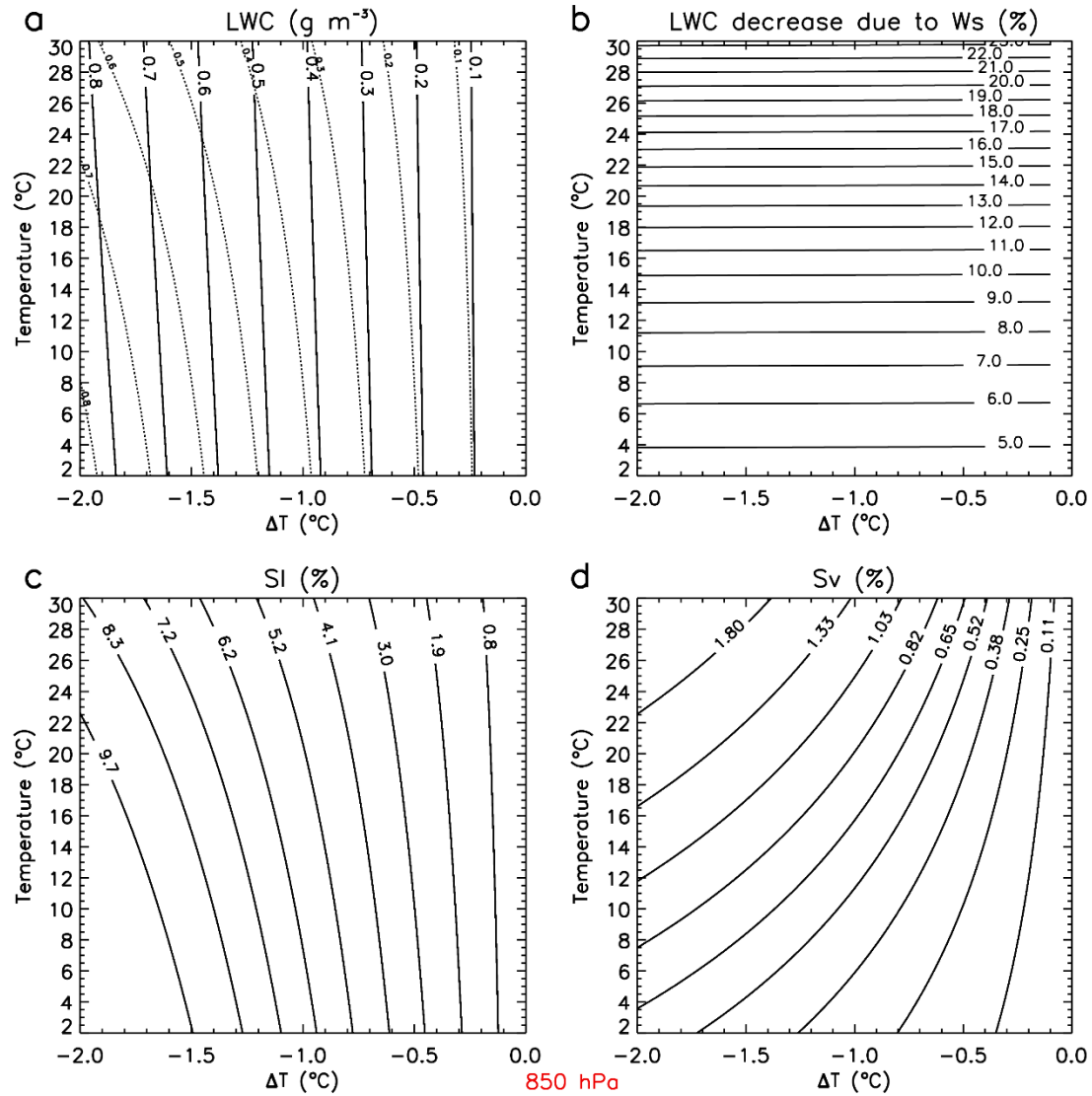
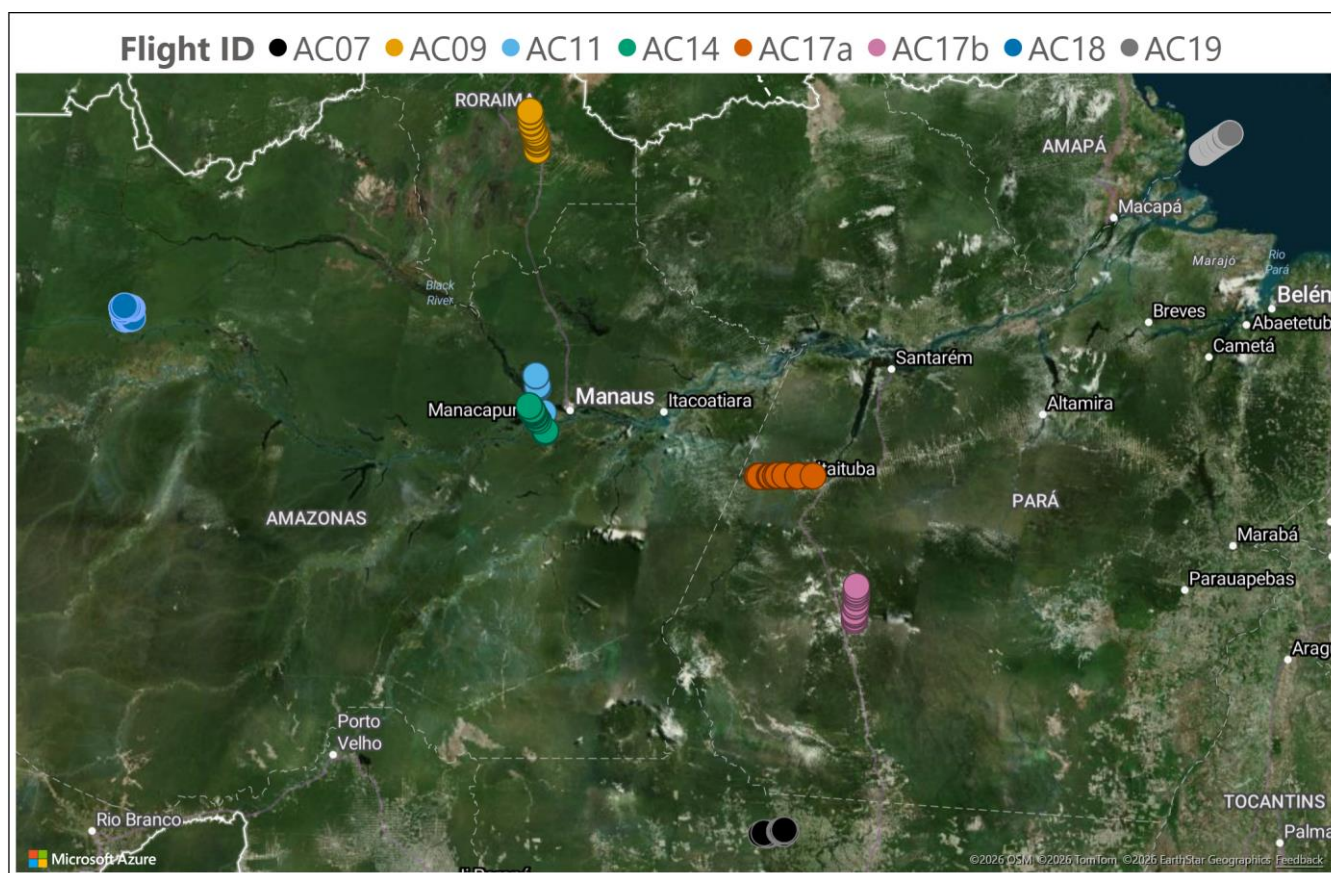
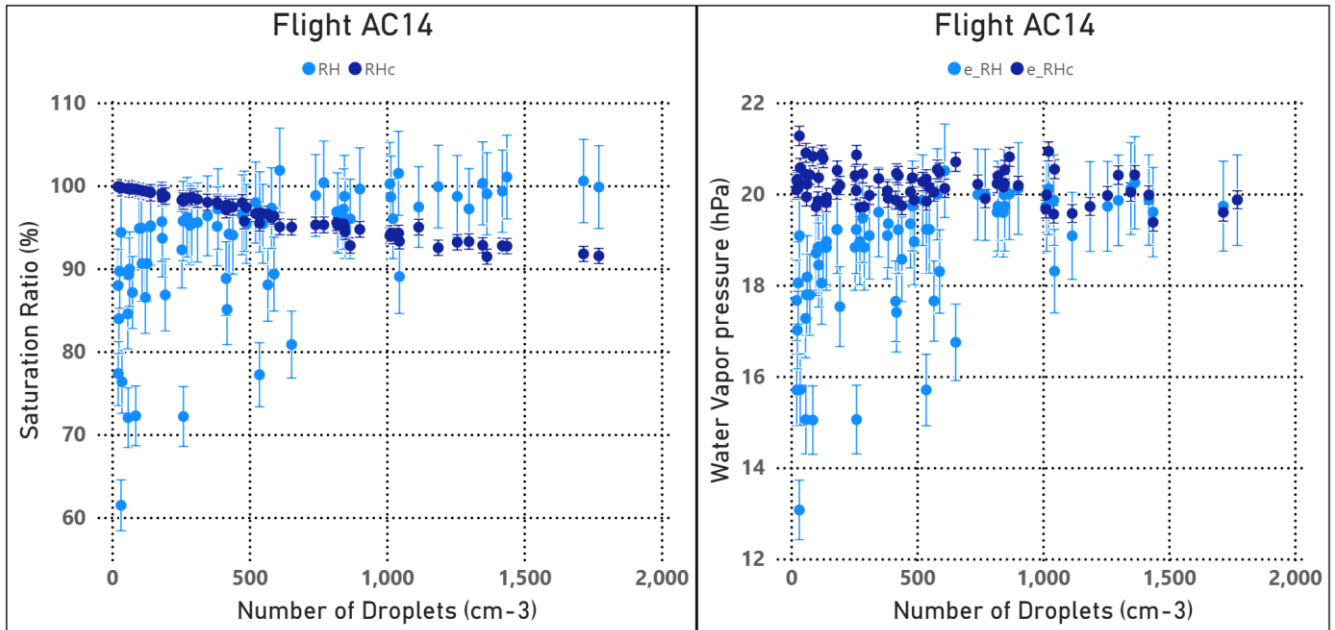


Figure S2. (a) Liquid water content (LWC) and its reduction due to vapor expansion work (W_s) at cloud base for a pressure of 850 hPa, shown as a function of cooling (ΔT) (x-axis) and initial temperature (T_0) (y-axis). LWC (g m^{-3}) calculated assuming all the sensible enthalpy change ($Q = dH$) contributes to condensational growth of droplets (solid contours), with the corresponding LWC including the effect of W_s shown as dotted contours. (b) similar to a) for fractional decrease in LWC (%) due to the inclusion of W_s ; c) and d) similar to a) for liquid supersaturation (S_l) and water vapor supersaturation (S_v), respectively.

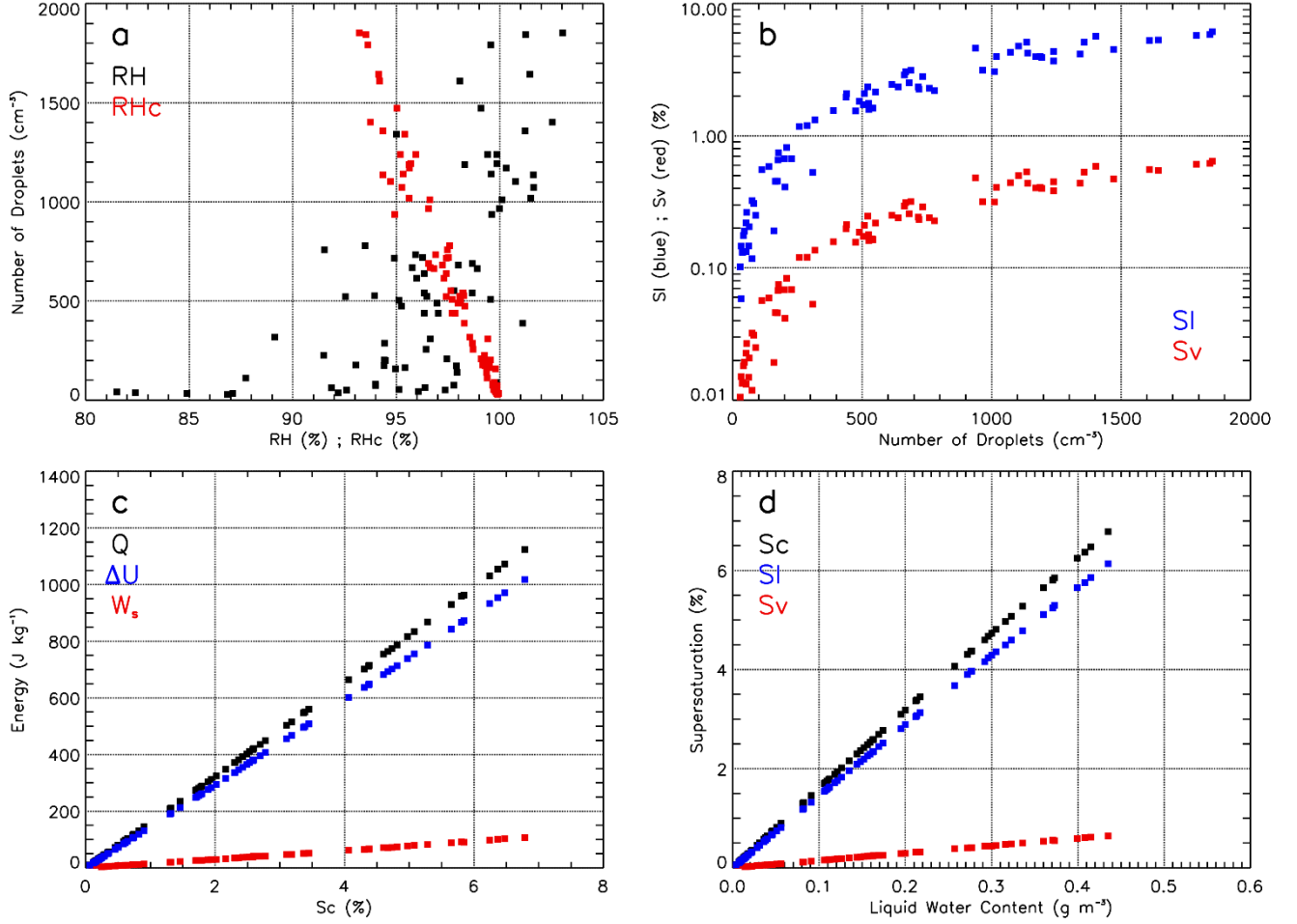


20 Figure S3. Regions of measurements at cloud bases of cumulus clouds during the ACRIDICON-CHUVA campaign. Flights are color-coded and labelled at the top.



30 Figure S4. [Left] Droplet number concentration - N_d (cm⁻³) vs. saturation ratio (%) RH and RH_c. [Right] N_d (cm⁻³) vs. water vapor pressure (hPa) calculated from RH and RH_c, where $e_{RH} = e_s(T) \cdot RH$, and $e_{RHc} = e_s(T_0) \cdot RHc$. Error bars indicate the estimated uncertainties in RH and e_{RH} (~5%), and in RH_c and e_{RHc} (typically <1%, with maximum values approaching ~1%). The corresponding flight segment is indicated at the top of each panel.

Flight AC07: Deforestation Arc



40 Figure S5. Cloud microphysical and thermodynamic properties measured at cloud bases of growing convective cumuli. (a) RH versus N_d (black) and RHc versus N_d (red). (b) N_d versus S_i (blue) and N_d versus S_v (red). (c) S_c versus Q , $|\Delta U|$, and W_s . (d) LWC versus S_c , S_i , and S_v . The corresponding flight segment is indicated at the top of the panel.

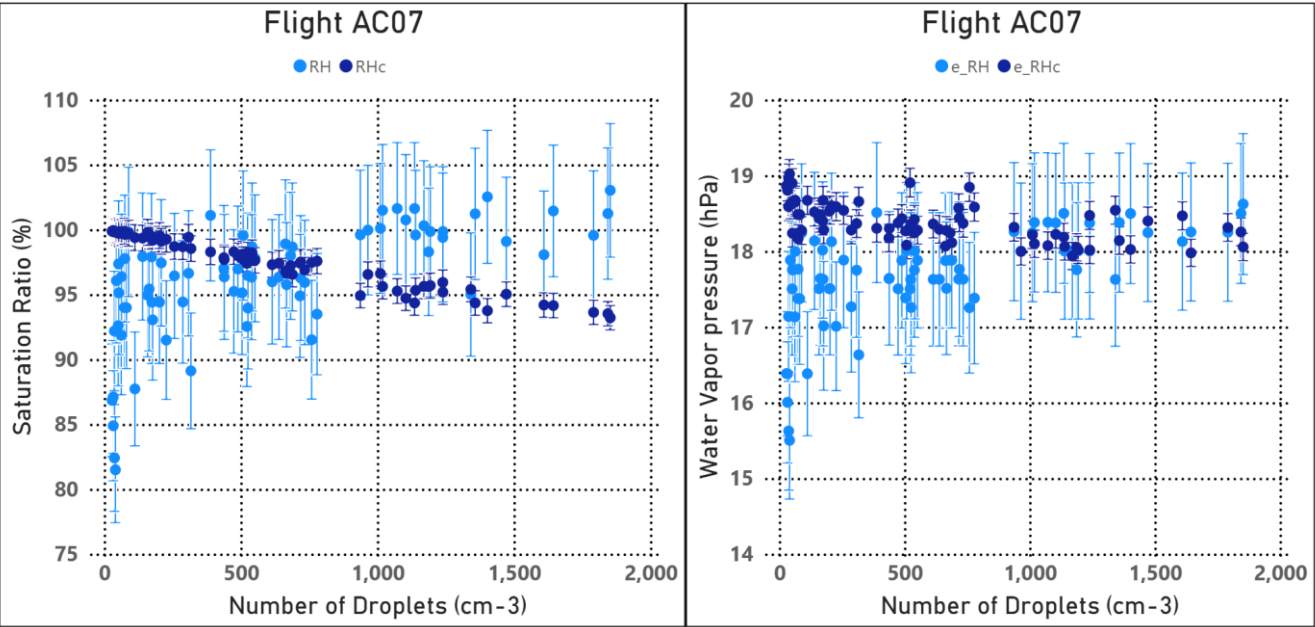


Figure S6. Similar to Figure S4 for flight segment AC07.

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Flight AC09: Northern Amazon

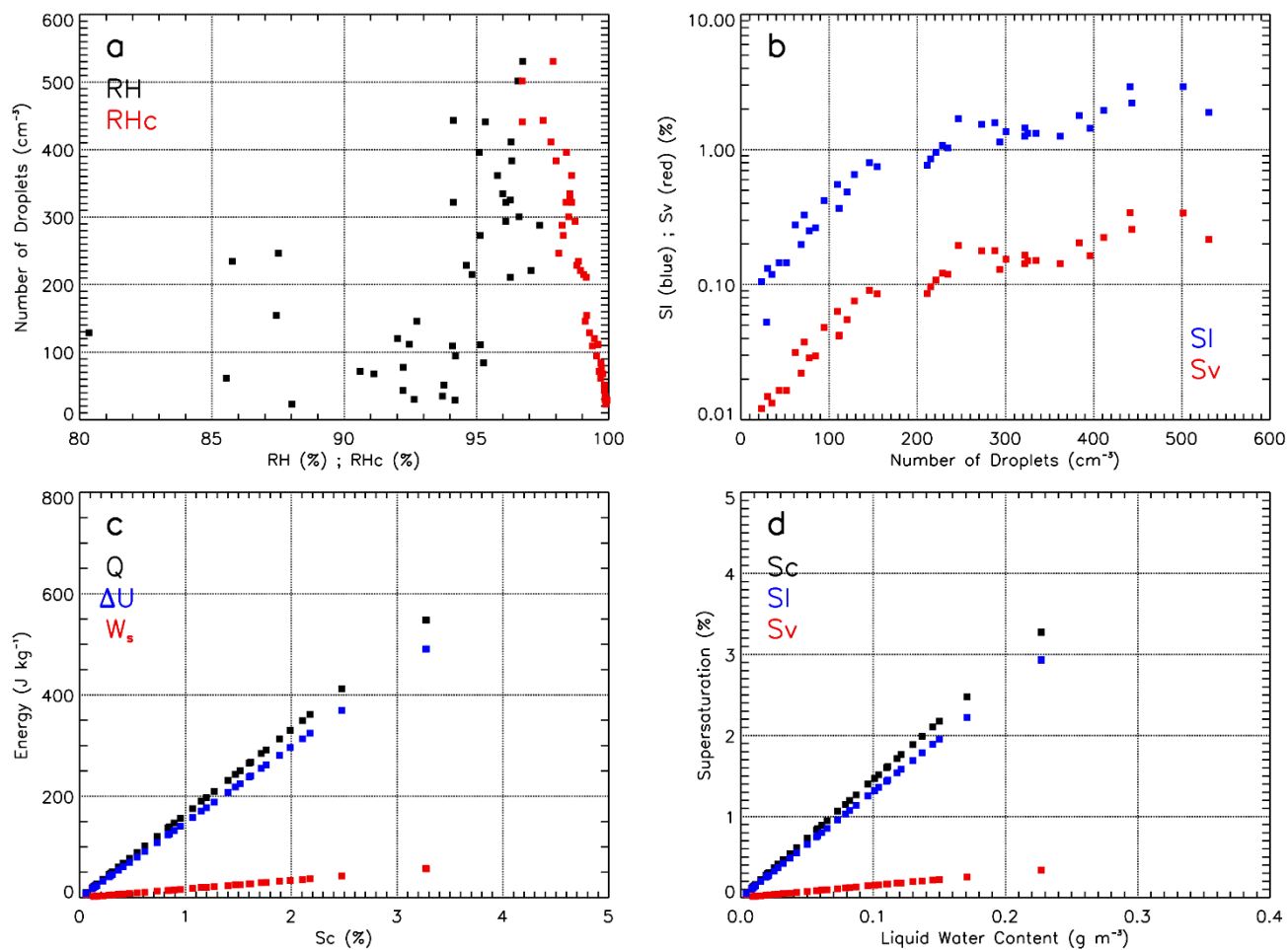


Figure S7. Similar to Figure S5 for flight segment AC09.

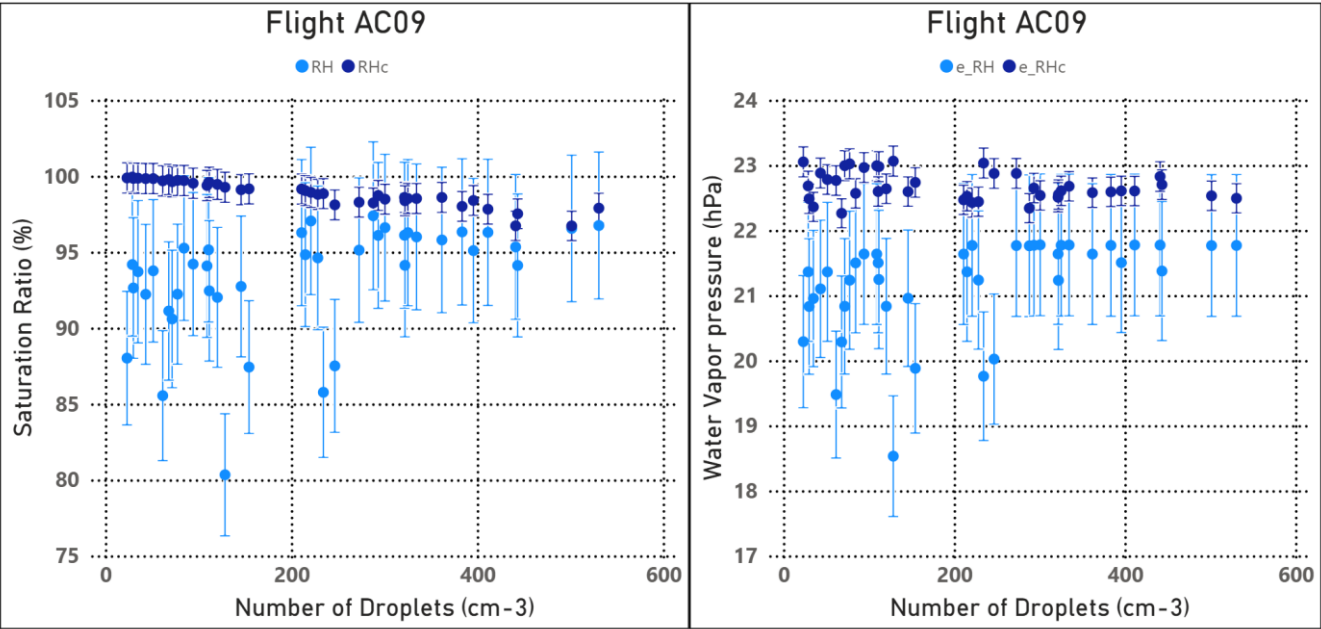


Figure S8. Similar to Figure S4 for flight segment AC09.

Flight AC11: Central Amazon

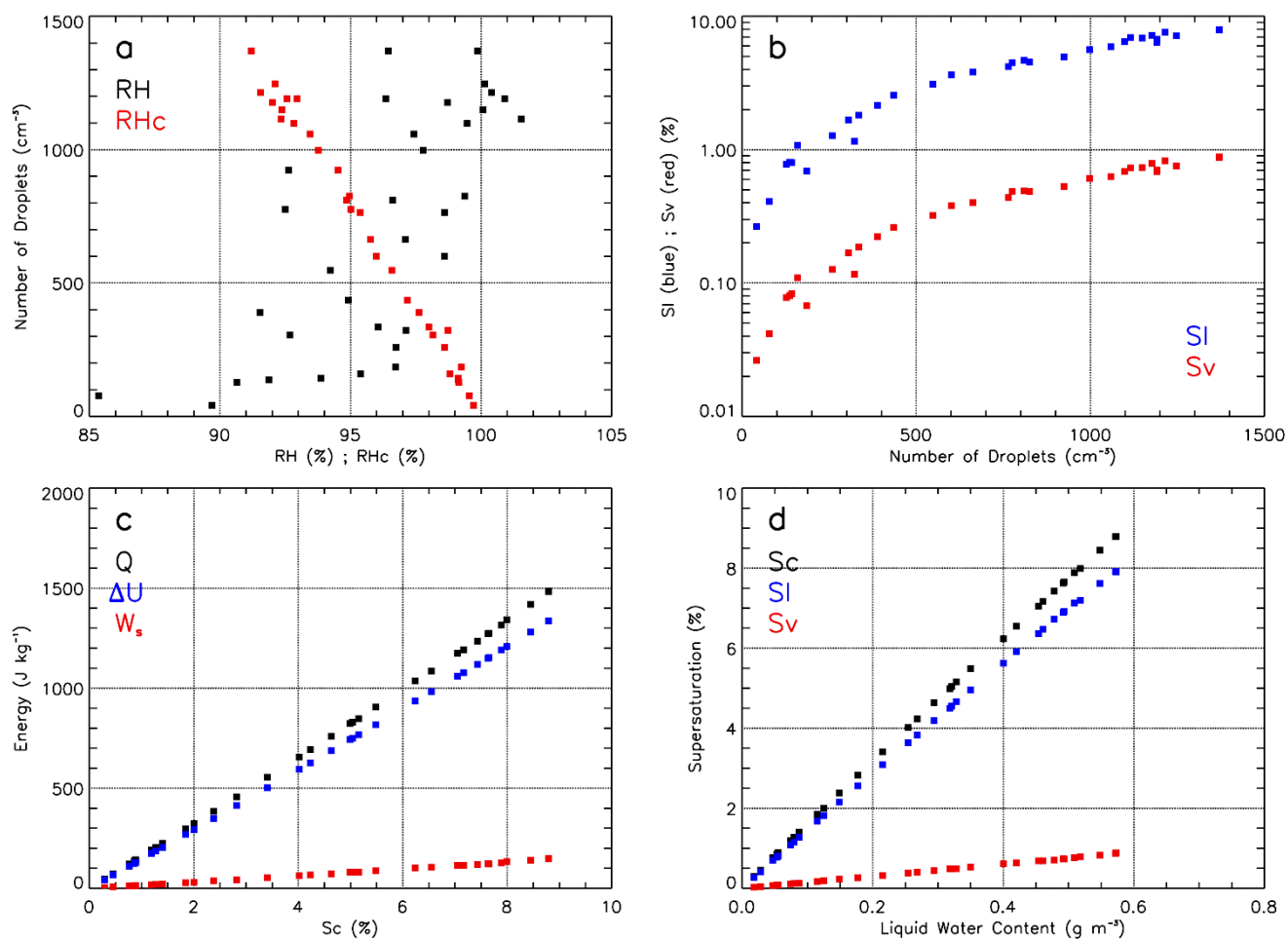


Figure S9. Similar to Figure S5 for flight segment AC11.

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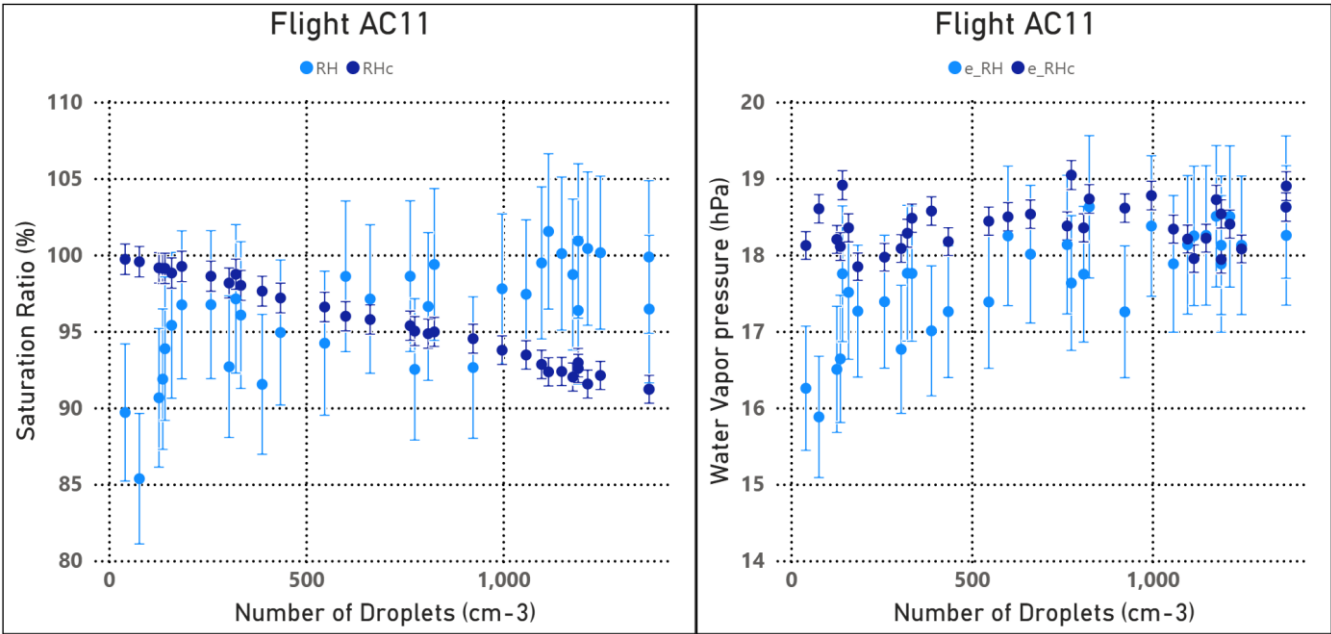


Figure S10. Similar to Figure S4 for flight segment AC11.

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Flight AC17a: Central Amazon

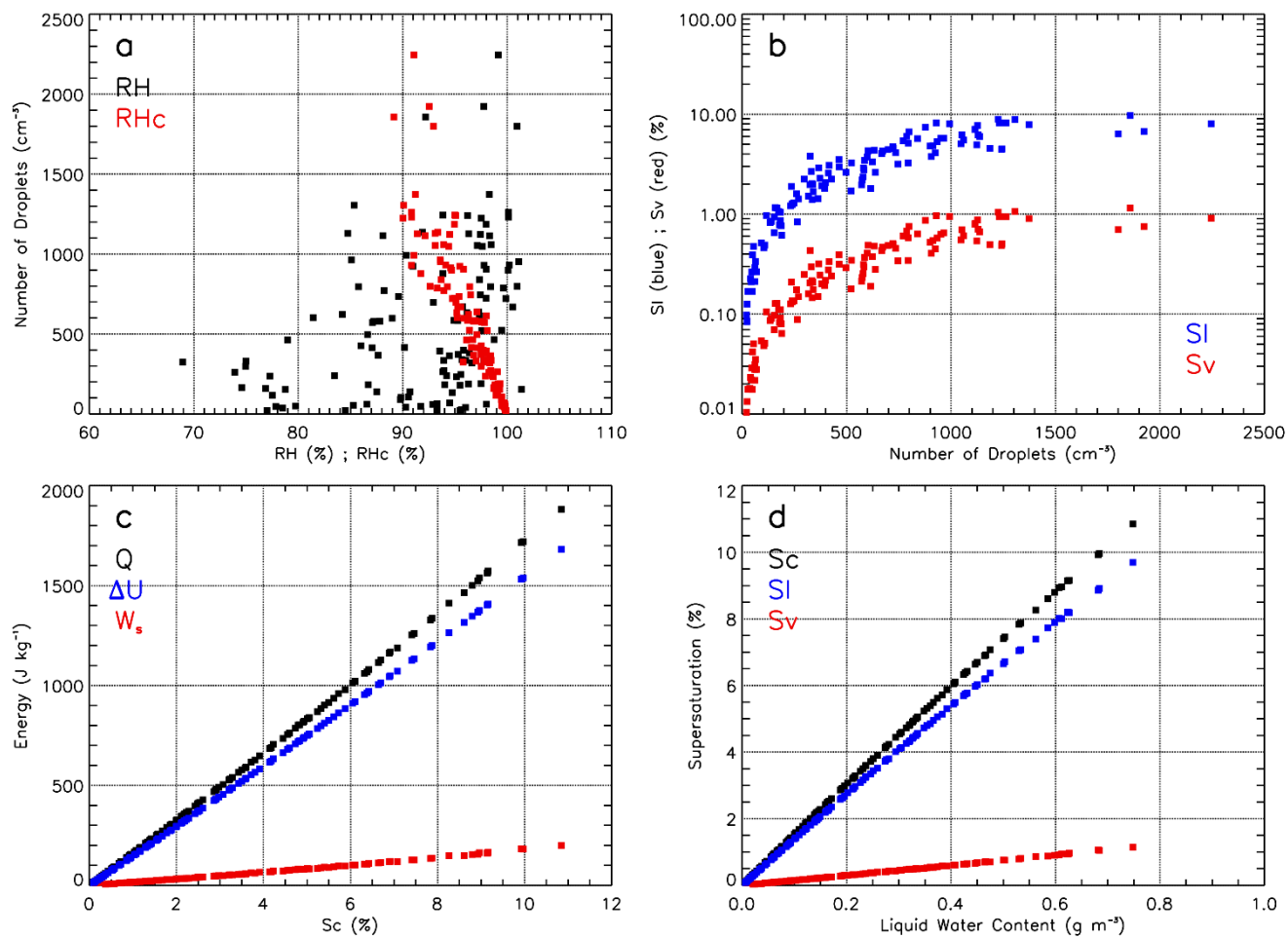
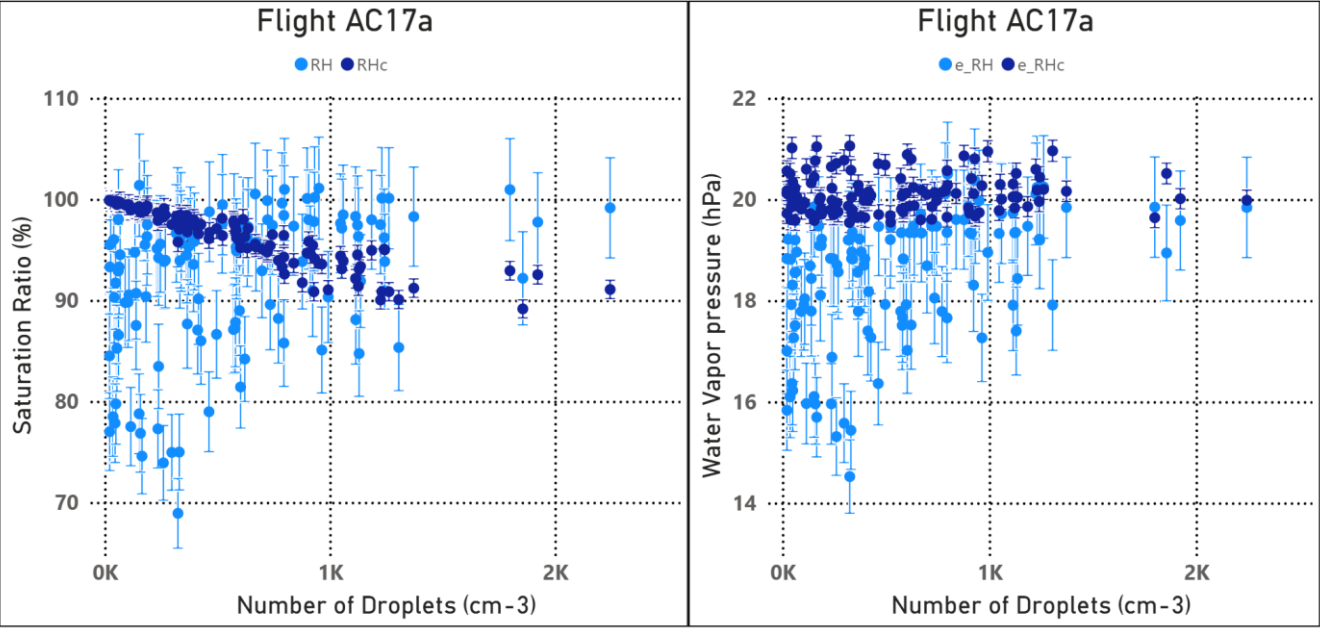


Figure S11. Similar to Figure S5 for flight segment AC17a.



105 Figure S12. Similar to Figure S4 for flight segment AC17a.

Flight AC17b: Central Amazon

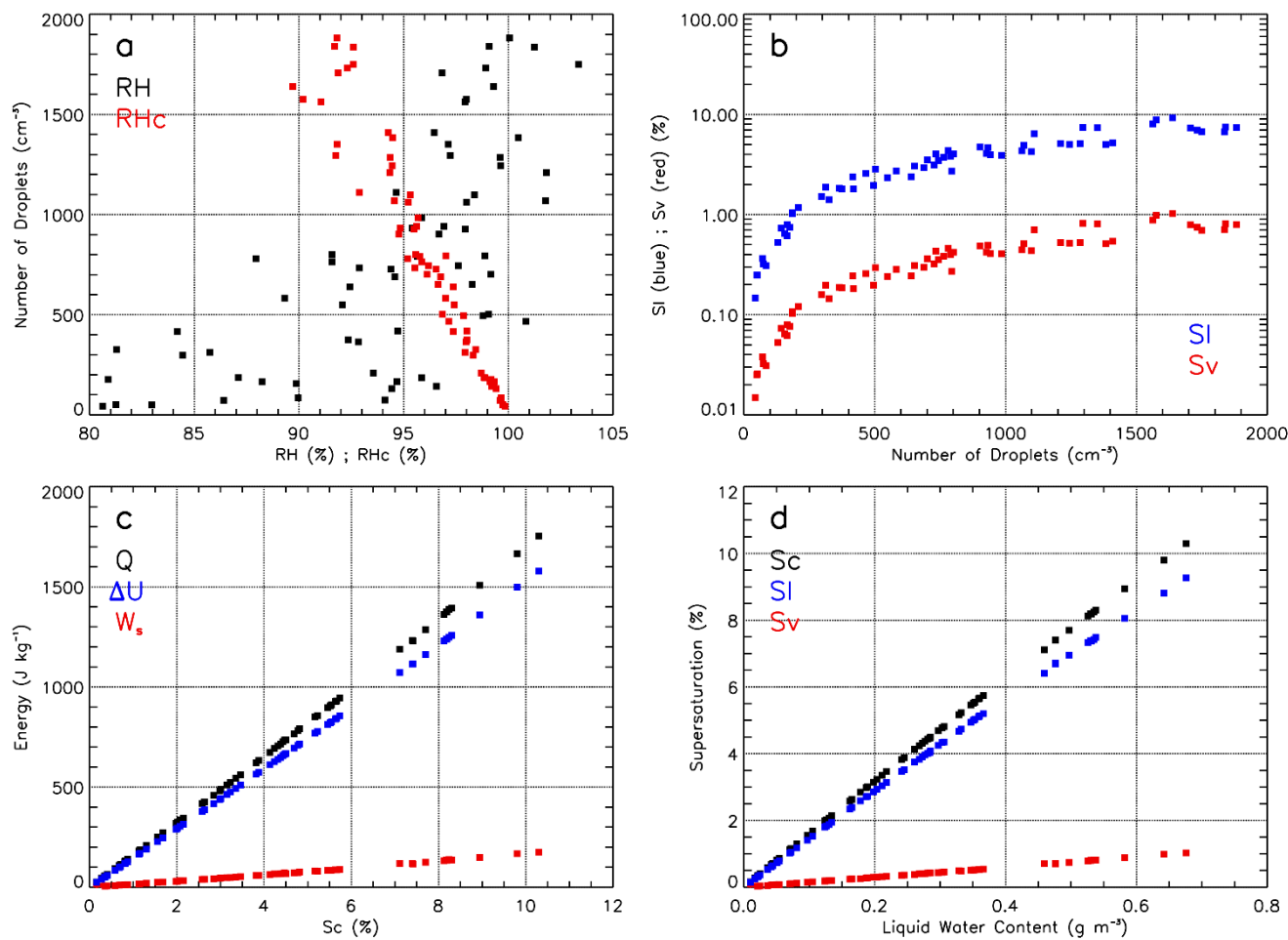
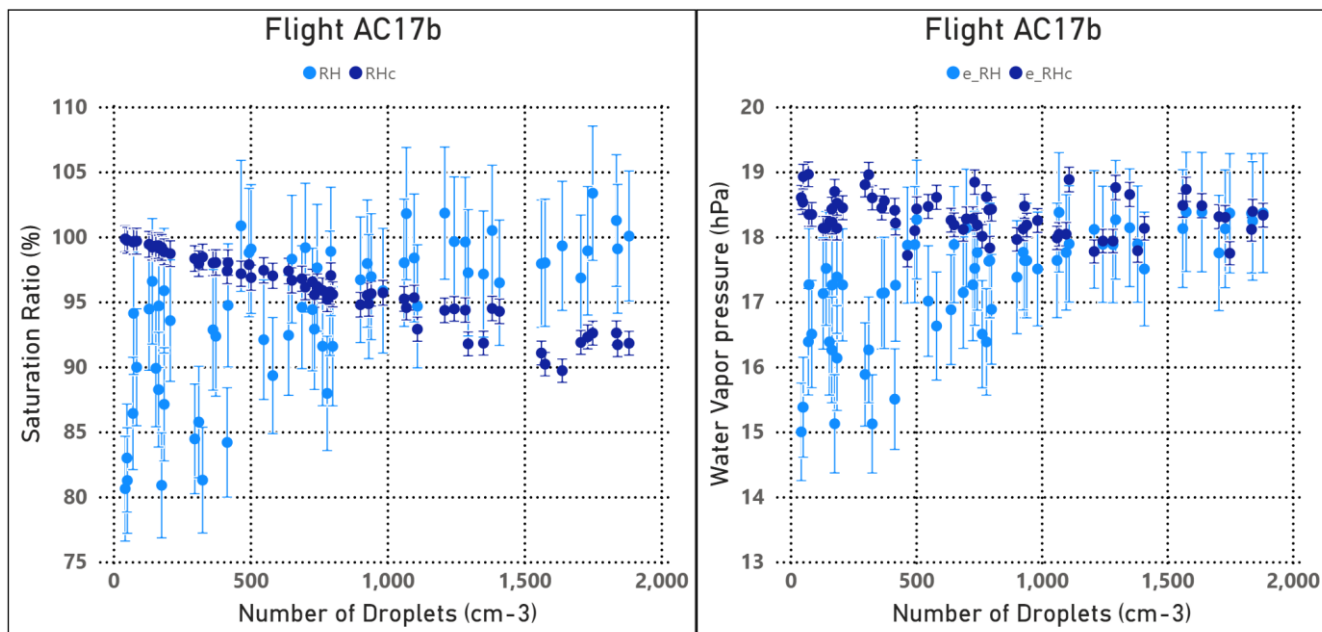
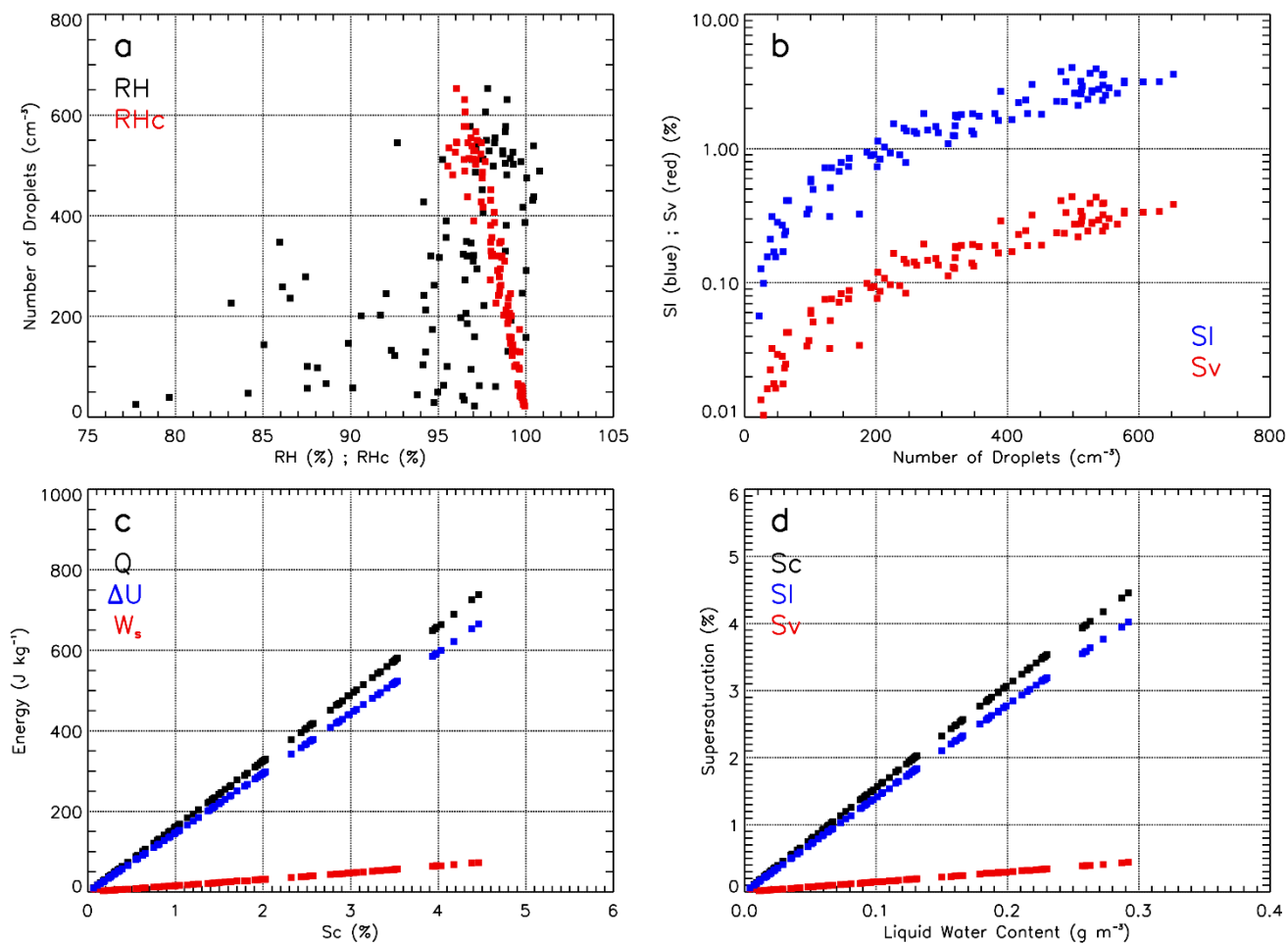


Figure S13. Similar to Figure S5 for flight segment AC17b.



120 Figure S14. Similar to Figure S4 for flight segment AC17b.

Flight AC18: Northern Amazon



125 Figure S15. Similar to Figure S5 for flight segment AC18.

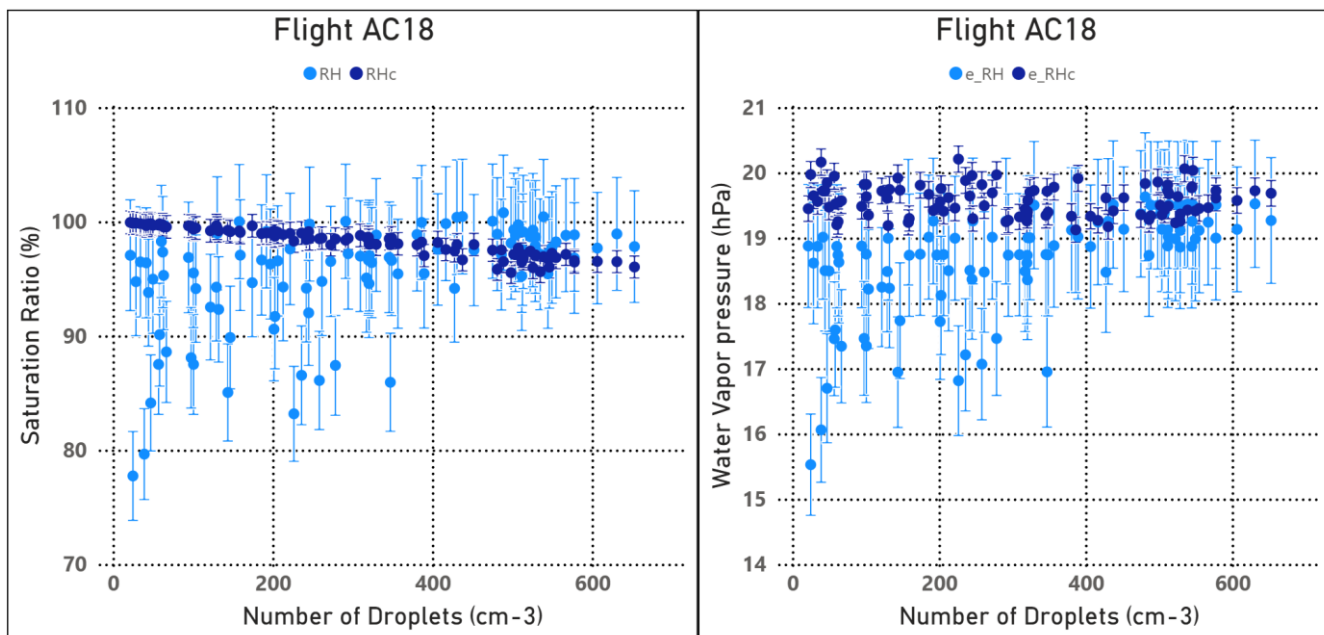


Figure S16. Similar to Figure S4 for flight segment AC18.

Flight AC19: Atlantic Ocean

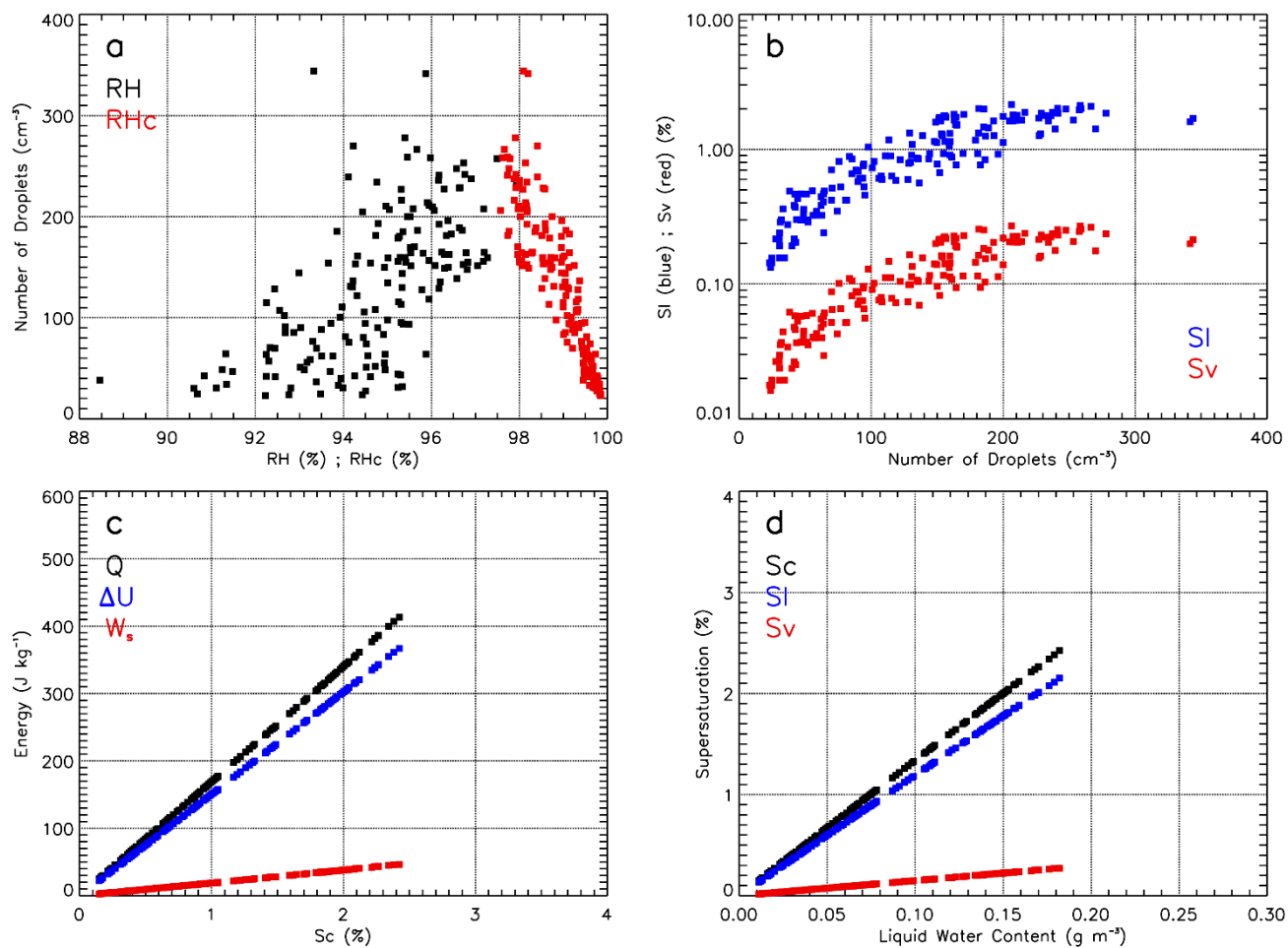


Figure S17. Similar to Figure S5 for flight segment AC19.

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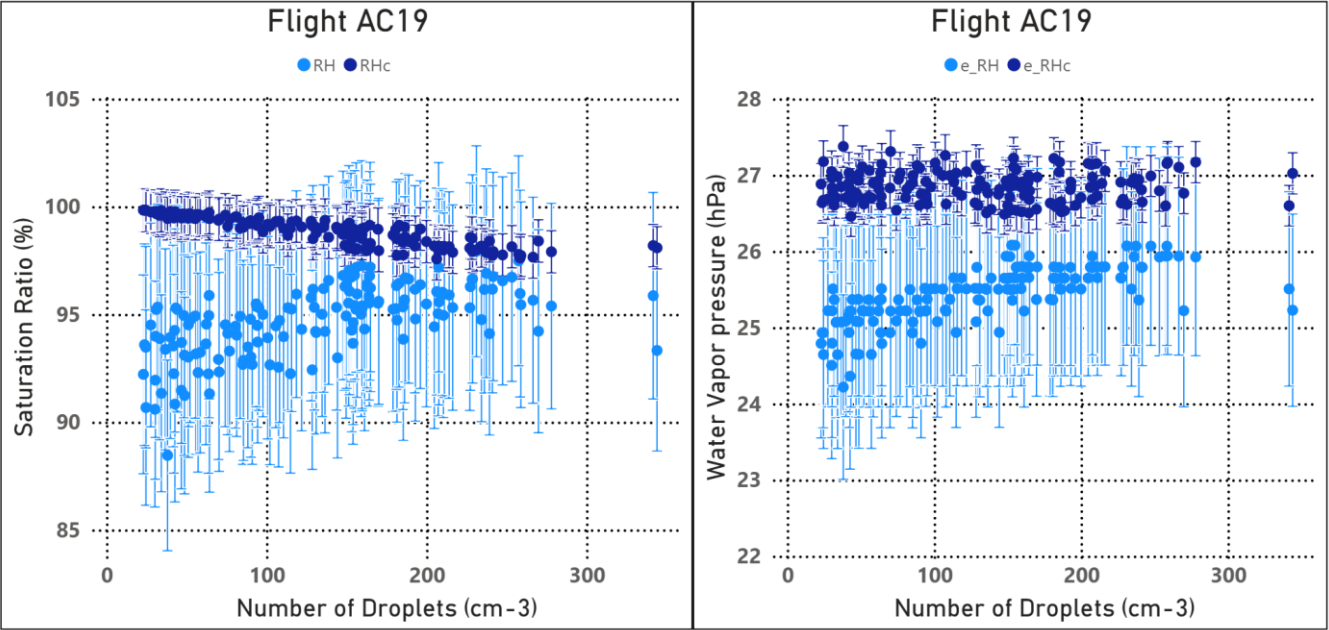


Figure S18. Similar to Figure S4 for flight segment AC19.

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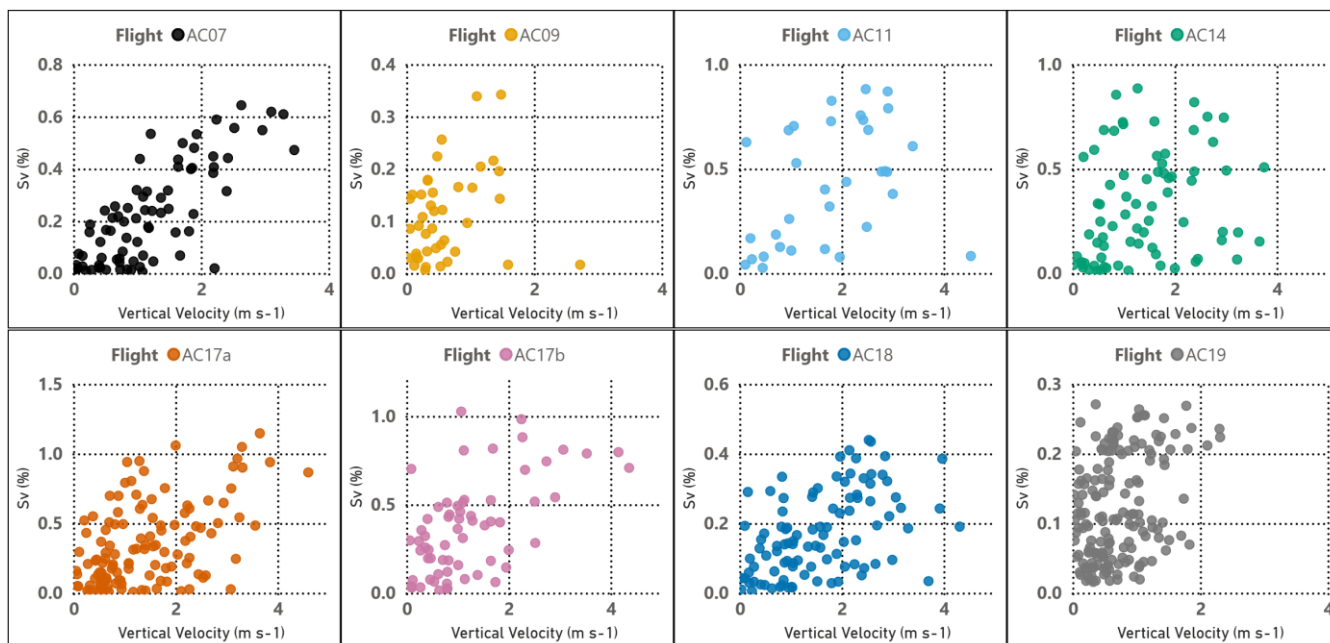


Figure S19. Vertical velocity (m s⁻¹) vs. water vapor supersaturation (S_v (%)). Flight segments are indicated in the legend.

S1. Statistical analysis of the reversibility of the energy and vapor budgets under isobaric conditions.

The reversibility analysis presented in this section evaluates the internal consistency of the energetic cloud-base framework under quasi-isobaric conditions. The calculations were conducted assuming an atmospheric pressure of 950 hPa, initial temperatures ranging from 2 °C to 30 °C, and adiabatic cooling between 0.1 °C and 2 °C with temperatures steps of 0.1°C. For each simulation, the thermodynamic and energetic variables recovered through the inversion procedure were compared with the corresponding variables obtained from the original calculations presented in the section 3 (Results). The objective of this analysis is to assess whether the energetic partition framework remains numerically stable and thermodynamically reversible over the analysed range of atmospheric conditions. The percentage bias was calculated for each quantity as:

$$Bias(\%) = 100 \times \frac{Recovered - Original}{Original} \quad (S1.1)$$

Table S1 presents the statistical differences between the original and recovered thermodynamic and energetic variables obtained from the reversibility analysis of the energetic framework under quasi-isobaric conditions. The results demonstrate that the inversion framework reproduces the original state variables with extremely small numerical deviations, supporting the reversibility, internal consistency, and numerical stability of the proposed energetic model. For all analysed variables, the mean absolute relative differences are generally below ~0.01 %, while the maximum deviations remain ~0.3 % for some variables. Slightly larger deviations were observed for variables such as S_v , T_{ws} , and $LWC2$, which are more sensitive to small residual numerical differences because they depend on nonlinear thermodynamic relationships associated with the Clausius–Clapeyron equation and the energetic partition between ΔU and W_s . Our sensitivity tests indicated that the initially larger recovery errors were primarily associated with the finite numerical precision of the temperature variables used in the original simulations. Therefore, the observed discrepancies mainly reflect finite numerical precision rather than limitations inherent to the inversion framework. For typical warm-cloud temperatures between 10 and 30 °C, a maximum recovery error of ~0.3 % corresponds to absolute temperature differences of only about ~0.03-0.09 °C, which are substantially smaller than the expected uncertainty of airborne temperature measurements (~ 0.5 K or °C). Similarly, the residual numerical differences associated with the energetic terms were much smaller than the observational uncertainties propagated from cloud microphysical measurements, particularly the uncertainties associated with LWC (~10%). From our sensitive tests, similar results were obtained for the lower pressure simulations.

190 Table S1. Statistical summary of the reversibility analysis of the energetic partition framework under quasi-isobaric conditions.
 The table presents the mean, median, standard deviation, minimum, and maximum absolute relative differences (%) between
 the original thermodynamic and energetic parameters calculated in the forward formulation and the corresponding parameters
 recovered through the inverse formulation. T_0 is the reference cloud-base temperature, T_{ws} is the saturated parcel temperature
 after accounting for the energetic contribution of water-vapor expansion work (W_s), and T_{air} is the resulting parcel air
 195 temperature. RH_c is the saturation ratio in the reference state of the saturated parcel, while S_c , S_l , and S_v are the cloud, liquid-
 phase, and vapor-phase supersaturations, respectively. Q is the latent-energy release associated with condensational growth,
 ΔU is the internal-energy change, and W_s is the water-vapor expansion work. $LWC1$ and $LWC2$ represent the liquid water
 contents calculated assuming complete latent-heat conversion into internal energy and after accounting for the energetic
 contribution of W_s , respectively.

Parameter	Mean_Bias (%)	Median_Bias (%)	Std_Bias(%)	Min_Bias (%)	Max_Bias(%)
T_0 (°C)	0.000457	0.000296	0.000755	0	0.005
T_{ws} (°C)	0.002053	0.001148	0.005553	0	0.251889
T_{air} (°C)	0.001605	0.001185	0.001671	0	0.018837
RH_c (%)	0.000323	0	0.000491	0	0.001155
S_c (%)	0.009512	0.004942	0.014646	0	0.115512
S_l (%)	0.009442	0.004864	0.01464	0	0.115875
S_v (%)	0.00982	0	0.025751	0	0.337838
Q (J kg ⁻¹)	0.009732	0.005155	0.01462	0	0.107059
ΔU (J kg ⁻¹)	0.009687	0.005102	0.014612	0	0.107617
W_s (J kg ⁻¹)	0.010053	0.005599	0.014752	0	0.118741
$LWC1$ (g m ⁻³)	0	0	0	0	0
$LWC2$ (g m ⁻³)	0.009453	0	0.030884	0	0.303951

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S2. Cloud identification threshold sensitivity, uncertainty analysis of the energy budget, and sensitivity of the N_d –S relationships to prescribed uncertainties in N_d and LWC .

S2.1 Cloud identification threshold sensitivity

Figures S20–S23 show the variability of LWC and N_d across all flight segments used in this study. Values of N_d greater than 5 cm^{-3} are shown to explore the minimum cloud identification threshold under different atmospheric conditions. The figures demonstrate that the proposed cloud identification threshold prevents the inclusion of well-mixed cloud elements while still allowing the analysis of different cloud environments. In practice, the adopted cloud-identification threshold ($N_d > 20 \text{ cm}^{-3}$) corresponds approximately to $LWC > \sim 0.01 \text{ g m}^{-3}$, which is a physically reasonable threshold for cloud initiation considering the sensitivity of the derived cloud supersaturations to LWC in this study. Tables S2–S7 present a sensitivity analysis of the derived supersaturation components (S_e , S_i , and S_v) using a range of cloud identification criteria based on varying thresholds of N_d and LWC . These results show that the derived supersaturations and their relationships, assuming the proposed threshold, are robust to reasonable variations in the cloud identification criteria. The remaining variability is primarily driven by environmental heterogeneity and turbulence rather than by the threshold definition itself.

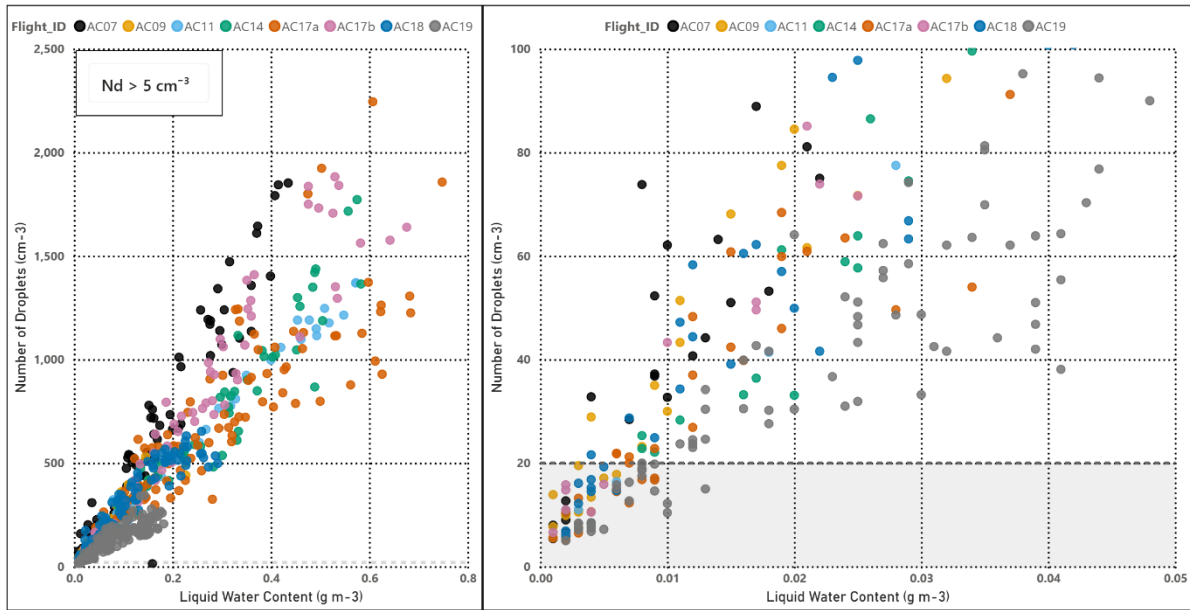


Figure S20. [left] LWC vs. N_d for all flight segments for $N_d > 5 \text{ cm}^{-3}$. [right] Similar for ($N_d < 100 \text{ cm}^{-3}$ & $LWC < 0.05 \text{ g m}^{-3}$). The threshold $N_d = 20 \text{ cm}^{-3}$ (dashed line) corresponds approximately to $LWC \gtrsim 0.01 \text{ g m}^{-3}$ for most flights. The shaded region shows the excluded 1 Hz cloud passes for the adopted criterion ($N_d > 20 \text{ cm}^{-3}$).

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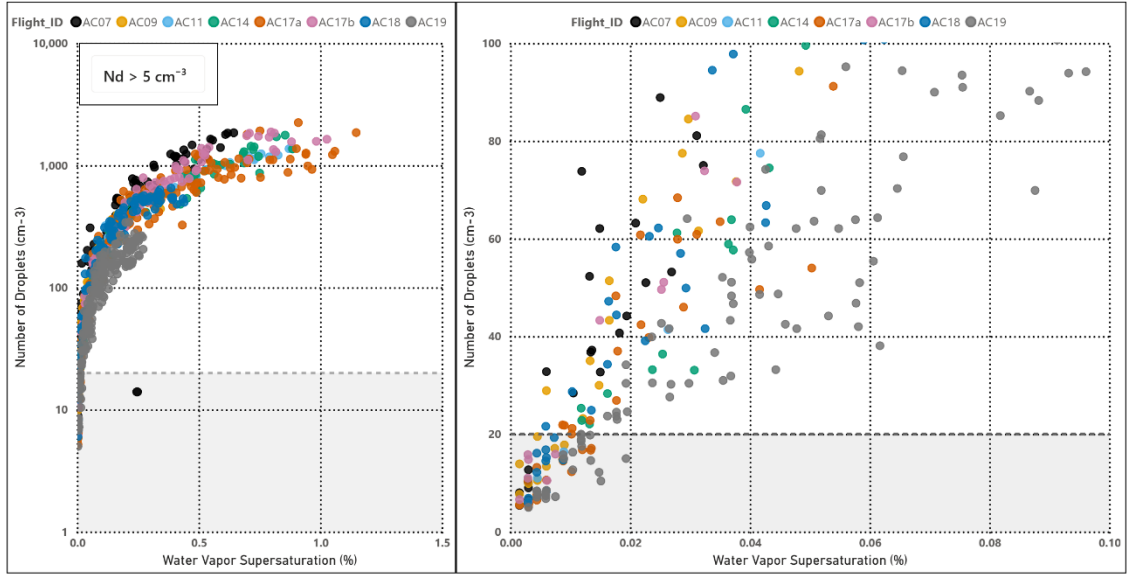


Figure S21. [left] S_v vs. N_d for all flight segments ($N_d > 5 \text{ cm}^{-3}$). [right] Similar for ($N_d < 100 \text{ cm}^{-3}$ & $S_v < 0.1 \%$). The shaded region shows the excluded 1 Hz cloud passes for the adopted criterion ($N_d > 20 \text{ cm}^{-3}$).

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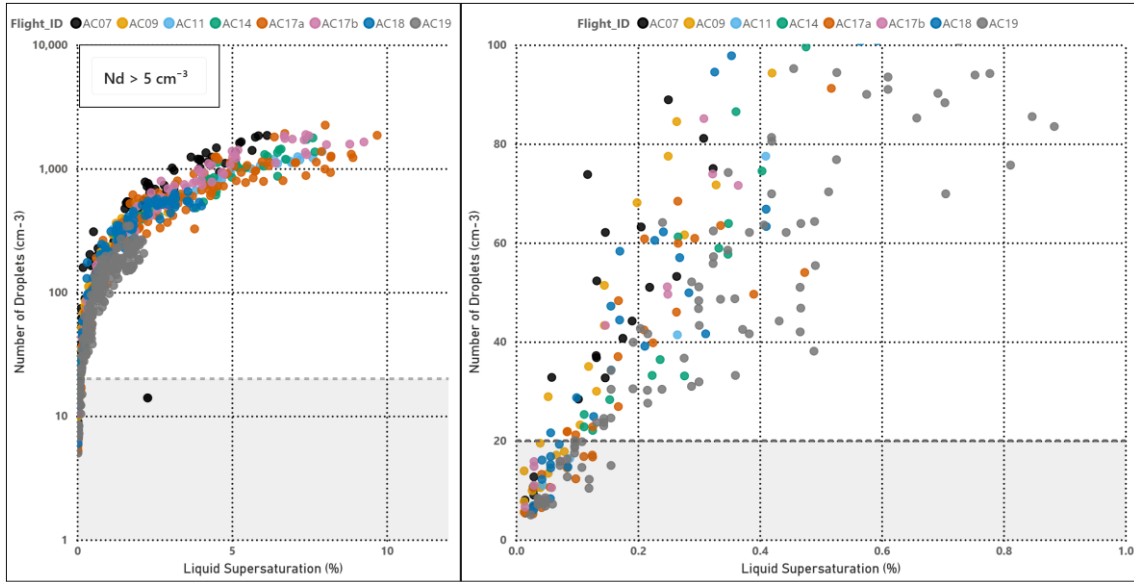


Figure S22. [left] S_l vs. N_d for all flight segments ($N_d > 5 \text{ cm}^{-3}$). [right] Similar for ($N_d < 100 \text{ cm}^{-3}$ & $S_l < 1 \%$). The shaded region shows the excluded 1 Hz cloud passes for the adopted criterion ($N_d > 20 \text{ cm}^{-3}$).

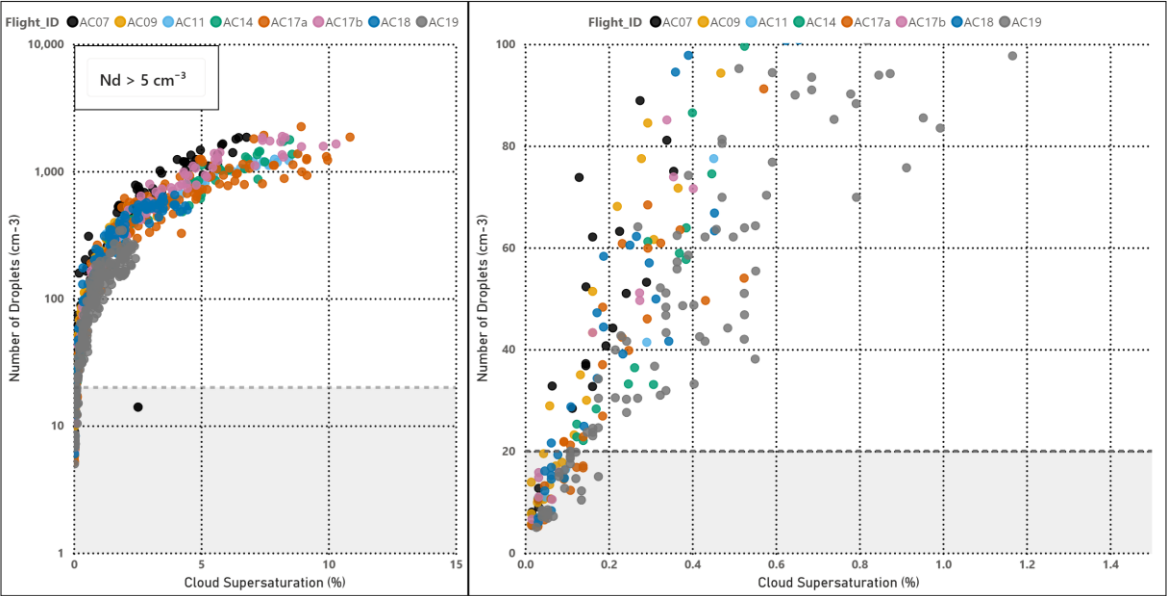


Figure S23. [left] S_c vs. N_d for all flight segments ($N_d > 5 \text{ cm}^{-3}$). [right] Similar for ($N_d < 100 \text{ cm}^{-3}$ & $S_c < 1.5 \%$). The shaded
240 region shows the excluded 1 Hz cloud passes for the adopted criterion ($N_d > 20 \text{ cm}^{-3}$).

S2.2. Uncertainties of the energy budget parameters

250 The measurement uncertainties of the energy budget parameters were calculated using a Monte Carlo method (JCGM, 2008), following the calculations of the energy budget described in Section 3. This approach enables a consistent estimation of uncertainties in both steps of the closure analysis: (1) latent energy closure and (2) energy partition. For each cloud pass (1 Hz data), 5000 realizations were generated by perturbing the measured inputs as follows:

$$T_i = T + \delta T, \quad p_i = p + \delta p, \quad LWC_i = LWC \cdot (1 + \delta_{LWC}) \quad (S.2.1)$$

Here, T_i , p_i , and LWC_i represent perturbed realizations of temperature, pressure, and liquid water content generated during
 255 the Monte Carlo uncertainty propagation, respectively. The perturbation terms δT , δp , and δ_{LWC} correspond to random realizations of the measurement uncertainties sampled from Gaussian distributions with standard deviations σ_T , σ_p , and σ_{LWC} , respectively. In our analysis, we adopted the measurement uncertainties of T (± 0.5 K) and p (3 Pa) from BAHAMAS data, and LWC (10% and 30%) calculated from CCP-CDP. The perturbations applied to T , p and LWC were randomly sampled from Gaussian distributions defined by mean $\mu = 0$ and standard deviations corresponding to the instrumental
 260 measurement uncertainties:

$$\delta T \sim \mathcal{N}(\mu, \sigma_T), \delta p \sim \mathcal{N}(\mu, \sigma_p), \delta_{LWC} \sim \mathcal{N}(\mu, \sigma_{LWC}) \quad (S.2.2)$$

For each cloud pass, the full thermodynamic calculation (step 1 and step 2) was recomputed. The uncertainty for each variable X was defined as:

$$\sigma_X = \sqrt{\frac{1}{Nr - 1} \sum_{i=1}^{Nr} (X_i - \bar{X})^2} \quad (S.2.3)$$

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Where, X_i denotes the value of variable X for realization i , $\bar{X}$ is the ensemble mean, Nr is the number of realizations, σ_X is the standard deviation of the realizations. The relative uncertainty in percentage was calculated with respect to the measured value:

$$\epsilon_X = \frac{\sigma_X}{X_{\text{nom}}} \cdot 100\% \quad (S.2.4)$$

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Where ϵ_X is the relative uncertainty expressed as a percentage of the nominal value X_{nom} . The figures below show the average relative uncertainties for the energy budget parameters as a function of flight leg.

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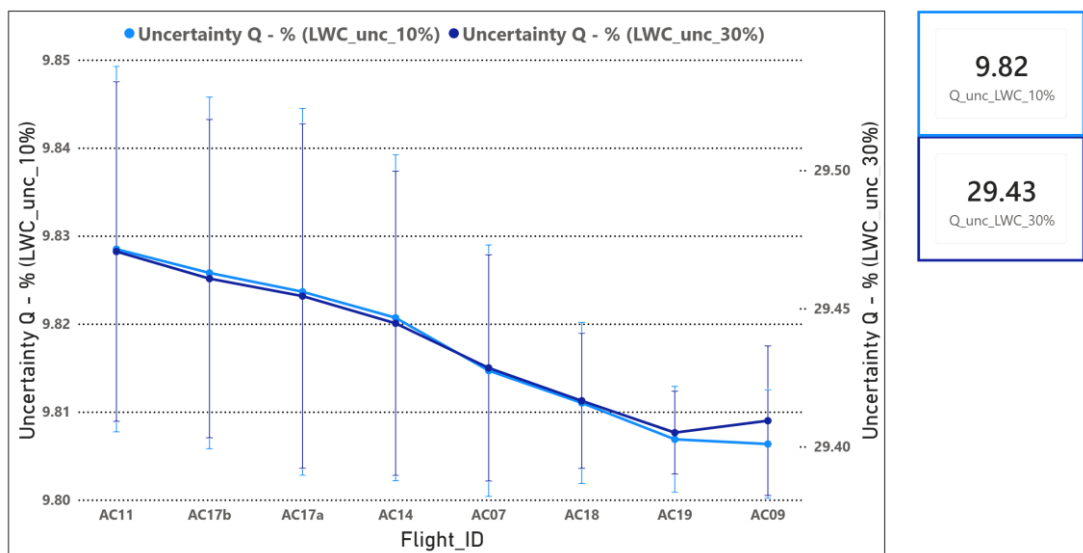


Figure S24. Relative uncertainty (%) of the latent heat term Q for each flight segment, derived from propagated liquid water content (LWC) uncertainties of 10% (left y-axis) and 30% (right y-axis). Markers indicate flight-mean values, and error bars represent $\pm 1\sigma$ variability within each segment. The right panel shows the mean uncertainty across all flights, highlighting the strong sensitivity of Q to LWC uncertainty.

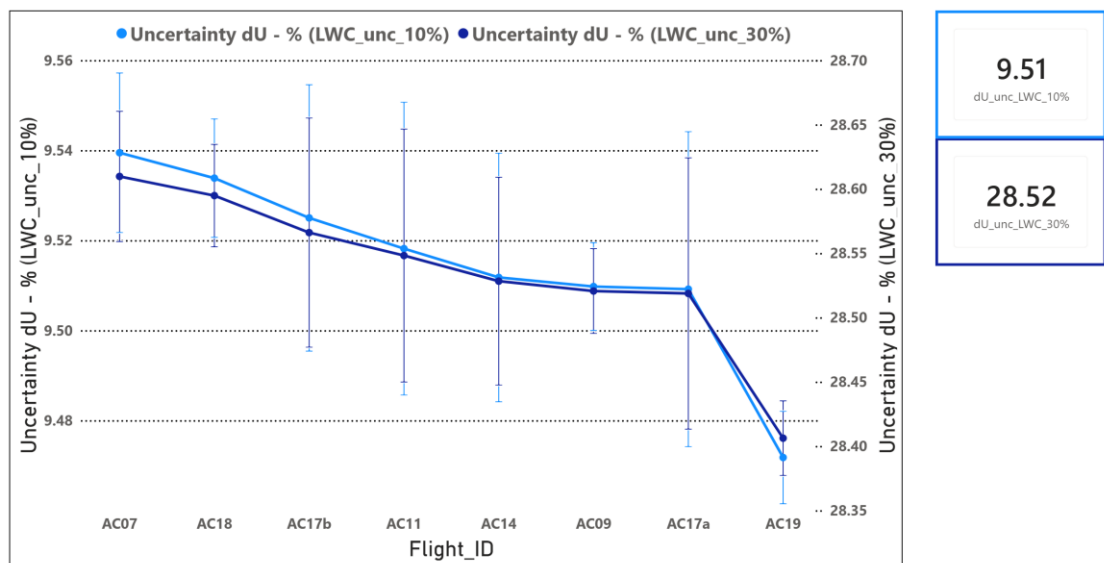


Figure S25. Similar to Figure S24 for the internal energy change ΔU for each flight segment. The right panel summarizes the mean uncertainty across all flights, showing similar sensitivity to LWC as observed for Q .

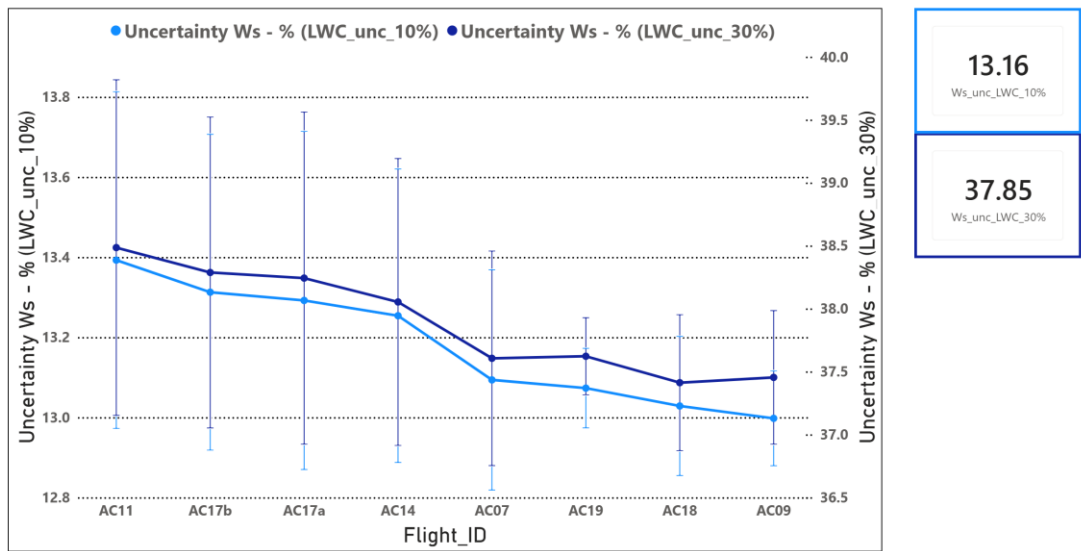


Figure S26. Similar to Figure S24 for the saturation work term W_s for each flight segment. The right panel indicates the mean uncertainty across flights, with greater sensitivity than Q and ΔU .

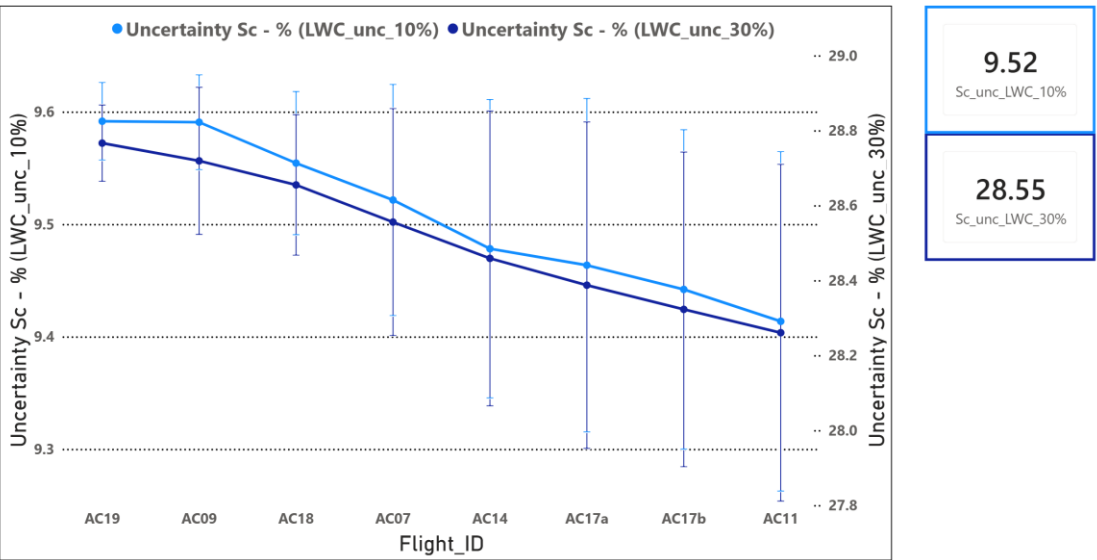


Figure S27. Similar to Figure S24 for cloud supersaturation S_c for each flight segment. The right panel presents the mean uncertainty across all flights.

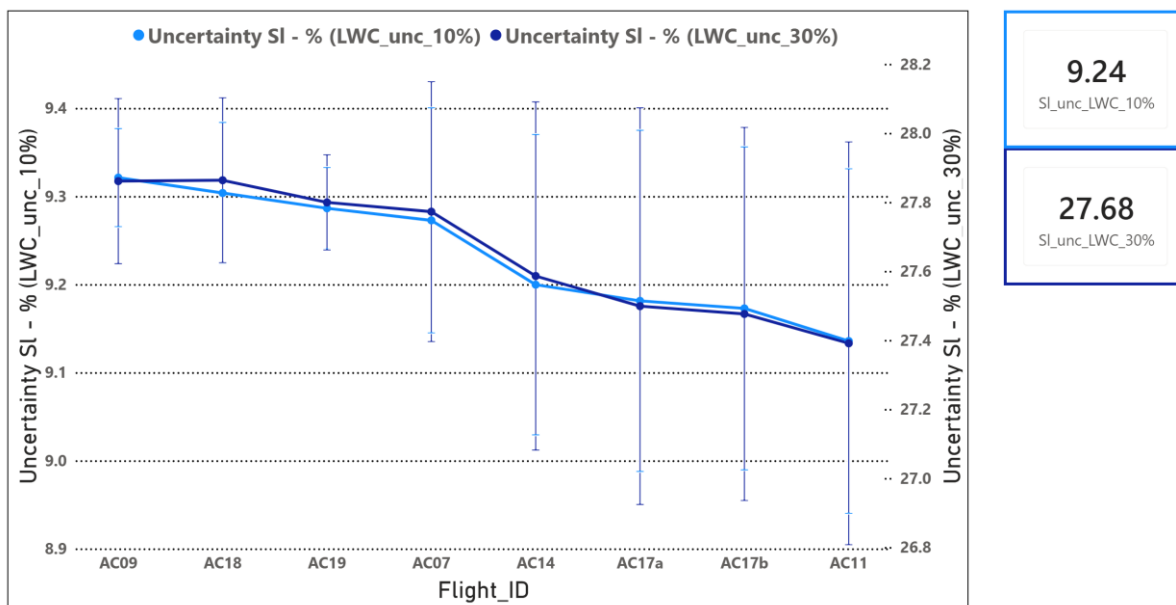


Figure S28. Similar to Figure S24 for liquid supersaturation S_l for each flight segment. The right panel presents the mean uncertainty across all flights.

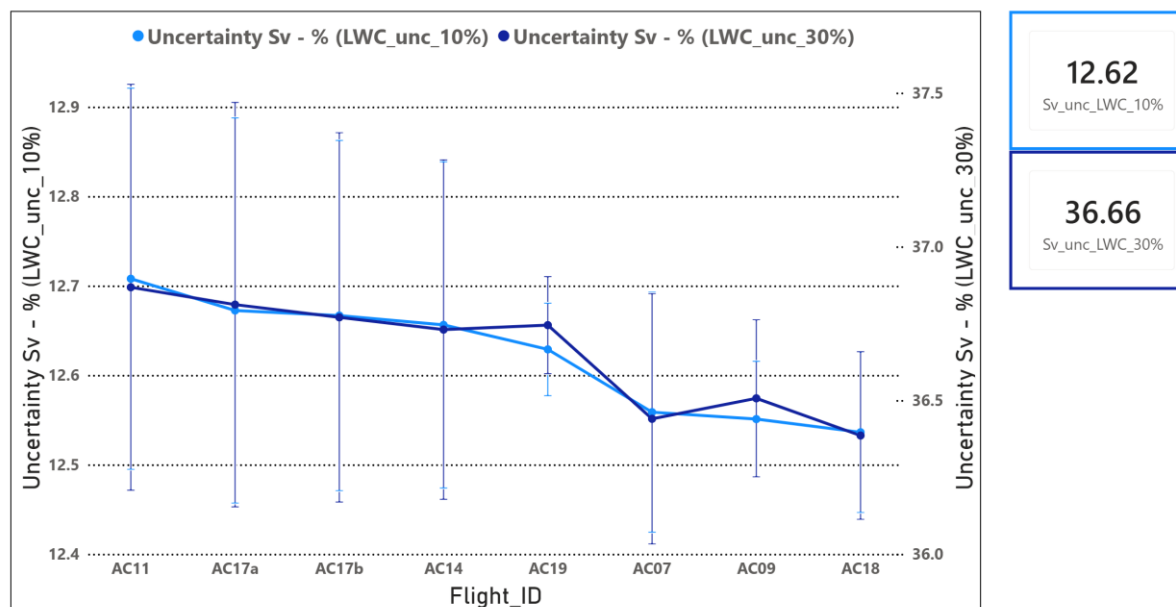


Figure S29. Similar to Figure S24 for water vapor supersaturation S_v for each flight segment. The right panel summarizes the mean uncertainty across all flights, indicating greater sensitivity than S_c and S_l .

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S2.3. Sensitivity of the N_d - S relationships to prescribed N_d and LWC uncertainties

To quantify the effect of measurement and retrieval uncertainties on the fitted relationship between droplet number concentration (N_d) and supersaturation (S), we performed a Monte Carlo uncertainty analysis of the power-law relation:

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$$N_d = C \cdot S^k$$

The regression was conducted in log-log space. Uncertainties in both variables were propagated by perturbing the observed values of S (S_c , S_l , and S_v) and N_d , assuming Gaussian-distributed errors. Figures S24-29 show the average uncertainties for the energetic budget components as a function of LWC uncertainty. The uncertainties for the best- and worst-case scenarios were assumed to be 10% and 30%, respectively. Two uncertainty scenarios were considered for each N_d and S value based on the average uncertainties:

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- *Best-case scenario*: 12.6 % uncertainty in S_v , 9.2 % uncertainty in S_l , 9.5 % uncertainty in S_c , and 10% in N_d .

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- *Worst-case scenario*: 36.6 % uncertainty in S_v , 27.6 % uncertainty in S_l , 28.5 % uncertainty in S_c , and 30% in N_d .

For each case, 5000 Monte Carlo realizations were performed, and the mean and standard deviation of the fitted parameters were calculated (JCGM, 2008). The results of these analyses are shown in Tables S2-7.

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Overall, the analysis shows that:

- The correlation coefficients (R^2) of the power-law fits ($N_d = C \cdot S^k$) are generally insensitive to the choice of threshold and remain comparable to those obtained using the baseline criterion ($N_d > 20 \text{ cm}^{-3}$).
- For most flight segments, variations in threshold produce only minor changes in the fitted parameters (C and k), which remain within the estimated uncertainty ranges.
- A notable exception occurs for flight AC07 (very polluted conditions), where lower $N_d (< 20 \text{ cm}^{-3})$ thresholds introduce increased variability and reduce correlation strength, likely due to inclusion of transitional particles.
- In contrast, for the cleanest case (AC19), slightly improved correlations are obtained when including lower N_d values ($< 20 \text{ cm}^{-3}$), consistent with relatively reduced CCN and buoyancy variability during cloud formation.

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These results show that the derived supersaturations and their relationships, assuming the proposed threshold ($N_d > 20 \text{ cm}^{-3}$), are robust with respect to reasonable variations in cloud identification criteria. The remaining variability is primarily driven by environmental heterogeneity and turbulence rather than by the threshold definition itself.

The conversion of N_d to Standard Temperature and Pressure (STP; $p = 1013.25 \text{ hPa}$ and $T = 273.15 \text{ K}$) when comparing with CCN(S_v) spectra measured below cloud bases introduces negligible additional uncertainty relative to the instrumental

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uncertainty in N_d . For typical cloud base conditions, a temperature uncertainty of 0.5 K contributes approximately 0.17%, while a pressure uncertainty of 3 Pa contributes less than 0.01%. Therefore, the total relative uncertainty in $N_{d\text{STP}}$ remains dominated by the assumed 10% uncertainty in N_d .

Table S2. Power-law fit parameters for the relationship $N_d = C \cdot S_v^k$ derived from log–log regression. The coefficients C and k are reported as Monte Carlo means $\pm 1\sigma$ ($n = 5000$ realizations). The coefficient of determination (R^2) and root-mean-square error (RMSE) are calculated from the nominal fit in logarithmic space. Data are filtered using threshold criteria $N_d > N_{d,\text{th}}$ and $LWC > LWC_{\text{th}}$. The uncertainties of N_d and S_v assumed in the calculations are 10% and 12.6%, respectively. Columns list: Flight ID, $N_{d,\text{th}}$, LWC_{th} , C , σC , k , σk , R^2 , RMSE, and sample size (N).

Flight	$N_{d,\text{th}}$	LWC_{th}	C	σC	k	σk	R^2	RMSE	N
AC07	10	0	2017.67	66.2	0.883	0.013	0.783	0.645	81
AC07	10	0.01	2068.73	82.01	0.899	0.02	0.657	0.678	71
AC07	10	0.02	2062.85	100.38	0.896	0.028	0.533	0.698	64
AC07	20	0	2414.84	82.16	0.922	0.014	0.958	0.249	77
AC07	20	0.01	2478.51	92.39	0.939	0.018	0.947	0.234	69
AC07	20	0.02	2441.89	102.81	0.926	0.024	0.943	0.193	62
AC07	30	0	2396.08	82.23	0.917	0.014	0.956	0.25	76
AC07	30	0.01	2478.51	92.39	0.939	0.018	0.947	0.234	69
AC07	30	0.02	2441.89	102.81	0.926	0.024	0.943	0.193	62
AC09	10	0	1347.58	67.91	0.832	0.015	0.956	0.244	48
AC09	10	0.01	1306.68	96.08	0.808	0.029	0.941	0.169	37
AC09	10	0.02	1406.77	132.03	0.845	0.042	0.91	0.173	32
AC09	20	0	1349.85	85.34	0.826	0.023	0.952	0.19	41
AC09	20	0.01	1306.68	96.08	0.808	0.029	0.941	0.169	37
AC09	20	0.02	1406.77	132.03	0.845	0.042	0.91	0.173	32
AC09	30	0	1326.86	93.99	0.815	0.028	0.949	0.167	38
AC09	30	0.01	1306.68	96.08	0.808	0.029	0.941	0.169	37
AC09	30	0.02	1406.77	132.03	0.845	0.042	0.91	0.173	32
AC11	10	0	1575.37	61.52	0.931	0.02	0.99	0.125	35
AC11	10	0.01	1565.84	67.87	0.924	0.028	0.982	0.125	33
AC11	10	0.02	1535.06	66.03	0.9	0.03	0.98	0.118	32

AC11	20	0	1565.84	67.87	0.924	0.028	0.982	0.125	33
AC11	20	0.01	1565.84	67.87	0.924	0.028	0.982	0.125	33
AC11	20	0.02	1535.06	66.03	0.9	0.03	0.98	0.118	32
AC11	30	0	1565.84	67.87	0.924	0.028	0.982	0.125	33
AC11	30	0.01	1565.84	67.87	0.924	0.028	0.982	0.125	33
AC11	30	0.02	1535.06	66.03	0.9	0.03	0.98	0.118	32
AC14	10	0	1595.69	49.67	0.951	0.015	0.979	0.18	74
AC14	10	0.01	1651.26	55.23	0.978	0.018	0.977	0.166	70
AC14	10	0.02	1604.71	54.92	0.948	0.02	0.972	0.153	65
AC14	20	0	1643.14	52.98	0.973	0.016	0.981	0.164	73
AC14	20	0.01	1651.26	55.23	0.978	0.018	0.977	0.166	70
AC14	20	0.02	1604.71	54.92	0.948	0.02	0.972	0.153	65
AC14	30	0	1650.02	54.81	0.977	0.019	0.975	0.167	69
AC14	30	0.01	1650.02	54.81	0.977	0.019	0.975	0.167	69
AC14	30	0.02	1604.71	54.92	0.948	0.02	0.972	0.153	65
AC17a	10	0	1501.55	31.98	0.928	0.009	0.961	0.271	131
AC17a	10	0.01	1460.03	34.89	0.899	0.013	0.934	0.263	117
AC17a	10	0.02	1449.64	37.55	0.891	0.017	0.9	0.267	108
AC17a	20	0	1465.85	34.06	0.903	0.012	0.947	0.26	121
AC17a	20	0.01	1460.03	34.89	0.899	0.013	0.934	0.263	117
AC17a	20	0.02	1449.64	37.55	0.891	0.017	0.9	0.267	108
AC17a	30	0	1449.85	35.18	0.892	0.014	0.931	0.263	116
AC17a	30	0.01	1449.85	35.18	0.892	0.014	0.931	0.263	116
AC17a	30	0.02	1449.64	37.55	0.891	0.017	0.9	0.267	108
AC17b	10	0	1893.74	53.69	0.916	0.013	0.978	0.201	69
AC17b	10	0.01	2002.72	67.55	0.963	0.02	0.973	0.159	63

AC17b	10	0.02	1978.54	68.29	0.95	0.023	0.967	0.16	61
AC17b	20	0	1983.55	64.99	0.954	0.019	0.975	0.161	64
AC17b	20	0.01	2002.72	67.55	0.963	0.02	0.973	0.159	63
AC17b	20	0.02	1978.54	68.29	0.95	0.023	0.967	0.16	61
AC17b	30	0	1983.55	64.99	0.954	0.019	0.975	0.161	64
AC17b	30	0.01	2002.72	67.55	0.963	0.02	0.973	0.159	63
AC17b	30	0.02	1978.54	68.29	0.95	0.023	0.967	0.16	61
AC18	10	0	1470.38	45.75	0.866	0.012	0.961	0.214	106
AC18	10	0.01	1405.33	53.13	0.84	0.017	0.927	0.212	96
AC18	10	0.02	1321.65	58.49	0.8	0.022	0.881	0.211	87
AC18	20	0	1417.28	51.16	0.845	0.016	0.94	0.213	99
AC18	20	0.01	1405.33	53.13	0.84	0.017	0.927	0.212	96
AC18	20	0.02	1321.65	58.49	0.8	0.022	0.881	0.211	87
AC18	30	0	1405.33	53.13	0.84	0.017	0.927	0.212	96
AC18	30	0.01	1405.33	53.13	0.84	0.017	0.927	0.212	96
AC18	30	0.02	1321.65	58.49	0.8	0.022	0.881	0.211	87
AC19	10	0	790.19	25.96	0.855	0.012	0.912	0.247	177
AC19	10	0.01	756.29	28.46	0.833	0.015	0.875	0.244	164
AC19	10	0.02	725.67	33.31	0.811	0.02	0.817	0.243	148
AC19	20	0	740.31	27.76	0.822	0.015	0.874	0.24	163
AC19	20	0.01	740.31	27.76	0.822	0.015	0.874	0.24	163
AC19	20	0.02	725.67	33.31	0.811	0.02	0.817	0.243	148
AC19	30	0	714.42	29.42	0.804	0.017	0.847	0.243	157
AC19	30	0.01	714.42	29.42	0.804	0.017	0.847	0.243	157
AC19	30	0.02	725.67	33.31	0.811	0.02	0.817	0.243	148

Table S3. Power-law fit parameters for the relationship $N_d = C \cdot S_l^k$ derived from log–log regression. The coefficients C and k are reported as Monte Carlo means $\pm 1\sigma$ ($n = 5000$ realizations). The coefficient of determination (R^2) and root-mean-square error (RMSE) are calculated from the nominal fit in logarithmic space. Data are filtered using threshold criteria $N_d > N_{d,\text{th}}$ and $LWC > LWC_{\text{th}}$. The uncertainties of N_d and S_l assumed in the calculations are 10% and 9.2%, respectively. Columns list: Flight ID, $N_{d,\text{th}}$, LWC_{th} , C , σC , k , σk , R^2 , RMSE, and sample size (N).

Flight	$N_{d,\text{th}}$	LWC_{th}	C	σC	k	σk	R^2	RMSE	N
AC07	10	0	268.66	4	0.895	0.011	0.788	0.638	81
AC07	10	0.01	263.48	4.7	0.919	0.017	0.664	0.671	71
AC07	10	0.02	262.04	6.19	0.926	0.024	0.543	0.691	64
AC07	20	0	293.63	4.51	0.932	0.012	0.958	0.249	77
AC07	20	0.01	288.53	5.22	0.954	0.016	0.948	0.233	69
AC07	20	0.02	290.75	6.53	0.948	0.021	0.942	0.194	62
AC07	30	0	294.76	4.63	0.927	0.012	0.956	0.249	76
AC07	30	0.01	288.53	5.22	0.954	0.016	0.948	0.233	69
AC07	30	0.02	290.75	6.53	0.948	0.021	0.942	0.194	62
AC09	10	0	221.36	4.72	0.838	0.013	0.957	0.243	48
AC09	10	0.01	225.78	4.88	0.821	0.026	0.944	0.166	37
AC09	10	0.02	223.16	5.14	0.866	0.037	0.913	0.169	32
AC09	20	0	224.22	4.88	0.836	0.02	0.953	0.188	41
AC09	20	0.01	225.78	4.88	0.821	0.026	0.944	0.166	37
AC09	20	0.02	223.16	5.14	0.866	0.037	0.913	0.169	32
AC09	30	0	225.51	4.88	0.829	0.024	0.951	0.164	38
AC09	30	0.01	225.78	4.88	0.821	0.026	0.944	0.166	37
AC09	30	0.02	223.16	5.14	0.866	0.037	0.913	0.169	32
AC11	10	0	187.75	5.06	0.953	0.017	0.99	0.121	35
AC11	10	0.01	186.53	6.54	0.959	0.025	0.983	0.12	33
AC11	10	0.02	192.65	7.45	0.936	0.027	0.981	0.114	32
AC11	20	0	186.53	6.54	0.959	0.025	0.983	0.12	33

AC11	20	0.01	186.53	6.54	0.959	0.025	0.983	0.12	33
AC11	20	0.02	192.65	7.45	0.936	0.027	0.981	0.114	32
AC11	30	0	186.53	6.54	0.959	0.025	0.983	0.12	33
AC11	30	0.01	186.53	6.54	0.959	0.025	0.983	0.12	33
AC11	30	0.02	192.65	7.45	0.936	0.027	0.981	0.114	32
AC14	10	0	190.43	3.25	0.969	0.013	0.981	0.173	74
AC14	10	0.01	185.13	3.59	1	0.015	0.979	0.157	70
AC14	10	0.02	190.9	4.37	0.975	0.018	0.974	0.147	65
AC14	20	0	186.46	3.33	0.993	0.014	0.983	0.157	73
AC14	20	0.01	185.13	3.59	1	0.015	0.979	0.157	70
AC14	20	0.02	190.9	4.37	0.975	0.018	0.974	0.147	65
AC14	30	0	185.14	3.78	1	0.016	0.977	0.158	69
AC14	30	0.01	185.14	3.78	1	0.016	0.977	0.158	69
AC14	30	0.02	190.9	4.37	0.975	0.018	0.974	0.147	65
AC17a	10	0	188.32	2.3	0.948	0.008	0.964	0.261	131
AC17a	10	0.01	193.17	2.94	0.929	0.012	0.938	0.254	117
AC17a	10	0.02	192.78	3.79	0.931	0.015	0.907	0.258	108
AC17a	20	0	193.22	2.72	0.928	0.01	0.951	0.251	121
AC17a	20	0.01	193.17	2.94	0.929	0.012	0.938	0.254	117
AC17a	20	0.02	192.78	3.79	0.931	0.015	0.907	0.258	108
AC17a	30	0	194.63	2.99	0.923	0.012	0.935	0.254	116
AC17a	30	0.01	194.63	2.99	0.923	0.012	0.935	0.254	116
AC17a	30	0.02	192.78	3.79	0.931	0.015	0.907	0.258	108
AC17b	10	0	234.08	4.08	0.931	0.011	0.979	0.195	69
AC17b	10	0.01	218.89	5.28	0.991	0.018	0.976	0.15	63
AC17b	10	0.02	221.56	5.91	0.982	0.02	0.97	0.151	61

AC17b	20	0	222.25	5.02	0.979	0.017	0.977	0.153	64
AC17b	20	0.01	218.89	5.28	0.991	0.018	0.976	0.15	63
AC17b	20	0.02	221.56	5.91	0.982	0.02	0.97	0.151	61
AC17b	30	0	222.25	5.02	0.979	0.017	0.977	0.153	64
AC17b	30	0.01	218.89	5.28	0.991	0.018	0.976	0.15	63
AC17b	30	0.02	221.56	5.91	0.982	0.02	0.97	0.151	61
AC18	10	0	208.28	2.6	0.878	0.01	0.962	0.21	106
AC18	10	0.01	210.65	2.84	0.858	0.015	0.93	0.208	96
AC18	10	0.02	215.24	3.39	0.826	0.02	0.885	0.207	87
AC18	20	0	210.1	2.8	0.861	0.014	0.942	0.209	99
AC18	20	0.01	210.65	2.84	0.858	0.015	0.93	0.208	96
AC18	20	0.02	215.24	3.39	0.826	0.02	0.885	0.207	87
AC18	30	0	210.65	2.84	0.858	0.015	0.93	0.208	96
AC18	30	0.01	210.65	2.84	0.858	0.015	0.93	0.208	96
AC18	30	0.02	215.24	3.39	0.826	0.02	0.885	0.207	87
AC19	10	0	133.38	1.43	0.869	0.011	0.914	0.244	177
AC19	10	0.01	133.61	1.42	0.852	0.013	0.878	0.241	164
AC19	10	0.02	133.78	1.42	0.838	0.018	0.82	0.241	148
AC19	20	0	133.75	1.41	0.841	0.014	0.876	0.238	163
AC19	20	0.01	133.75	1.41	0.841	0.014	0.876	0.238	163
AC19	20	0.02	133.78	1.42	0.838	0.018	0.82	0.241	148
AC19	30	0	133.96	1.41	0.825	0.015	0.85	0.24	157
AC19	30	0.01	133.96	1.41	0.825	0.015	0.85	0.24	157
AC19	30	0.02	133.78	1.42	0.838	0.018	0.82	0.241	148

365 **Table S4.** Power-law fit parameters for the relationship $N_d = C \cdot S_c^k$ derived from log–log regression. The coefficients C and k are reported as Monte Carlo means $\pm 1\sigma$ ($n = 5000$ realizations). The coefficient of determination (R^2) and root-mean-square error (RMSE) are calculated from the nominal fit in logarithmic space. Data are filtered using threshold criteria $N_d > N_{d,\text{th}}$ and $\text{LWC} > \text{LWC}_{\text{th}}$. The uncertainties of N_d and S_c assumed in the calculations are 10% and 9.5%, respectively. Columns list: Flight ID, $N_{d,\text{th}}$, LWC_{th} , C , σC , k , σk , R^2 , RMSE, and sample size (N).

Flight	$N_{d,\text{th}}$	LWC_{th}	C	σC	k	σk	R^2	RMSE	N
AC07	10	0	246.3	3.76	0.894	0.011	0.787	0.639	81
AC07	10	0.01	241.14	4.56	0.917	0.017	0.663	0.672	71
AC07	10	0.02	239.92	6.17	0.923	0.024	0.542	0.691	64
AC07	20	0	268.24	4.25	0.931	0.012	0.958	0.249	77
AC07	20	0.01	263.13	5.02	0.953	0.016	0.948	0.233	69
AC07	20	0.02	265.5	6.41	0.946	0.021	0.942	0.194	62
AC07	30	0	269.41	4.36	0.926	0.012	0.956	0.249	76
AC07	30	0.01	263.13	5.02	0.953	0.016	0.948	0.233	69
AC07	30	0.02	265.5	6.41	0.946	0.021	0.942	0.194	62
AC09	10	0	202.17	4.23	0.837	0.013	0.957	0.243	48
AC09	10	0.01	206.62	4.42	0.82	0.026	0.943	0.166	37
AC09	10	0.02	203.28	4.86	0.865	0.038	0.913	0.17	32
AC09	20	0	204.84	4.36	0.835	0.02	0.953	0.188	41
AC09	20	0.01	206.62	4.42	0.82	0.026	0.943	0.166	37
AC09	20	0.02	203.28	4.86	0.865	0.038	0.913	0.17	32
AC09	30	0	206.21	4.39	0.827	0.025	0.951	0.164	38
AC09	30	0.01	206.62	4.42	0.82	0.026	0.943	0.166	37
AC09	30	0.02	203.28	4.86	0.865	0.038	0.913	0.17	32
AC11	10	0	171.16	4.84	0.951	0.018	0.99	0.121	35
AC11	10	0.01	170.26	6.36	0.955	0.025	0.983	0.12	33
AC11	10	0.02	176.3	7.27	0.933	0.027	0.981	0.114	32

AC11	20	0	170.26	6.36	0.955	0.025	0.983	0.12	33
AC11	20	0.01	170.26	6.36	0.955	0.025	0.983	0.12	33
AC11	20	0.02	176.3	7.27	0.933	0.027	0.981	0.114	32
AC11	30	0	170.26	6.36	0.955	0.025	0.983	0.12	33
AC11	30	0.01	170.26	6.36	0.955	0.025	0.983	0.12	33
AC11	30	0.02	176.3	7.27	0.933	0.027	0.981	0.114	32
AC14	10	0	172.53	3.07	0.968	0.013	0.981	0.174	74
AC14	10	0.01	167.28	3.45	0.998	0.016	0.979	0.158	70
AC14	10	0.02	173.05	4.23	0.972	0.018	0.974	0.147	65
AC14	20	0	168.55	3.17	0.991	0.014	0.983	0.157	73
AC14	20	0.01	167.28	3.45	0.998	0.016	0.979	0.158	70
AC14	20	0.02	173.05	4.23	0.972	0.018	0.974	0.147	65
AC14	30	0	167.31	3.64	0.997	0.016	0.977	0.159	69
AC14	30	0.01	167.31	3.64	0.997	0.016	0.977	0.159	69
AC14	30	0.02	173.05	4.23	0.972	0.018	0.974	0.147	65
AC17a	10	0	171.04	2.16	0.946	0.008	0.964	0.262	131
AC17a	10	0.01	176.01	2.85	0.926	0.012	0.938	0.255	117
AC17a	10	0.02	175.85	3.71	0.927	0.015	0.906	0.259	108
AC17a	20	0	175.97	2.61	0.926	0.011	0.95	0.252	121
AC17a	20	0.01	176.01	2.85	0.926	0.012	0.938	0.255	117
AC17a	20	0.02	175.85	3.71	0.927	0.015	0.906	0.259	108
AC17a	30	0	177.47	2.9	0.919	0.012	0.935	0.254	116
AC17a	30	0.01	177.47	2.9	0.919	0.012	0.935	0.254	116
AC17a	30	0.02	175.85	3.71	0.927	0.015	0.906	0.259	108
AC17b	10	0	213.86	3.88	0.929	0.011	0.979	0.196	69
AC17b	10	0.01	199.07	5.12	0.988	0.018	0.976	0.151	63

AC17b	10	0.02	201.81	5.76	0.979	0.02	0.97	0.152	61
AC17b	20	0	202.3	4.86	0.976	0.017	0.977	0.153	64
AC17b	20	0.01	199.07	5.12	0.988	0.018	0.976	0.151	63
AC17b	20	0.02	201.81	5.76	0.979	0.02	0.97	0.152	61
AC17b	30	0	202.3	4.86	0.976	0.017	0.977	0.153	64
AC17b	30	0.01	199.07	5.12	0.988	0.018	0.976	0.151	63
AC17b	30	0.02	201.81	5.76	0.979	0.02	0.97	0.152	61
AC18	10	0	190.81	2.4	0.877	0.01	0.962	0.21	106
AC18	10	0.01	193.45	2.72	0.857	0.015	0.929	0.209	96
AC18	10	0.02	198.4	3.37	0.824	0.02	0.885	0.208	87
AC18	20	0	192.87	2.65	0.859	0.014	0.942	0.209	99
AC18	20	0.01	193.45	2.72	0.857	0.015	0.929	0.209	96
AC18	20	0.02	198.4	3.37	0.824	0.02	0.885	0.208	87
AC18	30	0	193.45	2.72	0.857	0.015	0.929	0.209	96
AC18	30	0.01	193.45	2.72	0.857	0.015	0.929	0.209	96
AC18	30	0.02	198.4	3.37	0.824	0.02	0.885	0.208	87
AC19	10	0	120.43	1.24	0.868	0.011	0.914	0.245	177
AC19	10	0.01	120.91	1.24	0.85	0.014	0.878	0.242	164
AC19	10	0.02	121.27	1.28	0.835	0.018	0.82	0.241	148
AC19	20	0	121.18	1.24	0.839	0.014	0.876	0.238	163
AC19	20	0.01	121.18	1.24	0.839	0.014	0.876	0.238	163
AC19	20	0.02	121.27	1.28	0.835	0.018	0.82	0.241	148
AC19	30	0	121.6	1.26	0.823	0.016	0.85	0.241	157
AC19	30	0.01	121.6	1.26	0.823	0.016	0.85	0.241	157
AC19	30	0.02	121.27	1.28	0.835	0.018	0.82	0.241	148

Table S5. Power-law fit parameters for the relationship $N_d = C \cdot S_v^k$ derived from log–log regression. The coefficients C and k are reported as Monte Carlo means $\pm 1\sigma$ ($n = 5000$ realizations). The coefficient of determination (R^2) and root-mean-square error (RMSE) are calculated from the nominal fit in logarithmic space. Data are filtered using threshold criteria $N_d > N_{d,\text{th}}$ and $LWC > LWC_{\text{th}}$. The uncertainties of N_d and S_v assumed in the calculations are 30% and 36.6%, respectively. Columns list: Flight ID, $N_{d,\text{th}}$, LWC_{th} , C , σC , k , σk , R^2 , RMSE, and sample size (N).

Flight	$N_{d,\text{th}}$	LWC_{th}	C	σC	k	σk	R^2	RMSE	N
AC07	10	0	1721.2	192.16	0.802	0.051	0.783	0.645	81
AC07	10	0.01	1646.06	218.54	0.765	0.075	0.657	0.678	71
AC07	10	0.02	1539.65	240.42	0.702	0.098	0.533	0.698	64
AC07	20	0	2014.97	231.49	0.827	0.055	0.958	0.249	77
AC07	20	0.01	1960.94	247.89	0.801	0.073	0.947	0.234	69
AC07	20	0.02	1813.89	252.3	0.731	0.09	0.943	0.193	62
AC07	30	0	1988.67	227.02	0.818	0.055	0.956	0.25	76
AC07	30	0.01	1960.94	247.89	0.801	0.073	0.947	0.234	69
AC07	30	0.02	1813.89	252.3	0.731	0.09	0.943	0.193	62
AC09	10	0	1103.63	193.86	0.758	0.059	0.956	0.244	48
AC09	10	0.01	900.9	217.98	0.642	0.103	0.941	0.169	37
AC09	10	0.02	839.96	246.48	0.593	0.137	0.91	0.173	32
AC09	20	0	1006.72	213.9	0.705	0.084	0.952	0.19	41
AC09	20	0.01	900.9	217.98	0.642	0.103	0.941	0.169	37
AC09	20	0.02	839.96	246.48	0.593	0.137	0.91	0.173	32
AC09	30	0	933.33	218.37	0.662	0.098	0.949	0.167	38
AC09	30	0.01	900.9	217.98	0.642	0.103	0.941	0.169	37
AC09	30	0.02	839.96	246.48	0.593	0.137	0.91	0.173	32
AC11	10	0	1414.52	175.01	0.844	0.075	0.99	0.125	35

AC11	10	0.01	1339.18	180.85	0.785	0.101	0.982	0.125	33
AC11	10	0.02	1294.82	175.18	0.742	0.11	0.98	0.118	32
AC11	20	0	1339.18	180.85	0.785	0.101	0.982	0.125	33
AC11	20	0.01	1339.18	180.85	0.785	0.101	0.982	0.125	33
AC11	20	0.02	1294.82	175.18	0.742	0.11	0.98	0.118	32
AC11	30	0	1339.18	180.85	0.785	0.101	0.982	0.125	33
AC11	30	0.01	1339.18	180.85	0.785	0.101	0.982	0.125	33
AC11	30	0.02	1294.82	175.18	0.742	0.11	0.98	0.118	32
AC14	10	0	1374.34	139.07	0.855	0.056	0.979	0.18	74
AC14	10	0.01	1366.47	152.1	0.844	0.072	0.977	0.166	70
AC14	10	0.02	1294.32	145.45	0.784	0.08	0.972	0.153	65
AC14	20	0	1390.5	146.66	0.862	0.061	0.981	0.164	73
AC14	20	0.01	1366.47	152.1	0.844	0.072	0.977	0.166	70
AC14	20	0.02	1294.32	145.45	0.784	0.08	0.972	0.153	65
AC14	30	0	1355.09	150.79	0.836	0.074	0.975	0.167	69
AC14	30	0.01	1355.09	150.79	0.836	0.074	0.975	0.167	69
AC14	30	0.02	1294.32	145.45	0.784	0.08	0.972	0.153	65
AC17a	10	0	1320.21	95.94	0.849	0.038	0.961	0.271	131
AC17a	10	0.01	1225.93	98.67	0.774	0.054	0.934	0.263	117
AC17a	10	0.02	1168.27	101.36	0.718	0.067	0.9	0.267	108
AC17a	20	0	1254.73	99.58	0.796	0.049	0.947	0.26	121
AC17a	20	0.01	1225.93	98.67	0.774	0.054	0.934	0.263	117
AC17a	20	0.02	1168.27	101.36	0.718	0.067	0.9	0.267	108
AC17a	30	0	1213.02	100.58	0.763	0.055	0.931	0.263	116
AC17a	30	0.01	1213.02	100.58	0.763	0.055	0.931	0.263	116
AC17a	30	0.02	1168.27	101.36	0.718	0.067	0.9	0.267	108

AC17b	10	0	1697.48	157.11	0.841	0.051	0.978	0.201	69
AC17b	10	0.01	1648.07	178.79	0.807	0.079	0.973	0.159	63
AC17b	10	0.02	1600.21	178.1	0.773	0.086	0.967	0.16	61
AC17b	20	0	1654.08	175.36	0.812	0.074	0.975	0.161	64
AC17b	20	0.01	1648.07	178.79	0.807	0.079	0.973	0.159	63
AC17b	20	0.02	1600.21	178.1	0.773	0.086	0.967	0.16	61
AC17b	30	0	1654.08	175.36	0.812	0.074	0.975	0.161	64
AC17b	30	0.01	1648.07	178.79	0.807	0.079	0.973	0.159	63
AC17b	30	0.02	1600.21	178.1	0.773	0.086	0.967	0.16	61
AC18	10	0	1176.15	130.06	0.765	0.048	0.961	0.214	106
AC18	10	0.01	1013.18	134.56	0.676	0.067	0.927	0.212	96
AC18	10	0.02	881.14	130.89	0.582	0.081	0.881	0.211	87
AC18	20	0	1063.6	131.77	0.706	0.06	0.94	0.213	99
AC18	20	0.01	1013.18	134.56	0.676	0.067	0.927	0.212	96
AC18	20	0.02	881.14	130.89	0.582	0.081	0.881	0.211	87
AC18	30	0	1013.18	134.56	0.676	0.067	0.927	0.212	96
AC18	30	0.01	1013.18	134.56	0.676	0.067	0.927	0.212	96
AC18	30	0.02	881.14	130.89	0.582	0.081	0.881	0.211	87
AC19	10	0	534	64.12	0.695	0.05	0.912	0.247	177
AC19	10	0.01	466.78	62.42	0.625	0.059	0.875	0.244	164
AC19	10	0.02	400.16	62.31	0.539	0.072	0.817	0.243	148
AC19	20	0	456.91	61.43	0.614	0.059	0.874	0.24	163
AC19	20	0.01	456.91	61.43	0.614	0.059	0.874	0.24	163
AC19	20	0.02	400.16	62.31	0.539	0.072	0.817	0.243	148
AC19	30	0	425.99	59.82	0.576	0.063	0.847	0.243	157
AC19	30	0.01	425.99	59.82	0.576	0.063	0.847	0.243	157
AC19	30	0.02	400.16	62.31	0.539	0.072	0.817	0.243	148

Table S6. Power-law fit parameters for the relationship $N_d = C \cdot S_l^k$ derived from log–log regression. The coefficients C and k are reported as Monte Carlo means $\pm 1\sigma$ ($n = 5000$ realizations). The coefficient of determination (R^2) and root-mean-square error (RMSE) are calculated from the nominal fit in logarithmic space. Data are filtered using threshold criteria $N_d > N_{d,th}$ and $LWC > LWC_{th}$. The uncertainties of N_d and S_l assumed in the calculations are 30% and 27.6%, respectively. Columns list:
385 Flight ID, $N_{d,th}$, LWC_{th} , C , σC , k , σk , R^2 , RMSE, and sample size (N).

Flight	$N_{d,th}$	LWC_{th}	C	σC	k	σk	R^2	RMSE	N
AC07	10	0	266.51	13.55	0.854	0.039	0.788	0.638	81
AC07	10	0.01	269	16.61	0.848	0.057	0.664	0.671	71
AC07	10	0.02	279.13	22.2	0.817	0.078	0.543	0.691	64
AC07	20	0	293.02	15.35	0.884	0.043	0.958	0.249	77
AC07	20	0.01	294.93	18.34	0.882	0.054	0.948	0.233	69
AC07	20	0.02	309.74	24.01	0.839	0.07	0.942	0.194	62
AC07	30	0	294.72	15.87	0.877	0.042	0.956	0.249	76
AC07	30	0.01	294.93	18.34	0.882	0.054	0.948	0.233	69
AC07	30	0.02	309.74	24.01	0.839	0.07	0.942	0.194	62
AC09	10	0	211.93	14.99	0.802	0.046	0.957	0.243	48
AC09	10	0.01	217.7	15.39	0.731	0.085	0.944	0.166	37
AC09	10	0.02	219.48	16.18	0.717	0.117	0.913	0.169	32
AC09	20	0	215.09	15.29	0.773	0.067	0.953	0.188	41
AC09	20	0.01	217.7	15.39	0.731	0.085	0.944	0.166	37
AC09	20	0.02	219.48	16.18	0.717	0.117	0.913	0.169	32
AC09	30	0	216.99	15.33	0.746	0.081	0.951	0.164	38
AC09	30	0.01	217.7	15.39	0.731	0.085	0.944	0.166	37
AC09	30	0.02	219.48	16.18	0.717	0.117	0.913	0.169	32
AC11	10	0	193.46	18.27	0.907	0.06	0.99	0.121	35

AC11	10	0.01	200.94	24.17	0.88	0.083	0.983	0.12	33
AC11	10	0.02	211.29	27.99	0.847	0.09	0.981	0.114	32
AC11	20	0	200.94	24.17	0.88	0.083	0.983	0.12	33
AC11	20	0.01	200.94	24.17	0.88	0.083	0.983	0.12	33
AC11	20	0.02	211.29	27.99	0.847	0.09	0.981	0.114	32
AC11	30	0	200.94	24.17	0.88	0.083	0.983	0.12	33
AC11	30	0.01	200.94	24.17	0.88	0.083	0.983	0.12	33
AC11	30	0.02	211.29	27.99	0.847	0.09	0.981	0.114	32
AC14	10	0	193.24	11.46	0.919	0.043	0.981	0.173	74
AC14	10	0.01	192.2	13.05	0.929	0.053	0.979	0.157	70
AC14	10	0.02	203.97	16.47	0.883	0.062	0.974	0.147	65
AC14	20	0	190.74	11.89	0.935	0.047	0.983	0.157	73
AC14	20	0.01	192.2	13.05	0.929	0.053	0.979	0.157	70
AC14	20	0.02	203.97	16.47	0.883	0.062	0.974	0.147	65
AC14	30	0	193.31	13.75	0.925	0.056	0.977	0.158	69
AC14	30	0.01	193.31	13.75	0.925	0.056	0.977	0.158	69
AC14	30	0.02	203.97	16.47	0.883	0.062	0.974	0.147	65
AC17a	10	0	189.08	7.94	0.909	0.029	0.964	0.261	131
AC17a	10	0.01	200.53	10.66	0.862	0.04	0.938	0.254	117
AC17a	10	0.02	208.84	14.3	0.831	0.052	0.907	0.258	108
AC17a	20	0	198.01	9.81	0.872	0.037	0.951	0.251	121
AC17a	20	0.01	200.53	10.66	0.862	0.04	0.938	0.254	117
AC17a	20	0.02	208.84	14.3	0.831	0.052	0.907	0.258	108
AC17a	30	0	202.61	10.9	0.853	0.042	0.935	0.254	116
AC17a	30	0.01	202.61	10.9	0.853	0.042	0.935	0.254	116
AC17a	30	0.02	208.84	14.3	0.831	0.052	0.907	0.258	108

AC17b	10	0	236.59	14.36	0.893	0.039	0.979	0.195	69
AC17b	10	0.01	234.73	19.91	0.906	0.061	0.976	0.15	63
AC17b	10	0.02	242.52	22.71	0.883	0.068	0.97	0.151	61
AC17b	20	0	235.49	18.71	0.903	0.057	0.977	0.153	64
AC17b	20	0.01	234.73	19.91	0.906	0.061	0.976	0.15	63
AC17b	20	0.02	242.52	22.71	0.883	0.068	0.97	0.151	61
AC17b	30	0	235.49	18.71	0.903	0.057	0.977	0.153	64
AC17b	30	0.01	234.73	19.91	0.906	0.061	0.976	0.15	63
AC17b	30	0.02	242.52	22.71	0.883	0.068	0.97	0.151	61
AC18	10	0	204.42	8.54	0.827	0.036	0.962	0.21	106
AC18	10	0.01	210.96	9.57	0.77	0.051	0.93	0.208	96
AC18	10	0.02	221.76	11.57	0.699	0.066	0.885	0.207	87
AC18	20	0	208.54	9.32	0.787	0.047	0.942	0.209	99
AC18	20	0.01	210.96	9.57	0.77	0.051	0.93	0.208	96
AC18	20	0.02	221.76	11.57	0.699	0.066	0.885	0.207	87
AC18	30	0	210.96	9.57	0.77	0.051	0.93	0.208	96
AC18	30	0.01	210.96	9.57	0.77	0.051	0.93	0.208	96
AC18	30	0.02	221.76	11.57	0.699	0.066	0.885	0.207	87
AC19	10	0	126.59	4.38	0.784	0.038	0.914	0.244	177
AC19	10	0.01	127.24	4.34	0.736	0.046	0.878	0.241	164
AC19	10	0.02	128.75	4.35	0.674	0.059	0.82	0.241	148
AC19	20	0	127.43	4.34	0.724	0.047	0.876	0.238	163
AC19	20	0.01	127.43	4.34	0.724	0.047	0.876	0.238	163
AC19	20	0.02	128.75	4.35	0.674	0.059	0.82	0.241	148
AC19	30	0	127.99	4.32	0.693	0.051	0.85	0.24	157
AC19	30	0.01	127.99	4.32	0.693	0.051	0.85	0.24	157
AC19	30	0.02	128.75	4.35	0.674	0.059	0.82	0.241	148

Table S7. Power-law fit parameters for the relationship $N_d = C \cdot S_c^k$ derived from log–log regression. The coefficients C and k are reported as Monte Carlo means $\pm 1\sigma$ ($n = 5000$ realizations). The coefficient of determination (R^2) and root-mean-square error (RMSE) are calculated from the nominal fit in logarithmic space. Data are filtered using threshold criteria $N_d > N_{d,\text{th}}$ and $LWC > LWC_{\text{th}}$. The uncertainties of N_d and S_c assumed in the calculations are 30% and 28.5%, respectively. Columns list: Flight ID, $N_{d,\text{th}}$, LWC_{th} , C , σC , k , σk , R^2 , RMSE, and sample size (N).

Flight	$N_{d,\text{th}}$	LWC_{th}	C	σC	k	σk	R^2	RMSE	N
AC07	10	0	246.01	12.94	0.85	0.04	0.787	0.639	81
AC07	10	0.01	249.2	16.59	0.841	0.058	0.663	0.672	71
AC07	10	0.02	260.33	22.76	0.807	0.081	0.542	0.691	64
AC07	20	0	269.85	14.76	0.879	0.044	0.958	0.249	77
AC07	20	0.01	272.28	18.1	0.875	0.056	0.948	0.233	69
AC07	20	0.02	288.17	24.58	0.83	0.072	0.942	0.194	62
AC07	30	0	271.65	15.23	0.872	0.044	0.956	0.249	76
AC07	30	0.01	272.28	18.1	0.875	0.056	0.948	0.233	69
AC07	30	0.02	288.17	24.58	0.83	0.072	0.942	0.194	62
AC09	10	0	194.3	13.42	0.798	0.047	0.957	0.243	48
AC09	10	0.01	201.34	14.07	0.723	0.087	0.943	0.166	37
AC09	10	0.02	203.71	15.63	0.706	0.12	0.913	0.17	32
AC09	20	0	197.88	13.73	0.767	0.068	0.953	0.188	41
AC09	20	0.01	201.34	14.07	0.723	0.087	0.943	0.166	37
AC09	20	0.02	203.71	15.63	0.706	0.12	0.913	0.17	32
AC09	30	0	200.33	13.94	0.739	0.082	0.951	0.164	38
AC09	30	0.01	201.34	14.07	0.723	0.087	0.943	0.166	37
AC09	30	0.02	203.71	15.63	0.706	0.12	0.913	0.17	32
AC11	10	0	178.21	18.02	0.901	0.061	0.99	0.121	35

AC11	10	0.01	186.53	24.5	0.871	0.085	0.983	0.12	33
AC11	10	0.02	197.14	28.73	0.837	0.092	0.981	0.114	32
AC11	20	0	186.53	24.5	0.871	0.085	0.983	0.12	33
AC11	20	0.01	186.53	24.5	0.871	0.085	0.983	0.12	33
AC11	20	0.02	197.14	28.73	0.837	0.092	0.981	0.114	32
AC11	30	0	186.53	24.5	0.871	0.085	0.983	0.12	33
AC11	30	0.01	186.53	24.5	0.871	0.085	0.983	0.12	33
AC11	30	0.02	197.14	28.73	0.837	0.092	0.981	0.114	32
AC14	10	0	176.81	11.14	0.914	0.045	0.981	0.174	74
AC14	10	0.01	176.04	12.88	0.922	0.055	0.979	0.158	70
AC14	10	0.02	188.13	16.62	0.875	0.064	0.974	0.147	65
AC14	20	0	174.38	11.65	0.929	0.049	0.983	0.157	73
AC14	20	0.01	176.04	12.88	0.922	0.055	0.979	0.158	70
AC14	20	0.02	188.13	16.62	0.875	0.064	0.974	0.147	65
AC14	30	0	177.23	13.63	0.917	0.057	0.977	0.159	69
AC14	30	0.01	177.23	13.63	0.917	0.057	0.977	0.159	69
AC14	30	0.02	188.13	16.62	0.875	0.064	0.974	0.147	65
AC17a	10	0	173.11	7.66	0.903	0.03	0.964	0.262	131
AC17a	10	0.01	185.21	10.82	0.853	0.043	0.938	0.255	117
AC17a	10	0.02	194.26	14.82	0.821	0.055	0.906	0.259	108
AC17a	20	0	182.36	9.74	0.865	0.038	0.95	0.252	121
AC17a	20	0.01	185.21	10.82	0.853	0.043	0.938	0.255	117
AC17a	20	0.02	194.26	14.82	0.821	0.055	0.906	0.259	108
AC17a	30	0	187.33	11.03	0.845	0.044	0.935	0.254	116
AC17a	30	0.01	187.33	11.03	0.845	0.044	0.935	0.254	116
AC17a	30	0.02	194.26	14.82	0.821	0.055	0.906	0.259	108

AC17b	10	0	217.89	13.9	0.889	0.04	0.979	0.196	69
AC17b	10	0.01	217.1	20.02	0.898	0.063	0.976	0.151	63
AC17b	10	0.02	225.27	23.23	0.873	0.07	0.97	0.152	61
AC17b	20	0	217.62	18.73	0.895	0.059	0.977	0.153	64
AC17b	20	0.01	217.1	20.02	0.898	0.063	0.976	0.151	63
AC17b	20	0.02	225.27	23.23	0.873	0.07	0.97	0.152	61
AC17b	30	0	217.62	18.73	0.895	0.059	0.977	0.153	64
AC17b	30	0.01	217.1	20.02	0.898	0.063	0.976	0.151	63
AC17b	30	0.02	225.27	23.23	0.873	0.07	0.97	0.152	61
AC18	10	0	188.64	8	0.822	0.037	0.962	0.21	106
AC18	10	0.01	196.19	9.42	0.762	0.053	0.929	0.209	96
AC18	10	0.02	208.19	11.92	0.688	0.068	0.885	0.208	87
AC18	20	0	193.44	9.02	0.781	0.048	0.942	0.209	99
AC18	20	0.01	196.19	9.42	0.762	0.053	0.929	0.209	96
AC18	20	0.02	208.19	11.92	0.688	0.068	0.885	0.208	87
AC18	30	0	196.19	9.42	0.762	0.053	0.929	0.209	96
AC18	30	0.01	196.19	9.42	0.762	0.053	0.929	0.209	96
AC18	30	0.02	208.19	11.92	0.688	0.068	0.885	0.208	87
AC19	10	0	115.44	3.86	0.777	0.039	0.914	0.245	177
AC19	10	0.01	116.76	3.86	0.726	0.048	0.878	0.242	164
AC19	10	0.02	119.15	4.02	0.661	0.061	0.82	0.241	148
AC19	20	0	117.09	3.86	0.715	0.048	0.876	0.238	163
AC19	20	0.01	117.09	3.86	0.715	0.048	0.876	0.238	163
AC19	20	0.02	119.15	4.02	0.661	0.061	0.82	0.241	148
AC19	30	0	118.08	3.91	0.683	0.052	0.85	0.241	157
AC19	30	0.01	118.08	3.91	0.683	0.052	0.85	0.241	157
AC19	30	0.02	119.15	4.02	0.661	0.061	0.82	0.241	148

