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# Effects of ozone–climate interactions on the long-term temperature trend in the Arctic stratosphere

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**Refractive index**

The quasi-geostrophic refractive index (RI) is used to diagnose the environment of wave propagation (Chen and Robinson, 1992), is calculated as:

$$RI = \frac{\overline{q}_\varphi}{\overline{u}} - \left(\frac{k}{a\cos\varphi}\right)^2 - \left(\frac{f}{2NH}\right)^2 \qquad \text{(S1)}$$

where the meridional gradient of the zonal mean potential vorticity is calculated as:

$$\overline{q}_\varphi = \frac{2\Omega}{a}\cos\varphi - \frac{1}{a^2}\left[\frac{(\overline{u}\cos\varphi)_\varphi}{a\cos\varphi}\right]_\varphi - \frac{f^2}{\rho_0}\left(\rho_0\frac{\overline{u}_z}{N^2}\right)_z \qquad \text{(S2)}$$

where $-\frac{f^2}{\rho_0}\left(\rho_0\frac{\overline{u}_z}{N^2}\right)_z = \left(\frac{f^2}{HN^2} + \frac{f^2}{N^4}\frac{\mathrm{d}N^2}{\mathrm{d}z}\right)\overline{u}_z - \frac{f^2}{N^2}\overline{u}_{zz}$ , and $H, q, k, N^2, \Omega, u_z$ are the scale height, potential vorticity, zonal wavenumber, buoyancy frequency, Earth's angular frequency, and zonal wind shear, respectively. The refractive index squared could be affected not only by atmospheric stability and wind shear, but also by the quadratic vertical shear of the zonal mean zonal wind. As discussed in Matsuno (1970), it is expected that planetary waves of wavenumber *k* tend to propagate toward regions where $n_k^2$>0 and are inhibited in regions where $n_k^2$<0.

**Takaya-Nakamura (T-N) wave activity flux**

The Takaya-Nakamura (T-N) wave activity flux (Takaya and Nakamura 1997; 2001; Nakamura et al., 2010) is used to represent the three-dimensional energy dispersion characteristics of the quasi-stationary Rossby wave with respect to climatological mean flow:

$$W=\frac{pcos\phi}{2|U|}\cdot\left\{\begin{array}{l}\frac{U}{a^2cos^2\phi}\left[\left(\frac{\partial\psi'}{\partial\lambda}\right)^2-\psi'\frac{\partial^2\psi'}{\partial\lambda^2}\right]+\frac{V}{a^2cos\phi}\left[\frac{\partial\psi'}{\partial\lambda}\frac{\partial\psi'}{\partial\phi}-\psi'\frac{\partial^2\psi'}{\partial\lambda\partial\phi}\right]\\ \frac{U}{a^2cos\phi}\left[\frac{\partial\psi'}{\partial\lambda}\frac{\partial\psi'}{\partial\phi}-\psi'\frac{\partial^2\psi'}{\partial\lambda\partial\phi}\right]+\frac{V}{a^2}\left[\left(\frac{\partial\psi'}{\partial\phi}\right)^2-\psi'\frac{\partial^2\psi'}{\partial\phi^2}\right]\\ \frac{f_0^2}{N^2}\left\{\frac{U}{acos\phi}\left[\frac{\partial\psi'}{\partial\lambda}\frac{\partial\psi'}{\partial z}-\psi'\frac{\partial^2\psi'}{\partial\lambda\partial z}\right]+\frac{V}{a}\left[\frac{\partial\psi'}{\partial\phi}\frac{\partial\psi'}{\partial z}-\psi'\frac{\partial^2\psi'}{\partial\phi\partial z}\right]\right\}\end{array}\right\} \qquad \text{(S3)}$$

where the superscript is the zonal deviation and where $\phi$, $\lambda$, $\Phi$, $f=2\Omega\sin\phi$, $a$, $\Omega$ are the latitude, longitude, geopotential height, Coriolis parameter, earth radius and Earth rotation rate, respectively. $\psi'=\frac{\phi'}{f}$ represents the perturbation of the quasi-geostrophic stream function relative to the climatological field, and $U=(U,V)$ represents the climatological basic flow fields.

In Eqs. (S1)–(S3), the overbar represents zonal mean, while the prime indicates deviations from zonal mean. The subscripts denote partial derivatives. The Fourier decomposition is used to extract components $u', v', and\ \theta'$ in Eqs. (S1)–(S3) and components $\psi'$ in Eqs. (S3) with different zonal wave numbers.

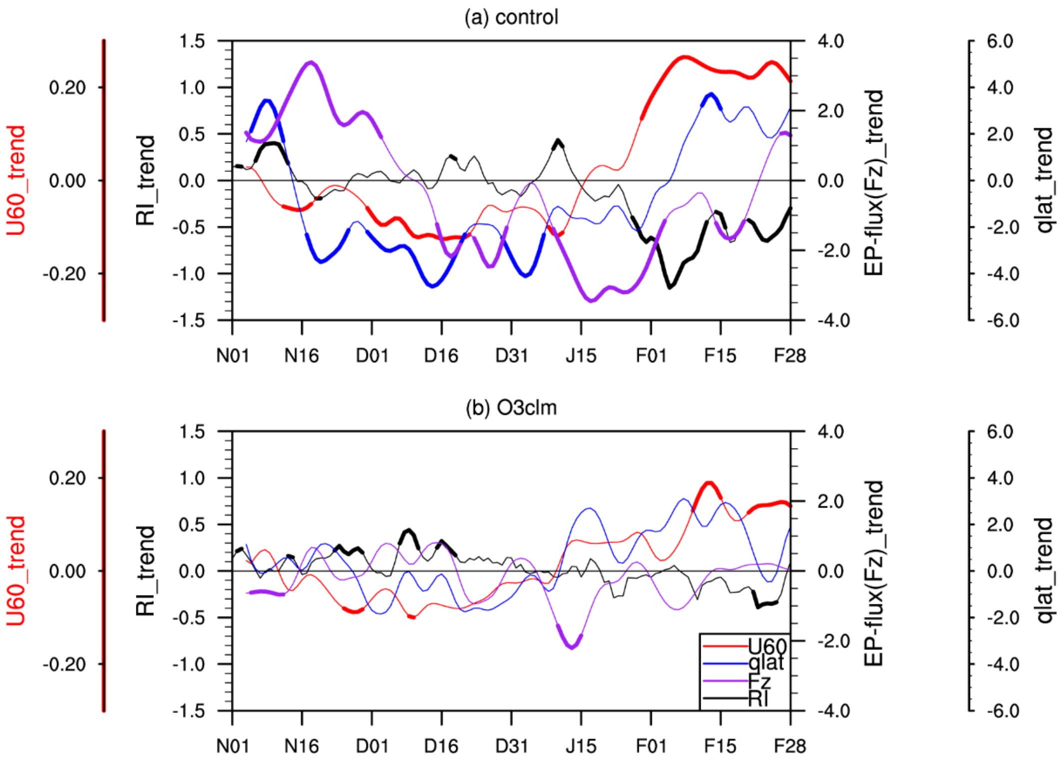


**Figure S1** Daily evolution of the trends in the RI (black lines), vertical component of the E-P flux ( $F_z$ ; purple lines), $\bar{q}_\varphi$ (blue lines), U60 (zonal wind at 60°N; red lines) before 2000 at 50–150 hPa averaged in mid-latitude (55°–70° N) from 1 November to 28 February, derived from (a) the ensemble mean of the control experiments and (b) O3clm experiments. The bold solid lines indicate the trends in the significant RI, vertical component of the E-P flux and $\bar{q}_\varphi$ at the 90% confidence level according to Student's *t* test (The daily data are first processed with a 7-day low-pass filter to remove high-frequency signals).

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