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Fragmentation of ice particles: laboratory experiments on graupel–graupel and graupel–snowflake collisions

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S1 Graupel free fall equations

The equation of motion for a spherical liquid drop from Pruppacher and Klett (2010) can be used to describe the motion of a lump graupel which is almost spherical with

$$m_g \frac{dv}{dt} = (m_g + m_a)g - \frac{1}{2}\rho_a C_d A v^2 = \left(1 - \frac{\rho_a}{\rho_g}\right) g - \frac{3}{8}\frac{C_d \rho_a}{\rho_g r} v^2. \tag{C1}$$

where m_g and m_a the graupel and air masses, ρ_g and ρ_a are the graupel and air densities, g the acceleration, A the graupel cross section, C_d the drag coefficient, r the radius of the graupel and v the graupel fall speed. The previous equation can be simplified with C_1 and C_2 terms:

$$\frac{dv}{dt} = C_1 - C_2 v^2. \tag{S2}$$

Equation S2 have to be integrated to deduce the speed and the position of the graupel during the fall speed as

$$t = \int \frac{1}{C_1 - C_2 v^2} dv = \int \frac{1}{(\sqrt{C_1} - \sqrt{C_2} v)(\sqrt{C_1} + \sqrt{C_2} v)} dv \tag{S3}$$

$$t = \int \frac{1}{2\sqrt{C_1}(\sqrt{C_1} + \sqrt{C_2} v)} + \frac{1}{2\sqrt{C_1}(\sqrt{C_1} - \sqrt{C_2} v)} dv \tag{S4}$$

$$t = \frac{1}{2\sqrt{C_1}} \left(\frac{\ln(\sqrt{C_1} + \sqrt{C_2} v)}{\sqrt{C_2}} - \frac{\ln(\sqrt{C_1} - \sqrt{C_2} v)}{\sqrt{C_2}} \right) = \frac{1}{2\sqrt{C_1 C_2}} \ln \left(\frac{1 + \sqrt{C_2}/\sqrt{C_1} v}{1 - \sqrt{C_2}/\sqrt{C_1} v} \right). \tag{S5}$$

From Eq. S5 and the arctanh function properties

$$t = \frac{1}{\sqrt{C_1 C_2}} \operatorname{arctanh} \left(\frac{\sqrt{C_2}}{\sqrt{C_1}} v \right). \tag{S6}$$

By integrating Eq. S6, one can deduce the speed of the graupel depending on the time

$$v = \frac{\sqrt{C_1}}{\sqrt{C_2}} \tanh \left(\sqrt{C_1}\sqrt{C_2} t \right). \tag{S7}$$

To know the position of the graupel according to the time, equation S7 have to be integrated. With the properties of the tanh function, the position of the graupel is expressed by

$$x = \frac{1}{\sqrt{C_1}} \ln \left(\cosh \left(\sqrt{C_1 C_2} t \right) \right). \tag{S8}$$

If $t \to \infty$, the fall velocity becomes equal to the terminal velocity v_T of the graupel:

$$v_T = \sqrt{\frac{8r(\rho_a/\rho_g - 1)g}{3C_d}}. \tag{S9}$$

S2 Fragments size distributions

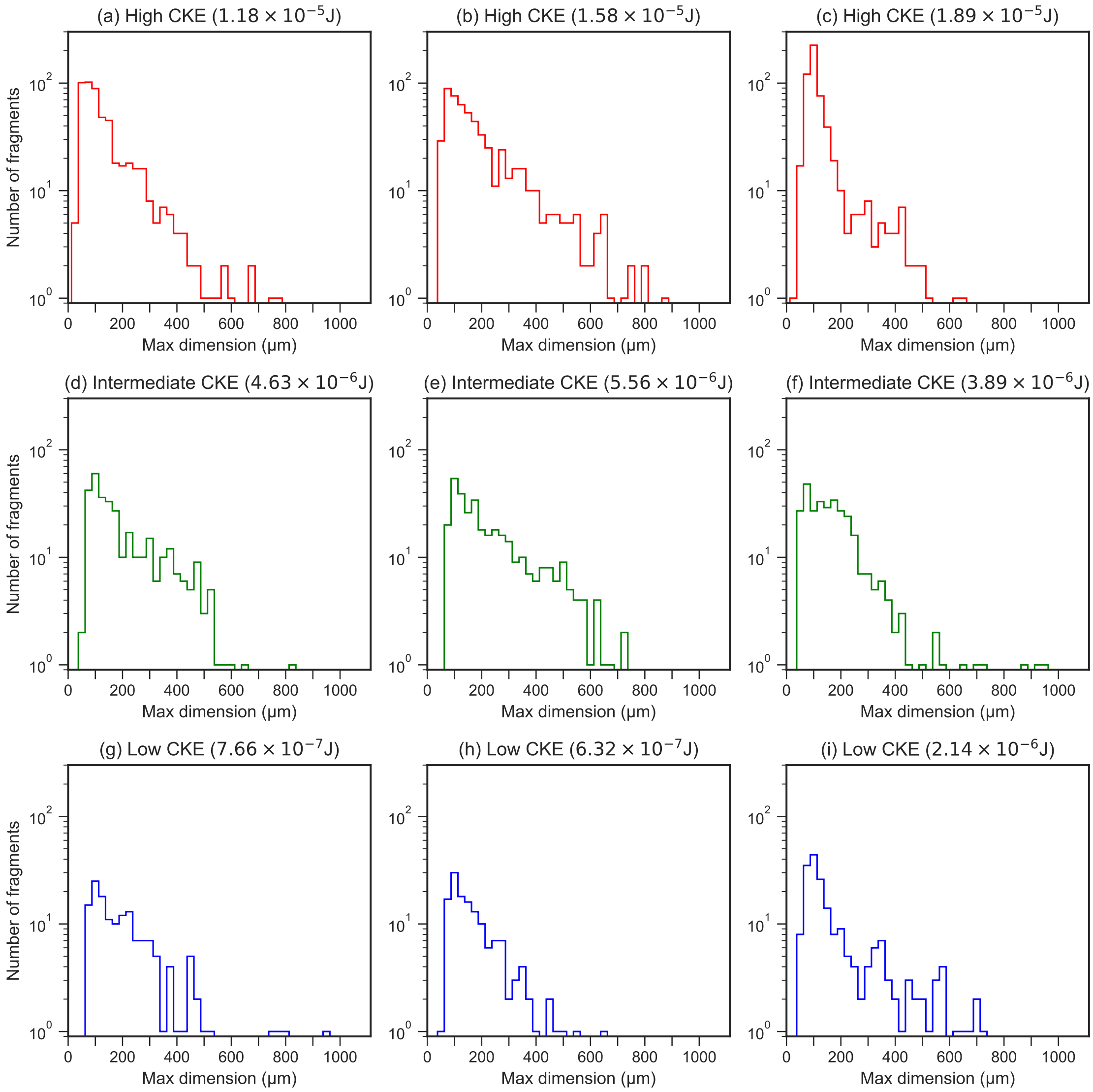

Figure S1: Size distributions of ice fragments produced by graupel-graupel with dendrites collisions. Low CKE in blue, intermediate CKE and high CKE in red.

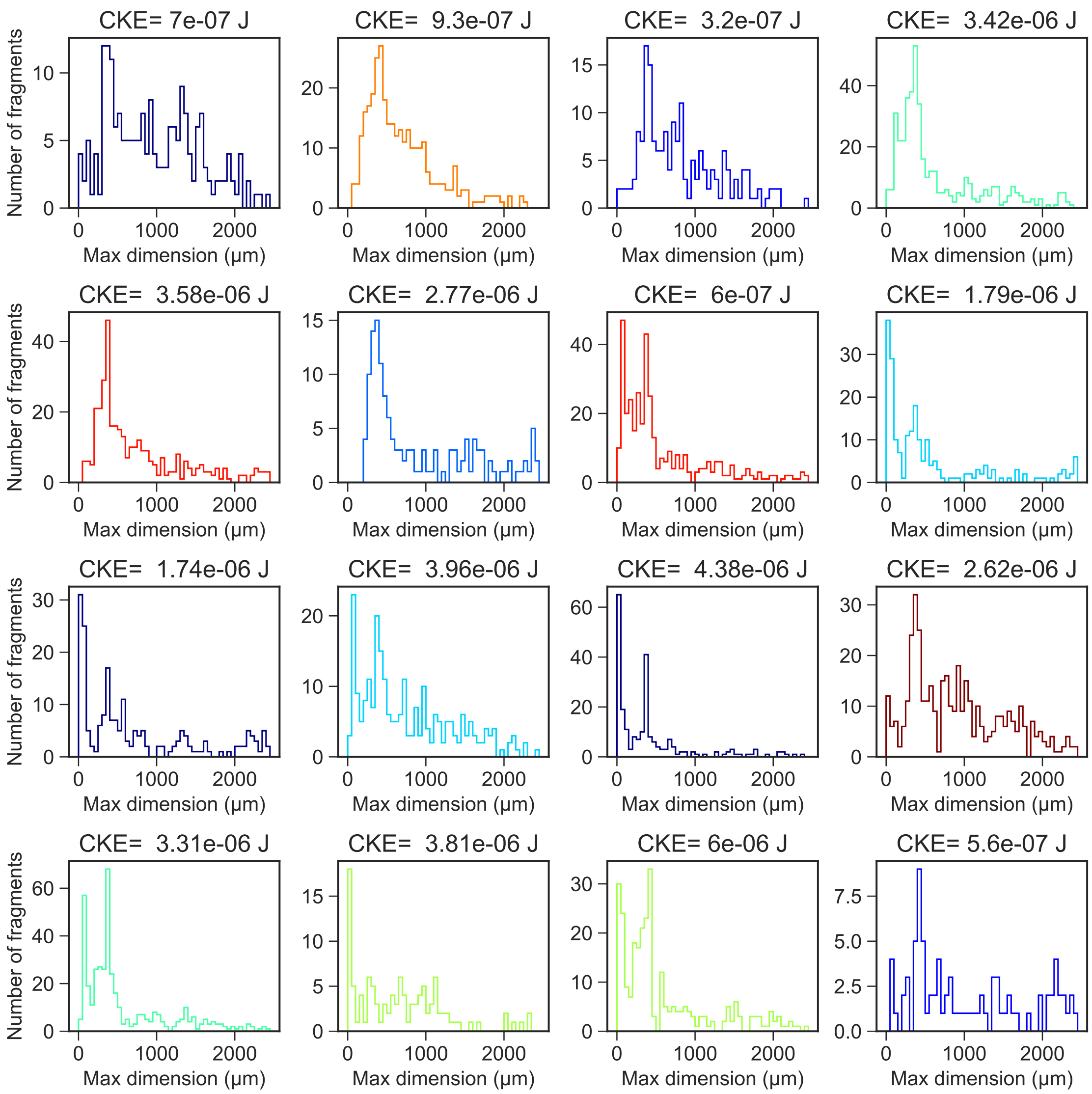

Figure S2: Size distributions of ice fragments produced by graupel-snowflake collisions for different impact positions (see color scale of Fig. 14).

S3 Distributions for the cross sectional areas of the fragments

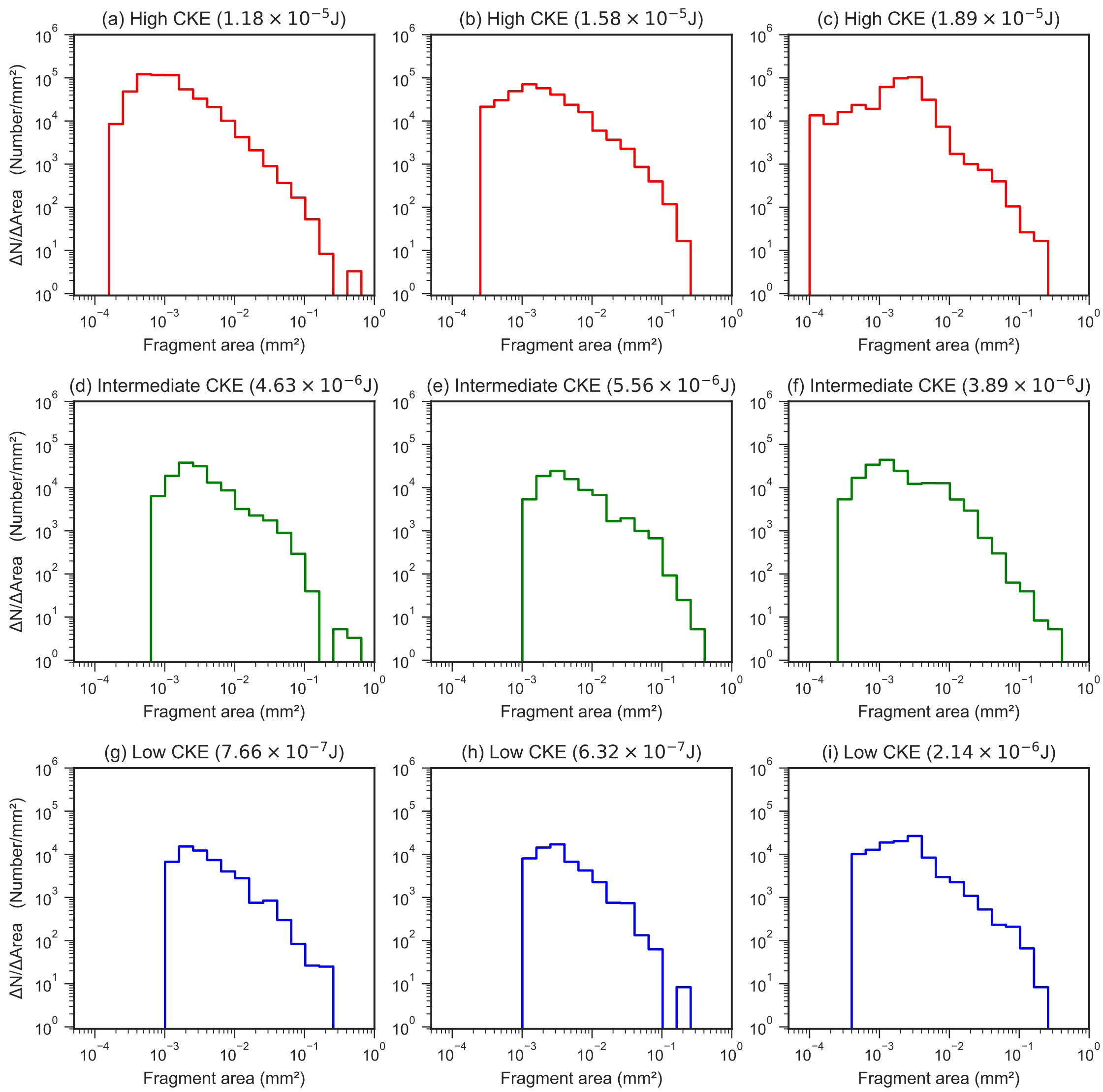

Figure S3: Area distributions of ice fragments produced by graupel-graupel with dendrites collisions. Low CKE in blue, intermediate CKE and high CKE in red.

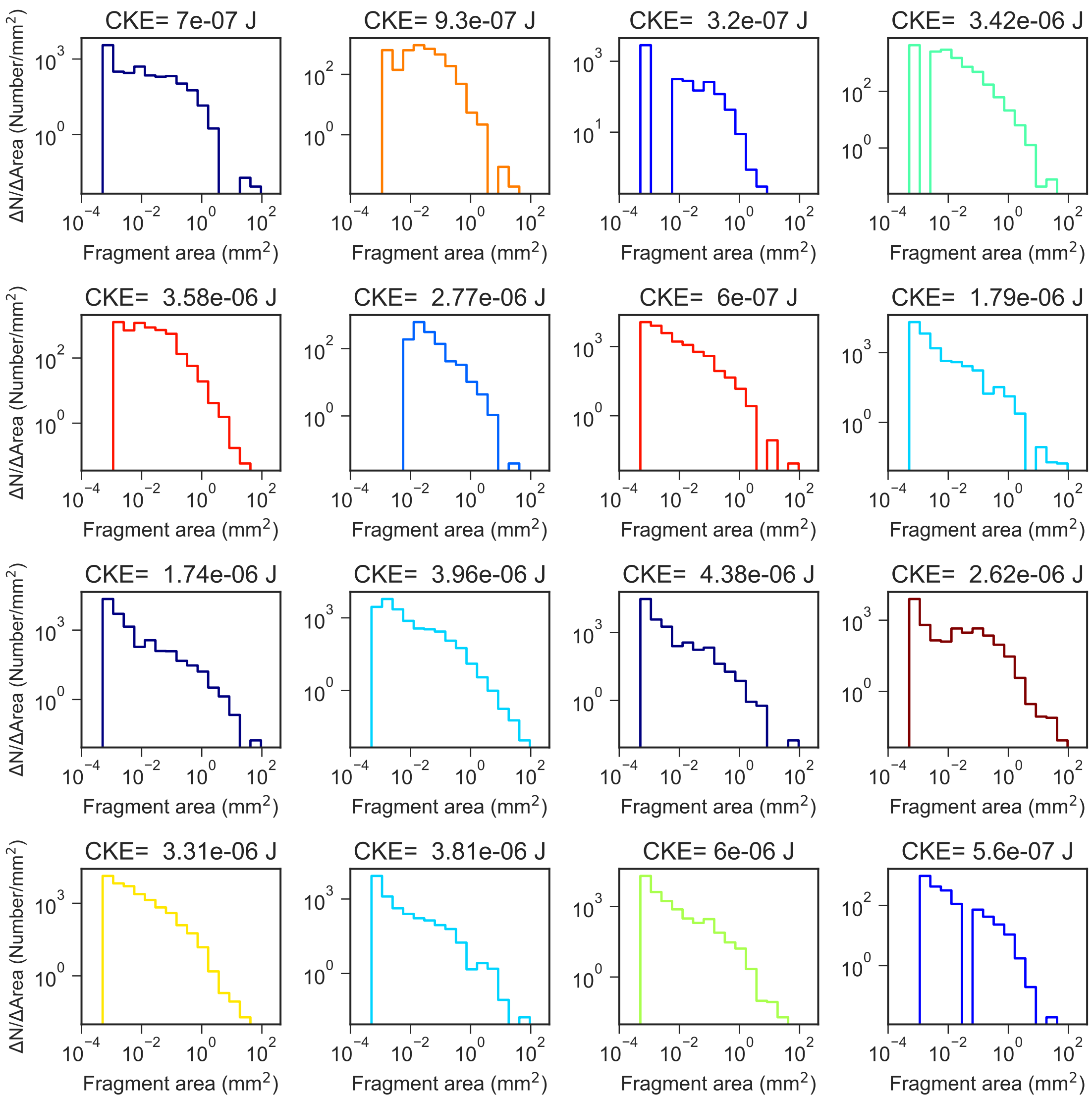

Figure S4: Area distributions of ice fragments produced by graupel-snowflake collisions for different impact positions (see color scale of Fig. 14).

S4 Distributions of fragments aspect ratio

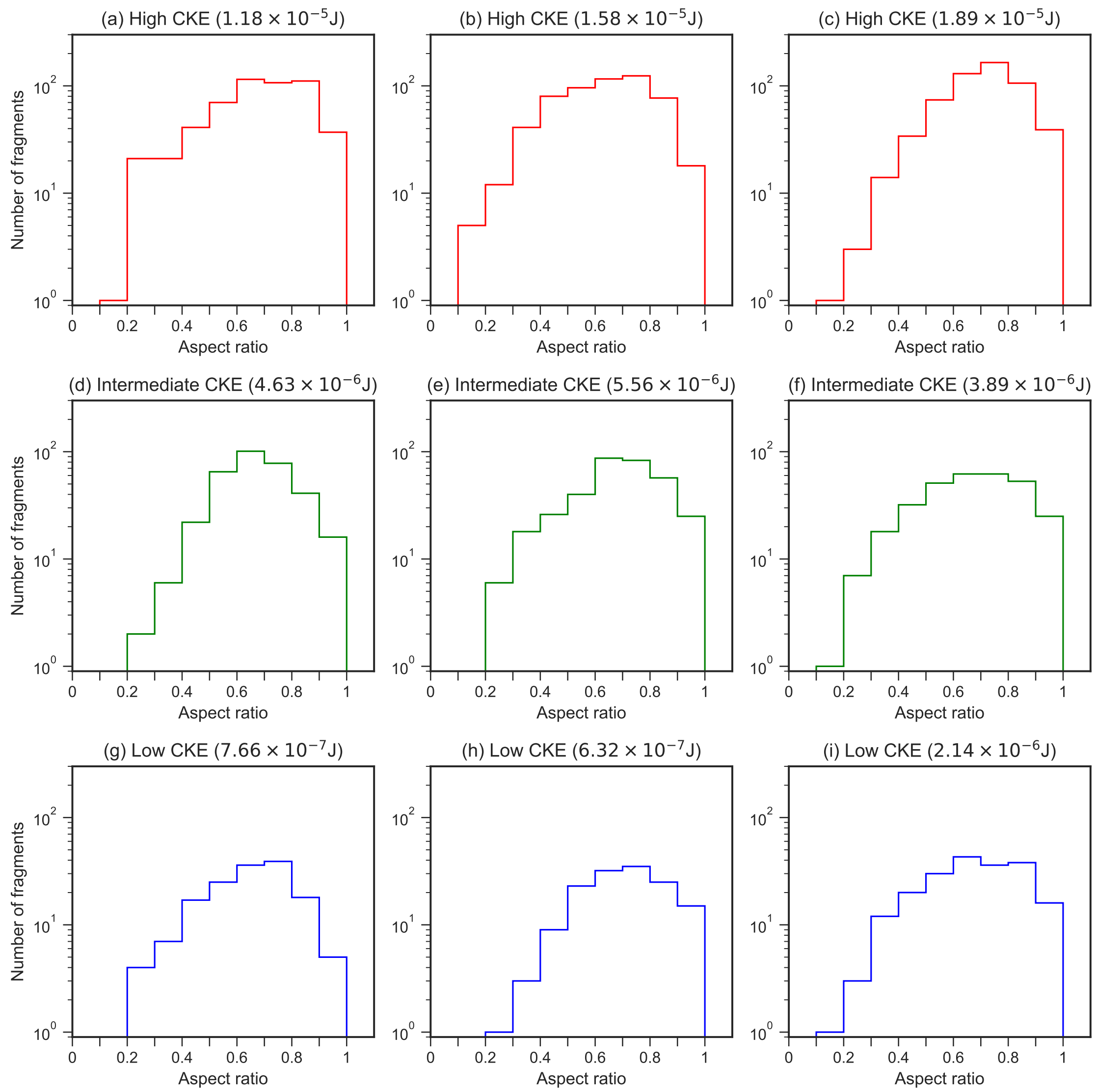

Figure S5: Aspect ratio (AR=D_{min}/D_{maj}) distributions of ice fragments produced by graupel-graupel with dendrites collisions. Low CKE in blue, intermediate CKE and high CKE in red.

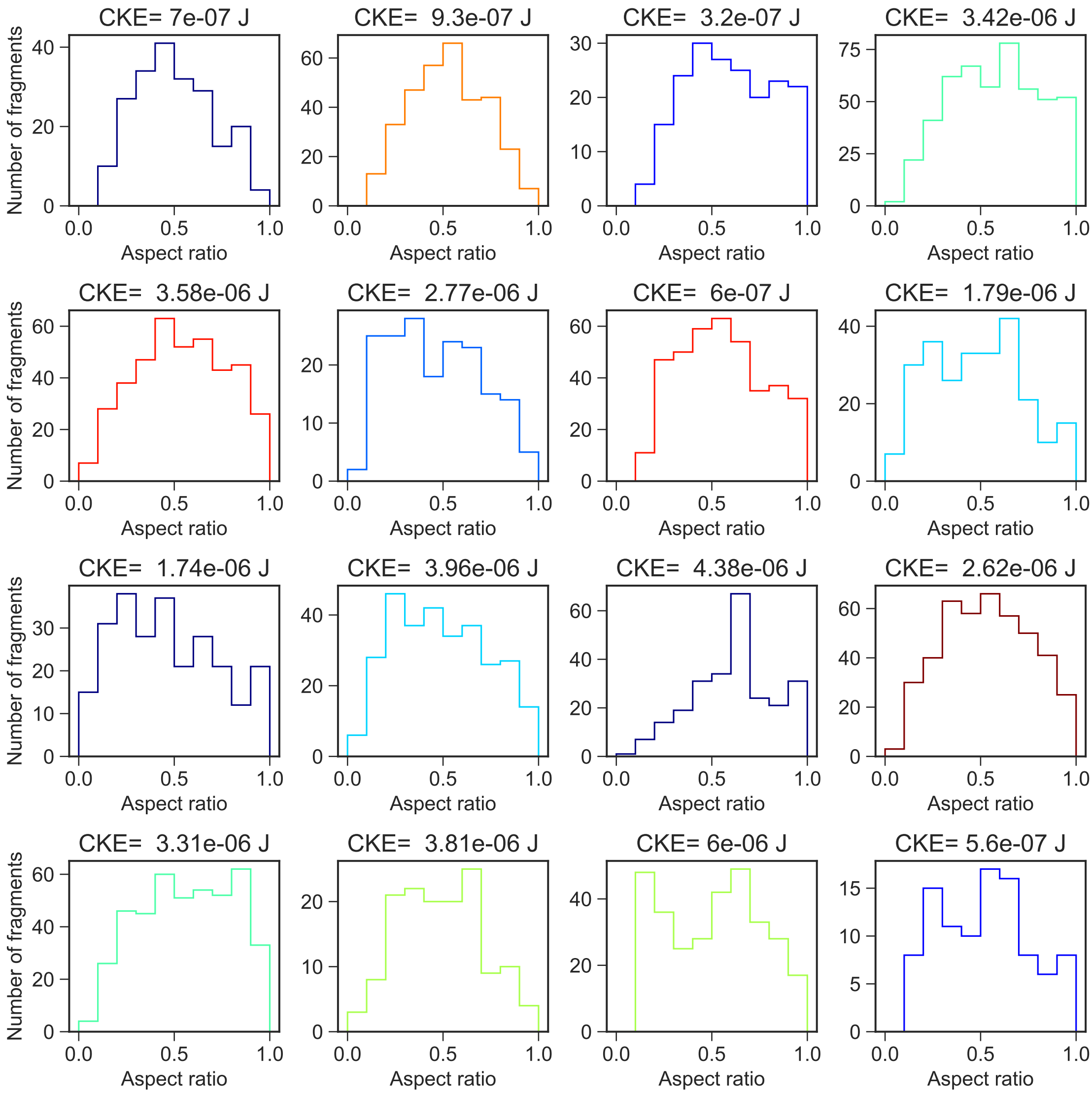

Figure S6: Aspect ratio (AR=D_{min}/D_{maj}) distributions of ice fragments produced by graupel-snowflake collisions for different impact positions (see color scale of Fig. 14).

References

H.R. Pruppacher and James Klett. *Microphysics of Clouds and Precipitation*, volume 18. 2010. ISBN 978-0-7923-4211-3. doi: 10.1007/978-0-306-48100-0.