Supplement of

Dependency of particle size distribution at dust emission on friction velocity and atmospheric boundary-layer stability

Yaping Shao et al.

Correspondence to: Jie Zhang (zhang-j@lzu.edu.cn) and Ning Huang (huangn@lzu.edu.cn)

The copyright of individual parts of the supplement might differ from the CC BY 4.0 License.
We use a Lagrangian stochastic model for saltation in turbulent flow to examine the intensity of saltation bombardment. The model combines the equation of sand motion with a stochastic equation for fluid velocity fluctuations along the saltation trajectories. Following Thomson (1987, 1990), the turbulent motion of fluid elements can be modelled with

\[ dU_i = a_i(U, X, t)dt + b_{ij}(U, X, t)\, d\omega_{ij} \]  
\[ dX_i = U_i(X, t)dt \]

where \( U \) is fluid element Lagrangian velocity (\( U_i \) its \( i \) component), \( X \) fluid element position, \( a_i \) drift coefficient, \( b_{ij} \) diffusion coefficient and \( d\omega_{ij} \) increment of the Wiener process. Sand particle and fluid element follow different trajectories due to the trajectory-crossing effect (Yudine, 1959; Csanady, 1963).

The model used in this study is two dimensional, with \( x_1 \) aligned in the horizontal mean wind direction and \( x_3 \) in the vertical direction. We denote the sand particle position as \( Y \), its velocity as \( V \), and the fluid element velocity at \( Y \) as \( U^* \).

The sand-particle to fluid-element relative velocity is \( V_R = V - U^* \).

The equation of sand particle motion is written as

\[ \frac{dv_i}{dt} = -\frac{V_{Ri}}{\tau_p} - \delta_{i3}g \quad (i = 1, 3) \]  

with \( \tau_p \) being the sand particle response time (Morsi and Alexander, 1972). \( V_{Ri} \) is given by \( V_{R1} = V_1 - \bar{U}_1^{*} - u_1^{*} \) and \( V_{R3} = V_3 - u_3^{*} \), where \( \bar{U}_1^{*} \) is the mean wind speed at sand particle location. The influences of turbulence on sand particle motion are embedded in \( u_1^{*} \) and \( u_3^{*} \). These are calculated using a modified Thomson (1987) model. Note that \( U = \bar{U} + u \) and \( u = (u_1, u_3) \). \( \bar{U} \) is assumed to be known, and the fluid element motion fluctuations \( (u_1, u_3) \) are calculated by using Equations (a1) and (a2). The diffusion coefficients \( b_{ij} \) are given by

\[ b_{ij} = \delta_{ij}\sqrt{C_0\varepsilon} \]

where \( \delta_{ij} \) is Kronecker delta, \( C_0 \) a constant and \( \varepsilon \) the dissipation rate for turbulent kinetic energy. The determination of \( a_i \) uses the well-mixed condition of Thomson (1987), which leads to

\[ a_iP = \frac{1}{2} \frac{\partial C_0\varepsilon}{\partial u_i} + \varphi_i \]

and

\[ \frac{\partial \varphi_i}{\partial u_i} = -\frac{\partial P}{\partial \bar{U}} - \frac{\partial u_i P}{\partial x_i} \]

with \( P \) being the phase-space probability density function \( P(U, X, t) \). The well-mixed condition requires that \( P \) equals to the probability density function of the Eulerian velocity \( U(x=X, t) \).

The increment \( du_i^* \) is expressed as

\[ du_i^* = du_i + \delta u_i \]

where \( du_i \) is the fluid-element velocity increment between \( t \) and \( t+dt \), computed using Equation (s1), and \( \delta u_i \) the spatial velocity increment at \( t+dt \) between the two points separated by \( V_R \, dt \). While the structure function of \( du_i \) satisfies

\[ \langle du_i du_i \rangle = C_0\varepsilon dt, \]

that of \( \delta u_i \) satisfies

\[ \langle \delta u_i \delta u_i \rangle = C_1\varepsilon^{2/3}V_R^{2/3} dt^{2/3}. \]

Due to its fractional nature, \( \delta u_i \) is difficult to generate stochastically and it is in this study assumed to be
\[ \langle \delta u_i \delta u_i \rangle = C_1 \epsilon^{2/3} V_l l^{-1/3} dt \quad (s10) \]

with \( l \) being a fixed scaling length. Following Hanna (1981) and Stull (1988), \( C_0 = 5 \) and \( C_1 = 2 \).

Sand particles are randomly lifted from the surface with velocity \((V_{1o}, V_{3o})\). The PDF of \( V_{1o} \) is assumed to be Gaussian and that of \( V_{3o} \) Weibull (to avoid negative liftoff speed). The sand-particle liftoff angle is confined to 0° and 180° and Gaussian distributed with a mean liftoff angle of 55° and a standard deviation of 5°. The sand particles are allowed to rebound from the surface with the rebounding kinetic energy half the impacting kinetic energy and a mean rebounding angle of 40°. If the kinetic energy of a sand particle becomes lower than a critical value, its motion is stopped.

References:


Thomson, D. J., A Stochastic Model for the Motion of Particle Pairs in Isotropic High-Reynolds-Number Turbulence, and its Application to the Problem of Concentration Variance. J. Fluid Mech., 210(-1), 113-153, [Link](https://doi.org/10.1017/S0022112090001239), 1990.

Yudine, M. I., Physical Considerations on Heavy-Particle Diffusion. Advances in Geophysics, 6, 185-191, [Link](https://doi.org/10.1016/S0065-2687(08)60106-5), 1959.