



Supplement of

Analysis of isothermal and cooling-rate-dependent immersion freezing by a unifying stochastic ice nucleation model

Peter A. Alpert and Daniel A. Knopf

Correspondence to: Peter A. Alpert (peter.alpert@ircelyon.univ-lyon1.fr) and Daniel A. Knopf (daniel.knopf@stonybrook.edu)

The copyright of individual parts of the supplement might differ from the CC-BY 3.0 licence.

References

- Hartmann, S., Wex, H., Clauss, T., Augustin-Bauditz, S., Niedermeier, D., Rösch, M., and Stratmann, F.: Immersion freezing of kaolinite: Scaling with particle surface area, J. Atmos. Sci., 73, 263–278, doi:10.1175/JAS-D-15-0057.1, 2016.
- Knopf, D. A. and Alpert, P. A.: A water activity based model of heterogeneous ice nucleation kinetics for freezing of water and aqueous solution droplets, Farad. Discuss., 165, 513–534, doi:10.1039/c3fd00035d, 2013.
- Wiedensohler, A. and Fissan, H. J.: Aerosol charging in high purity gases, J. Aerosol Sci., 19, 867–870, doi:10.1016/0021-8502(88)90054-7, 1988.



Figure S1. Example of random sampled surface areas from the lognormal distributions used in Iso1-4. The number of sampled droplets and lognormal distribution parameters for each simulation are given in Table 1. Solid lines are histograms of the sampled surface areas. Dashed lines are lognormal probability density functions scaled with the sampled frequency. The distribution used in Iso3 is the same for model simulation Cr1 and Cr2. The distribution used in Iso1 is the same for model simulation Cr3 and Cr4.



Figure S2. Example of random sampled surface areas from the lognormal distribution used in IsoWr. The number of sampled droplets and lognormal distribution parameters for each simulation are given in Table 1. The solid line is a histogram of the sampled surface areas. The dashed line is the lognormal probability density function scaled with the sampled frequency. The grey region shows the range in surface area spanning a factor of 125.



Figure S3. Example of random sampled surface areas from the distribution used in IsoBr. Distribution parameters and the number of sampled droplets are given in Table 1. The solid line is a histogram of the sampled surface areas. The dashed line is the uniform probability density function scaled with the sampled frequency.



Figure S4. Example of random sampled surface areas from the lognormal distribution used in IsoHE1. Distribution parameters and the number of sampled droplets are given in Table 1. The solid line is a histogram of the sampled surface areas. The dashed line is the lognormal probability density function scaled with the sampled frequency.



Figure S5. Example of random sampled surface areas from the lognormal distribution used in IsoHE2. Lines are the same as in Fig. S4.



Figure S6. Example of random sampled surface areas from the lognormal distributions used in IsoDI1-3. Lines are the same as in Fig. S4.



Figure S7. Sensitivity calculations of heterogeneous ice nucleation rate coefficients, J_{het} , and frozen droplet fractions, f_{frz} , on cooling rate, r, derived from model simulations Cr1 (orange), Cr2 (blue), Cr3 (black) and Cr4 (green). For Cr1 and Cr3, r = 0.5 and for Cr2 and Cr4, $r = 5.0 \text{ K min}^{-1}$. Panels (**a**) and (**b**) are the same as Fig. 4(**a**) and (**b**). J_{het} as a function of temperature, T, are shown in (**a**) and (**c**) assuming uniform ice nuclei surface area (ISA) per droplet yielding $J_{het}^{apparent}$, and (**b**) and (**d**) accounting for individual ISA yielding J_{het}^{actual} . The dashed lines in (**a**) and (**c**), and and (**b**) and (**d**) are $\overline{J}_{het}^{actual}$ and $\overline{J}_{het}^{actual}$, respectively. Shadings in (**a**)-(**d**) correspond to upper and lower fiducial limits with x = 0.999 confidence and the solid red line is calculated from Eq. (6) for illite (Knopf and Alpert, 2013). Frozen droplet fractions, f_{frz} , are shown in (**e**) and (**f**) where dashed lines and shadings represent \overline{f}_{frz} and 5th and 95th percentile bounds, respectively. Parameter values for Cr1, Cr2, Cr3 and Cr4 are given in Table 2.



Figure S8. Example of random sampled surface areas from the lognormal distributions used in CrDI1 and 2. Lines are the same as in Fig. S4. Distribution parameters and the number of sampled droplets are given in Table 2.



Figure S9. Example of random sampled discrete particle diameters, D_p , used in model simulations IsoCFDC and IsoLACIS from the probability density function of multiple charged particles following (Wiedensohler and Fissan, 1988) with a constant electrical mobility diameter equal to 300 nm. The frequency of occurrence for the random sample is given by bars at 300, 510, 720, 920, 1100, 1300 and 1500 nm in diameter representing particles with increasing charge, i.e. +1, +2, +3, +4, +5 and +6, respectively. Symbols shows the distribution function scaled to the frequency. The top and bottom panel shows 833 and 21 randomly sampled D_p , respectively. The measured multiple charge distribution by (Hartmann et al., 2016) is shown as dashed grey lines for comparison.



Figure S10. Example of random sampled surface areas from the lognormal distributions used in CrNI1 and 2. Lines are the same as in Fig. S4.