

Main comment:

I appreciate the tremendous amount of work forming the basis of the manuscript and the very comprehensive answers to the reviewers' comments. Nevertheless, I apologize for still not agreeing with the basic idea of the manuscript. The authors have not yet convinced me that, in the case of **one** variable of interest (ozone), the singular vector analysis can yield more powerful results than, e.g., the adjoint sensitivity method. I am going to explain my concern in more detail:

Let us assume that ozone corresponds to the first component of the vector of $\mathbf{c}(t)$ and related vectors and matrices. Then, if we are interested in ozone at time t_F , the projection matrix \mathbf{P}_{t_F} in Equ. (M-16) ("M" $\hat{=}$ Manuscript) is

$$\mathbf{P}_{t_F} = \begin{pmatrix} 1 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 0 \end{pmatrix} .$$

If \mathbf{e} denotes the first unit vector,

$$\mathbf{e} = \begin{pmatrix} 1 \\ 0 \\ \dots \\ 0 \end{pmatrix} ,$$

then

$$\mathbf{P}_{t_F} = \mathbf{e}\mathbf{e}^T . \quad (1)$$

Now the singular vector analysis requires the solution of the eigenvalue problem (M-26):

$$\mathbf{B}^T \mathbf{B} \mathbf{v}_g(t_I) = \lambda_g \mathbf{v}_g(t_I) \quad (2)$$

with the matrix \mathbf{B} defined in Equ. (M-24):

$$\mathbf{B} = \mathbf{P}_{t_F} \mathbf{L}_{t_I, t_F} \quad (3)$$

(Equ. (M-24) is applicable, because the projection operator is a special family operator; and condition (M-25) is automatically fulfilled if the corresponding family operator is a projection operator).

Omitting subscripts and "(t_I)" (for simplicity), we obtain by inserting (3) in (2):

$$\begin{aligned} \mathbf{L}^T \mathbf{P}^T \mathbf{P} \mathbf{L} \mathbf{v} &= \lambda \mathbf{v} \\ \mathbf{L}^T \mathbf{P} \mathbf{L} \mathbf{v} &= \lambda \mathbf{v} , \quad \text{because } \mathbf{P}^T \mathbf{P} = \mathbf{P} \\ \mathbf{L}^T \mathbf{e} \mathbf{e}^T \mathbf{L} \mathbf{v} &= \lambda \mathbf{v} \quad \text{because of (1)} \end{aligned} \quad (4)$$

The matrix $\mathbf{L}^T \mathbf{P} \mathbf{L} = \mathbf{L}^T \mathbf{e} \mathbf{e}^T \mathbf{L}$ in (4) has rank 1. That is why it has only one non-zero eigenvalue - this is the one that will be determined by the singular

vector analysis. The corresponding eigenvector is (the normalized version of) $\mathbf{L}^T \mathbf{e}$, because:

$$\begin{aligned} & \mathbf{L}^T \mathbf{e} \mathbf{e}^T \mathbf{L} \cdot \mathbf{L}^T \mathbf{e} \\ = & \mathbf{L}^T \mathbf{e} \cdot (\mathbf{e}^T \mathbf{L} \cdot \mathbf{L}^T \mathbf{e}) \\ = & \lambda \cdot \mathbf{L}^T \mathbf{e} \end{aligned}$$

with

$$\lambda = \mathbf{e}^T \mathbf{L} \mathbf{L}^T \mathbf{e} .$$

On the other hand, $\mathbf{L}^T \mathbf{e}$ is the result of the adjoint model, starting (at $t = t_F$) with the initial value \mathbf{e} (expressing that we are interested in sensitivities of species no. 1, i.e. ozone, with respect to the initial concentrations of all species).

This means that, in the case of **one** species of interest, the singular vector analysis yields the same result as the adjoint model (with the only difference, i.e. disadvantage, that the information on absolute values is lost because of the normalization of the eigenvector).